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A Comparison Study for Channel Capacity of MIMO Systems with Nakagami-M, Weibull, and Rice Distributions

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Abstract-The growing interest in improving the performance and capacity of wireless communication systems has led to new approaches and solutions to solve the problems countered many operation scenarios and environments. Multi-input multi-output (MIMO) is one of these approaches which aimed to increase the spectral efficiency of the communication system besides numerous other advantages comparing to the traditional single antenna systems. This paper offers analyses and simulations to the behavior of MIMO system and its expected capacity for various channel distribution under flat fading. Several types of distributions (Nakagami-m, Weibull, and Rice) are considered with different parameters to generate the channel matrix and determine the capacity for several cases of antenna numbers in both transmitter and receiver sides.

Keywords- *MIMO; channel matrix; capacity; Nakagami-m distribution; Weibull distribution; Rice distribution; fading.*

I. INTRODUCTION

The communications channel is a physical transmission medium that is used to send the signal from the transmitter to the receiver. In many cases, it's convenient to construct mathematical models that reflect the most important characteristics of the transmission channel. Then the mathematical model for the channel is used in the design of channel encoder and modulator at the transmitter and the demodulator and the channel decoder at the receiver. When a signal is transmitted through the communication channel, it undergoes two main imperfections that made signal different from its original form. These imperfections can be divided into deterministic in nature (such as linear and nonlinear distortion, intersymbol, etc.) and nondeterministic (such as the additional noise, multipath fading, etc.) [1]. Many approaches and solutions were suggested to overcome to these imperfections such as the use of multi-input multi- output (MIMO) antenna system[2,3]. MIMO system improves communication performance with the use of multiple antennas at both the transmitter and receiver for multiple transmitted data streams. Significant increase in data throughput and link range can be observed in applying MIMO techniques, without additional cost of bandwidth or transmission power; benefiting from antenna diversity and spatial multiplexing [4]. MIMO techniques may strongly improve the spectral efficiency and/or decrease the error rate, keeping the transmitting power and the bandwidth constant. The total channel capacity is increased by creating a number of preferably non-interacting paths between the transmitter and the receiver. The channel capacity increases with the number of paths and decreases with increase in mutual correlations between them. The availability of the channel state information and the effect of characteristic of the environment influence the channel capacity [5]. This work considers the effect of channel distribution with the present of multipath flat fading on the MIMO system capacity. The achieved results show that selection of channel distribution approximation led to obvious differences in the expected capacity of the MIMO system which can be explored for different operation scenarios. The rest of the paper is organized as follows: section 2 presents a theoretical background for the main considered distribution in the communication channels. Section 3 illustrates the achieved simulation results. Finally, some concluding remarks are presented in section 4.

II. THEORY

The general MIMO system is shown in Fig. 1 with N_T transmits antennas and N_R receiver antennas. The signal model represented as:

$$r = Hx + n \tag{1}$$

where r is (N_R x 1) received signal vector, x is (N_T x 1) transmitted signal vector, n is (N_T x 1) complex additive white Gaussian noise (AWGN) vector with variance σ, and H is the (N_R x N_T) channel matrix.

The channel matrix H represents the effect of the medium on the transmitter –receiver links. The channel matrix H can be represented as follows:

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \tag{2}$$

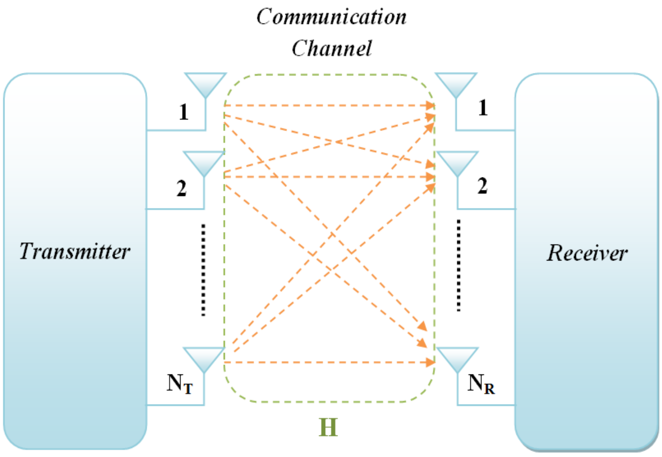

Figure 1. General MIMO system model

Channel matrix may offer K equivalent parallel sub channels with different mean gains [6], where

$$K = RANK\ (HH^H) \leq MIN\ (N_T, N_R) \tag{3}$$

Singular value decomposition (SVD) simplification can be used to demonstrate the effect of channel matrix H on the capacity. Then, channel matrix H can be expressed as:

$$H = UBV^H \tag{4}$$

With the columns of the unitary matrix U (N_R x N_R) contains the eigenvectors of HH^H and the columns of the unitary matrix V (N_T x N_T) contains the eigenvectors of H^HH. The diagonal matrix B (N_R x N_T) has nonnegative, real valued elements (called singular values) equal to the square roots of the Eigen values λ of HH^H [7].

Assuming that the channel is known at both T_X and R_X (full or prefect channel sensing information CSI) then the maximum normalized capacity with respect to bandwidth (in term of b/s/Hz spectral efficiency) of parallel sub channels equals [8]:

$$C = \sum_{i=1}^{K} \log_2 (1 + \lambda_i \frac{P_i}{\sigma_N^2}) \tag{5}$$

where P_i is the power allocated to each sub channel i and can be determined to maximize the capacity using water filling theorem such that each sub channel was filled up to a common level D [6]:

$$\frac{1}{\lambda_1} + P_1 + \cdots + \frac{1}{\lambda_k} + P_k = D \tag{6}$$

Or

$$P_i = D - \frac{1}{\lambda_i} \tag{7}$$

Such that it satisfies the following condition that sums of all sub channels power equal to the total transmitted power or :

$$\sum_{i=1}^{K} P_i = P_{TX} \tag{8}$$

and if $\frac{1}{\lambda_i} > D$ then P_i is set to zero.

A brief overview of the random distributions used in this work is as following:

A. Rice Distribution

Rice distribution (sometimes written as Rician or Ricean) is important to communication engineers in characterizing wireless (i.e., radio and optical) channels. The Rice distribution is appropriate to use when the receiver's position is on a line of sight (LOS) with respect to the transmitter, thus there will be an LOS signal component in the received signal due the multipath [9]. The density function for this distribution is given by:

$$f(x) = \frac{x}{b^2} e^{\left(\frac{-(x^2+s^2)}{2b^2}\right)} I_o\left(\frac{xs}{b^2}\right) \tag{9}$$

where I_0 is the zero-order modified Bessel function of the first kind , s (s ≥ 0) non-centrality parameter and b (b > 0) scale parameter. The Rice distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{\text{Rice}} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \tag{10}$$

B. Weibull Distribution

The Weibull distribution has been used extensively in recent years to deal with multipath problems. The Weibull distribution is also applied to reliability and life-testing problems such as the time to failure or life length of a component, measured from some specified time until it fails [10]. A random variable *x* is said to have a Weibull distribution with parameters α and β (and shifting equal to zero) if the probability density function of *x* is given by:

$$f(x) = \begin{cases} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^\alpha} &, \ x > 0 \\ 0 &, \ elsewhere \end{cases} \tag{11}$$

Where β (β > 0) is the scale parameter and α (α > 0) is the shape parameter. Both α and β can be varied to obtain a number of different-looking density curves. In some situations, there are theoretical justifications for the appropriateness of the Weibull distribution, but in many applications *f(x)* simply provides a good fit to observed data for particular values of α and β. The parameters of this distribution offer a vast flexibility to model systems. When α=1 the probability density function reduces to the exponential distribution so the exponential distribution is a special Weibull distributions. [11]. The Rayleigh distribution is also a special case of the two-parameter Weibull distribution with α=2 and β =$\sqrt{2}\delta$ where δ is the scale parameter of Rayleigh distribution which probability density function is given by [9]:

$$f(x) = \frac{x}{\delta^2} e^{-\frac{x^2}{2\delta^2}} \tag{12}$$

This Weibull distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{\text{Weibull}} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \tag{13}$$

C. *Nakagami-m Distribution*

The Nakagami-m distribution is another important distribution used in communication field to model the statistical fading of the multipath scenarios and was developed from experimental measurements. The probability density function for this Nakagami-m is given by:

$$f(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mx^2}{\Omega}} = \frac{2x^{2m-1}}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m e^{-\frac{mx^2}{\Omega}} \qquad (14)$$

Where Ω is the second moment and represent the scale parameter, m ($m \geq 0.5$) is known as the Nakagami fading parameter or shape parameter, and $\Gamma(.)$ is the standard Gamma function. The Nakagami-m distribution covers a wide range of fading conditions; when m=0.5 it is a one-sided Gaussian distribution and when m=1, it is a Rayleigh distribution and when m < 1, the Nakagami model applies a fading scenario that is more severe than Rayleigh fading [12,13]. The Nakagami-m distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{\text{Nakagami}} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \qquad (15)$$

III. Simulation Results

In this work, MATLAB m-file is used to verify the model and simulate the effects of several types of distributions (Nakagami-m, Weibull, and Rice) for a MIMO system under flat fading to generate the channel matrix.

Water filling theorem with its concept is considered to determine the power allocation for the equivalent parallel subchannel and determine the capacity for a wide range of SNR (-10 dB to 30 dB) with a resolution step of 2dB and noise equal to 0.0001. The simulation is done for several pairs of N_R and N_T as detailed in Table 1.

TABLE I. Number of transmitter and receiver antennas

Case No.	*Number of transmitter antennas (N_T)*	*Number of receiver antennas (N_R)*
1st	1	1
2nd	2	2
3rd	3	3
4th	4	4
5th	5	5
6th	6	6
7th	7	7
8th	8	8

A. *Rice Distribution*

The first distribution considered is Rice distribution with three different sets of non-centrality parameter s and scale parameter σ. The capacity of the system (in term of b/s/Hz), for each set of the Rice distribution parameters, is calculated for each case in Table 1 over a wide range of SNR (-10 dB to 30 dB). Each of the eight cases is represented with capacity curves using different colors and special marker symbols. The first set of parameters is unity non-centrality parameter (s = 1) and unity scale parameter (b = 1). The achieved results are shown in Fig. 2.

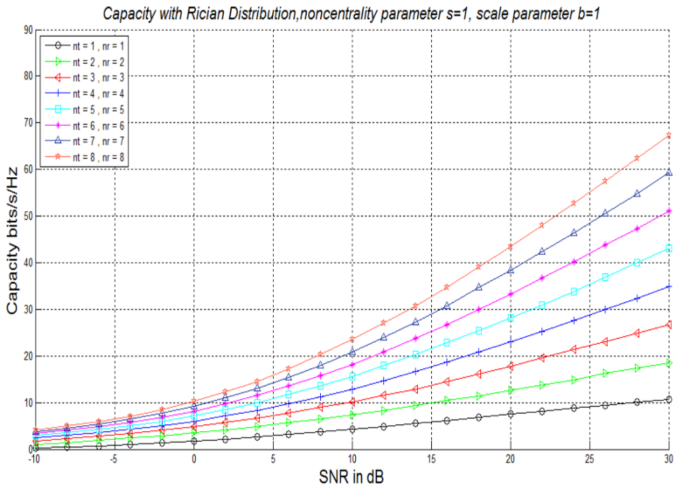

Figure 2. The channel capacity with Rice distribution (s = 1, b = 1)

From the inspection of the Fig .2 , and for the 1st curve (N_T = 1, N_R = 1), it's obvious that the capacity is increased as signal to noise ratio (SNR) increases with respect to eq. (5) which is relate to the generating channel matrix H by Rice distribution as in eq. (9).

For the 2nd case (N_T = 2, N_R = 2), it's noticeable that the capacity is improved for the same values of SNR comparing to the1st one because of increasing number of antennas in both transmitter and receiver sides.

The 3rd case (N_T = 3 and N_R = 3) shows that the capacity is increased for the same values of SNR comparing to the first and 2nd case. The capacity increasing corresponds to the H_{Rice} in approximating exponential manner.

The 4th case (N_T = 4 and N_R = 4) shows that the capacity is increased for the same values of SNR comparing to previous cases in more approximating exponential behavior. These observations are still similar for the rest of the cases (5th to 8th). It's clear that the capacity still improved comparing to the previous cases even for small value of SNR, while the curve shape becomes more and more resembling an exponential behavior.

The second set of parameters is non-centrality parameter (s = 2) and scale parameter (b = 2). The analysis leads to the results shown in Fig. 3. It is clear that these results are look like those presented in Fig. 2 (the capacity of channel is increasing with number of antennas increasing in both of transmitter and receiver sides). But its noticeable the raise in capacity for the same number of receiving and transmitting antenna and SNR which is related to the values of non-centrality and scale parameters (s = b = 2 for this case).

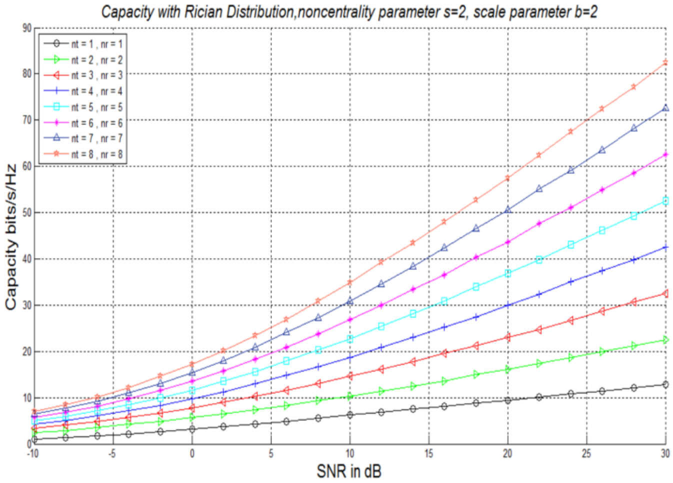

Figure 3. The channel capacity with Rice distribution (s = 2, b = 2)

The third set of parameters is non-centrality parameter (s = 3) and scale parameter (b = 3). The achieved results are depicted in Fig. 4. The results in this figure indicate that with adapting this case (s = b = 3) there is improvement in the channel capacity compared with that achieved in cases 2 and 3.

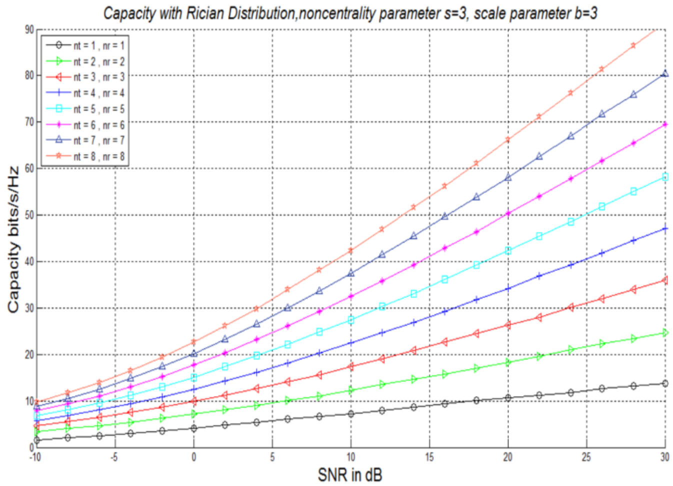

Figure 4. The channel capacity with Rice distribution (s = 3, b = 3)

B. Weibull Distribution

The second considered distribution is Weibull distribution; with three different sets of the scale parameter (β) and the shape parameter (α). The capacity of the system (in term of b/s/Hz), for each set of the Weibull distribution parameters is calculated for each case in Table 1 over a wide range of SNR (-10 dB to 30 dB). The first set of parameters is unity scale parameter (β) and the unity shape parameter (α). The achieved results are illustrated in Fig. 5.

The results of Fig. 5, illustrates variation of capacity with number of employed antennas. The capacity is increasing function to the number of antennas in both transmitter and receiver sides and manner similar to that of Rice distribution. Comparing with results in Fig. 2, the capacity with Weibull distribution (with β =1, α = 1) is lower in value comparing to that with Rice distribution (s =1, b = 1) for the 1st to 6th cases while its higher in other cases (7th & 8th).

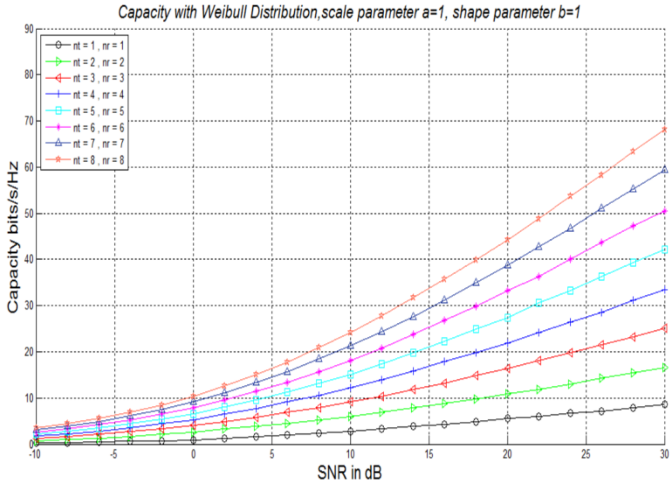

Figure 5. The channel capacity with Weibull distribution (β =1, α = 1)

While comparing results in Fig. 5 with that in Fig. 3 & 4 indicate that the capacity with Weibull distribution (with β =1, α = 1) is lower in value comparing to capacity with Rice distribution (s = 2, b = 2) & (s = 3, b = 3).

The second set of parameters is the scale parameter (β = 2) and the shape parameter (α = 2). The achieved results are given in Fig. 6. It is clear that the capacity is increasing function to the number of antennas in both transmitter and receiver sides, as that achieved in Fig. 5, but with this case a little increase in the capacity is achieved for the same SNR and no. of antenna pairs (as both β & α increased by 1 comparing with last values of them).

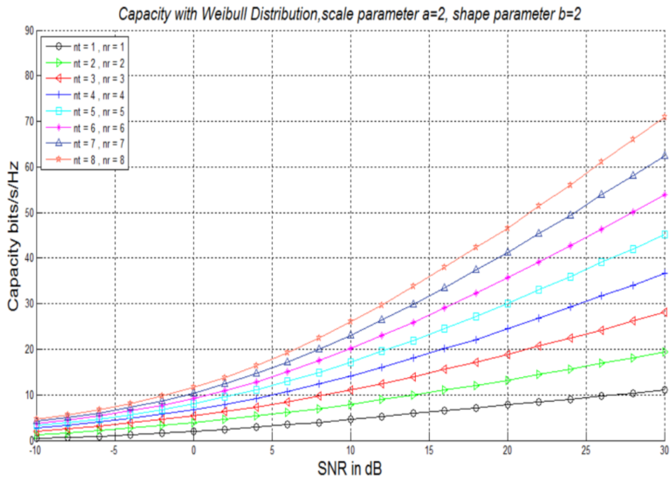

Figure 6. The channel capacity with Weibull distribution (β = 2, α = 2)

By comparing results in Fig. 6 with that in Fig. 2, it's noticeable that the capacity with Weibull distribution (with β =

2, α = 2) has a little improvement than the capacity of Rice distribution (s = 1, b = 1). On the other hand, the results in Fig. 6 indicate that the capacity with Weibull distribution (with β = 2, α = 2) is lower in value comparing to capacity with Rice distribution (s = 2, b = 2) & (s = 3, b = 3) respectively that given in Fig. 3 and 4.

The third set of the evaluation parameters is scale parameter (β = 3) and the shape parameter (α = 3). The achieved results are shown in Fig. 7.

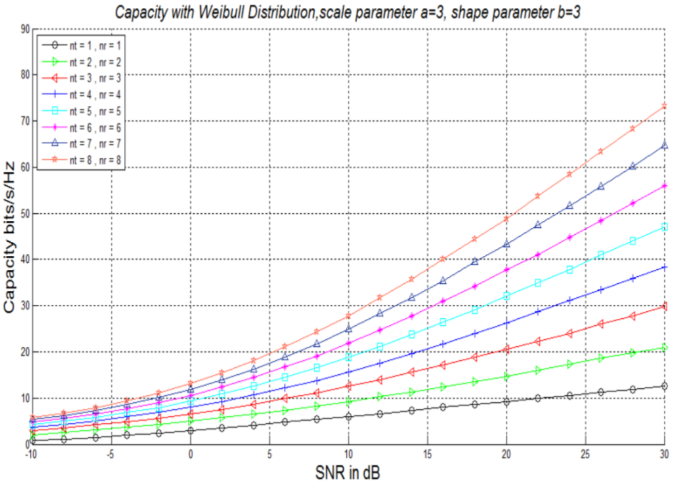

Figure 7. The channel capacity with Weibull distribution (β = 3, α = 3)

By examine Fig. 7, its clear that increasing amount of the channel capacity is related to increasing number of antennas in transmitter and receiver for the same SNR, yet there is a little increase in the capacity in Fig. 7 comparing to that given in Fig.5 & 6 for the same SNR and number of antenna pairs. While comparing with that in Fig. 2, it's noticeable that the capacity with Weibull distribution (with β = 3, α = 3) is greater than the capacity of Rice distribution (s = 1, b = 1). And the capacity with Weibull distribution is lower than the capacity with Rice distribution (s = 2, b = 2) and (s = 3, b = 3) respectively, with using that in Fig. 3 and 4

C. Nakagami-m Distribution

The third considered distribution is Nakagami-m distribution; as for both Rice and Weibull distributions, the capacity of the system (in term of b/s/Hz) is calculated for each case in Table 1 over a wide range of SNR (-10 dB to 30 dB). Each of the eight cases is represented with capacity curves using different colors and special marker symbols shown in the upper left corner of the figures. The first set of evaluation parameters is the scale parameter (Ω = 1), m shape parameter (m = 1). The obtained results are depicted in Fig. 8.

The channel capacity, shown in Fig. 8, is improved as the no. of receiving and transmitting antennas is increased for the same amount SNR. The capacity with Nakagami-m (Ω = 1, m = 1) is lower than that in Fig. 2, 3, and 4 for all cases of no. of antennas and the same SNR, comparing with that obtained under the Rice distribution (s = 1, b = 1) & (s = 2, b = 2) & (s = 3, b = 3) respectively.

However, comparing the results with that in Fig.5, its notable that the capacity of first case (N_T = 1, N_R = 1) with Nakagami-m is greater than that with weibull distribution for the same value of SNR. While, the capacity for the (2nd up to 8th cases) are lower than those with weibull distribution at the same SNR. And, it is lower than that in Fig. 6 and 7 for all the cases at the same SNR.

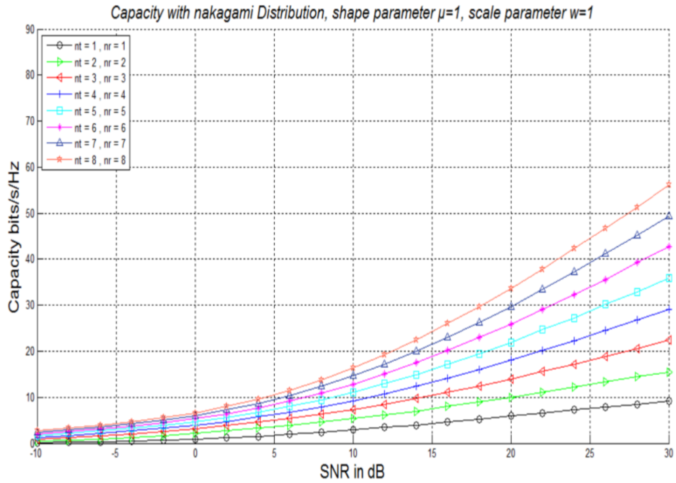

Figure 8. The channel capacity with Nakagami-m distribution (Ω = 1, m = 1)

The second set of parameters is the scale parameter (Ω = 2) and shape parameter (m = 2). The achieved results are illustrated in Fig. 9.

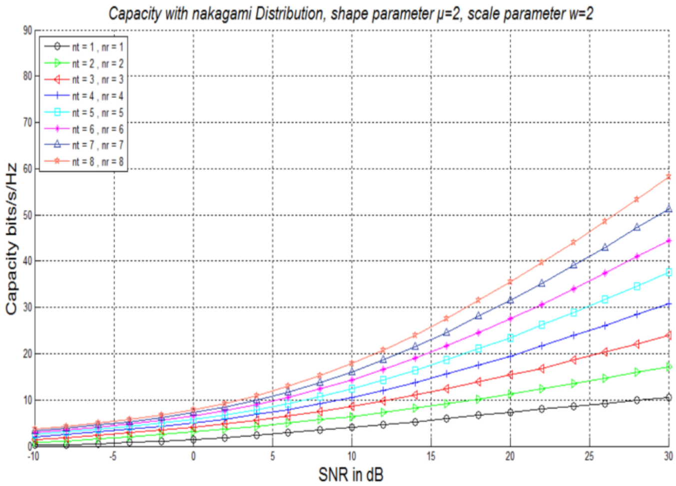

Figure 9. The channel capacity with Nakagami-m distribution (Ω = 2, m = 2)

Fig. 9 shows that the capacity is increasing function to the number of antennas in both transmitter and receiver sides, as that in Fig. 8, but with a slightly increase in the capacity for the same SNR and number of antenna pairs.

With judging results of this case, Fig. 9 against that in Fig. 2, it's notable that the 1st capacity case (N_T = 1, N_R = 1) is almost equal. While the capacity of the rest (2nd up to 8th) is lower for those with Nakagami-m distribution (Ω = 2, m = 2) comparing to the ones with Rice distribution. Also, the capacity is lower in value, for all cases of number of antennas and the same SNR, than that obtained under the Rice distribution with parameters (s = 2, b = 2) and (s = 3, b = 3) respectively.

The capacity for the case ($N_T = 1$, $N_R = 1$) is greater than that with Weibull distribution (Fig. 5). While the capacity almost equal for the 2nd case ($N_T = 2$, $N_R = 2$) with both distribution. The rest of the capacity cases (3rd up to 8th), the capacity with Nakagami-m distribution is lower than that with Weibull distribution for the same SNR. By the comparing with Fig. 6 & 7, the capacity is lower for all the cases at the same SNR and no. of antennas pair.

The other set of the evaluation parameters is the scale parameter (Ω = 3) and shape parameter (m = 3) are examined in Fig. 10. It indicates that the capacity increasing to the number of antenna in transmitter and receiver for the same value of SNR, and there is a marginable increase in the capacity comparing to the results in Fig. 8 & 9.

Also the capacity with Nakagami-m distribution is greater in value comparing to the capacity with Rice distribution (s = 1, b = 1) for the first case (see Fig. 2) while its lower in value in other cases (2nd up to 8th). And the capacity is lower comparing to that in Fig. 3 & 4 for all cases.

The capacity with Nakagami-m distribution (Ω = 3, m = 3) is greater in value comparing to the that with Weibull distribution (s = 1,b = 1) for the 1st and 2nd cases while its lower in value in other cases (3rd up to 8th).

While, the capacity with Nakagami-m distribution (Ω = 3, m = 3) is slightly greater than to the capacity with Weibull distribution (s = 2, b = 2) for the first case while it's lower in value for the rest of cases, shown in Fig. 6. And, lower in comparing with Weibull distribution (s = 3, b = 3), shown in Fig. 7.

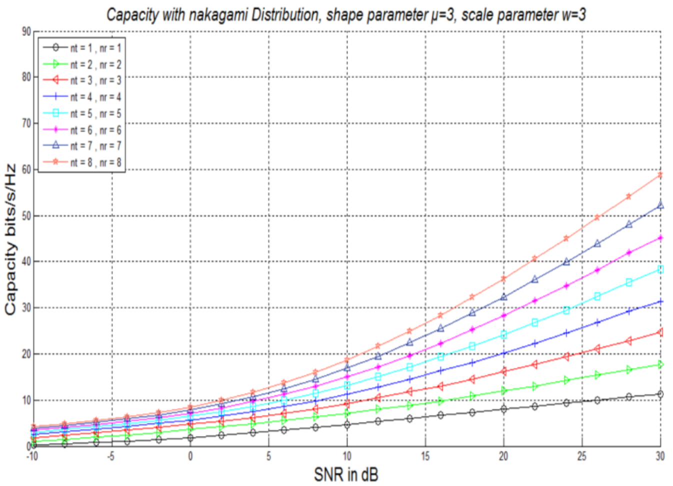

Figure 10. The channel capacity with Nakagami-m distribution (Ω = 3, m = 3)

IV. CONCLUSIONS

The obtained results give an inspection to the influence of the distribution selection over the capacity of multi-input multi-output MIMO system estimation and led to better understanding of the effect of each distribution and how it can be used to approximate different environments. The change of the evaluation parameters of each distribution, for the same number of antenna pair at receiver and transmitter and SNR, led to different value of capacity since its effect the generating of H matrix. Also, the investigating of more channel distributions is benefit led to better modeling of channel for different operation scenarios and various environments.

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