On The Weak Stability Theorems For Jungck Picard Iteration Procedures

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Abstract:

In this paper, we obtain some weak stability results for Picard iteration processes in metric space using different contraction conditions.

Our results are generalization of some results of Harder and Hicks 1988a,b, Rhoades 1993, Osilike 1995, Berinde 2002, Imoru and Olatinwa 2003 and Imoru et al. 2006.

Mathematics Subject Classification(2000) :**47H10** , **54H25**

Keywords: Metric space , Fixed points ,Multi-valued maps, Stable iteration, weak stable iteration.

1-Introduction :

The first result on stability of iteration procedures on metric spaces was given by M.Ostrowski 1967. Czerwik 1998 has extended Ostrowski's classical theorem for stability of iterative procedures for multi-valued maps in b- metric space.

S.L Singh etal 1995, S.L Singh et al. 2005 have been establish several stability result for multi-valued operators.

For any $x_0 \in X$, and $x_{n+1} \in f(T, x_n)$, $n = 1, 2, \ldots$. (1)

Let the sequence $\{x_n\}$ be converges to a fixed point p of T .Let $\{y_n\}$ be an arbitrary sequence in X and set $\varepsilon_n = H\left(y_{n+1}, f\left(T, y_n\right)\right)$, $n = o, 1, 2, \ldots$

If $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = p$, Then the iteration process defined in (1) is said to be T- stable or stable with respect to T.

Hashim 2011 obtained stability results for hybrid contraction maps involving single-valued and multi-valued maps in b-metric spaces.

However, formal definition of stability of general iterative procedures is due to Harder and Hicks 1988a,b, Rhoades 1993 generalized the results of Harder and Hicks 1988a,b to more general contractive map. In Osilike 1995, a generalization of some results of Harder and Hicks 1988a,b and Berinde 2007 obtained some stability results for the same iteration procedures using the same contractive conditions, but applied a different method which is similar to that used in Osilike and Udomene 1999.

Recently Timis and Berinde 2010 observed that the concept of stability is not very precise because of choice of an arbitrary sequence in place of approximate sequence with this notation they introduced a weaker concept of stability called weak stability. We remarked that every stable iteration is weakly stable but the reverse may not be true (see Timis 2010).

2- preliminaries

Consistent with Berinde 2007 and Hashim 2011, we use the following notion and definitions.

Let (X, d) be a metric space and let Y be an arbitrary nonempty set and R^+ the set of nonnegative real numbers. $CL(X) = \{A; A$ is nonempty closed subset of $X\}$

$$H(A,B) = \max\{\sup\{d(a,B); a \in A\}, \sup\{d(A,b); b \in B\}\}$$

H is called a generalized Haudorff on $CL(X)$.

Definition 2.1 [Berinde 2007**]**

Let (X, d) be a metric space and $\{x_n\}_{n=1}^{\infty} \subset X$ be a given sequence. We shall say that $\{y_n\}_{n=0}^{\infty} \subset X$ is an approximate sequence of $\{x_n\}$ if, for any $k \in N$, there exists $\eta = \eta(k)$ such that $d(x_n, y_n) \le \eta$ for all $n \ge k$.

Lemma 2.2 [Berinde 2007**]**

If δ is a real number such that $0 \le \delta < 1$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ is a sequence of position numbers such that $\lim_{x\to\infty} \varepsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfy $u_{n+1} \le \delta u_n + \varepsilon_n$, $n = 0,1,2,...$.

we have $\lim_{x\to\infty} u_n = 0$

Lemma 2.3[Berinde 2007**]**

Let $\{\varepsilon_n\}$ be a sequence of nonnegative real numbers and $\delta_n = \sum_{i=0}^{n} k^{n-i}\varepsilon_i$, where $0 \le k \le 1$. Then $\lim_{x\to\infty}\varepsilon_n = 0$ if ,and only if, $\lim_{x\to\infty}\delta_n = 0$.

Definition 2.4 [Timis 2010]

Let (X, d) be a metric space and $Y \subseteq X$ .let $S : Y \to X$, $T : Y \to CL(X)$ be such as $TY \subseteq SY$ and z is a coincidence point of δ and T ,that is $u = Sz \in Tz$.

For any $x_0 \in Y$, let the sequence $\{Sx_n\}$ be generated by the general iterative procedure $Sx_{n+1} \in f(T, x_n),\ n = 0,1,2,....$ (2) Converges to u.

Let $\{Sy_n\}_{n=0}^{\infty} \subseteq X$ be an approximate sequence of $\{Sx_n\}_{n=1}^{\infty}$,We have that

$\lim_{x\to\infty} H(Sy_{n+1}, f(T, y_n)) = 0$ Implies $\lim_{x\to\infty} Sy_n = u$, (3)

Then we shall say that (3) is weakly (S,T) stable.

We remark that, if $f(T, x_n) = Tx_n$ in (3) then the iteration procedure is called Jungck Picard iteration see [9]and[10].

Consider the following conditions for $T : Y \to CL(X)$ and $S : Y \to X$, for all $x, y \in X$.

$$H(Tx, Ty) \le q\max\{d(Sx, Sy), d(Sx, Tx), d(Sy, Ty), \frac{1}{2}[d(Sx, Ty) + d(Sy, Tx)]\}, \qquad (4)$$

Where $q \in [0,1)$.

$$H(Tx, Ty) \le Ld(Sx, Tx) + qd(Sx, Sy), \qquad (5)$$

Where $L \ge 0$ and $q \in [0,1)$.

$$H(Tx, Ty) \le \phi d(Sx, Tx) + qd(Sx, Sy), \qquad (6)$$

Where $\phi : R^+ \to R^+$ a monotone increasing function with $\phi(0) = 0$ and $\lim_{n\to\infty}\phi^n(t) = 0,$ for all $t \ge 0$.

$$H(Tx,Ty) \le qd(Sx,Sy)\Phi\left(d(Sx,Tx)\right), \tag{7}$$

Where $\Phi : R^+ \to R^+$ is a monotone increasing function with $\Phi(0) = 1$.

$$H(Tx,Ty) \le \psi\left(d(Sx,Sy)\right)e^{Ld(Sx,Tx)} \tag{8}$$

Where $\psi : R^+ \to R^+$, is a monotone increasing function with $\psi(0) = 0$ and $L \ge 0$.

We remark that i- (4) and (5) are independents, (7) and (8) are independent also.

ii- (7) is more general than (5) in the following sense,

If in (7) $\Phi(u) = 1 + \dfrac{ku}{d(Sx,Sy)}$, $\quad k \ge 0, \quad d(x,y) \ne 0, \ \forall \ x,y \in X, \quad x \ne y, \quad u \in R^+$;

Also if in (7), we put $\Phi(u) = 1 + \dfrac{\Phi(u)}{d(Sx,Sy)}$ we obtain condition (6)

Similarly condition (8) is more general than (5) in the sense that if in (8)

$$\psi(u) = \left(qu + Ld(Sx,Tx)\right)e^{-Ld(Sx,Tx)},$$

Where $q \in [0,1), L \ge 0$; $u \in R^+, \ \forall x \in X$, then we have obtained (5).

3- Main Results

Theorem 3.1

Let (X,d) be a metric space and $Y \subseteq X$. Let $T : Y \to CL(X)$, $S : Y \to X$ such that $TY \subseteq SY$ and one of SY or TY is a complete subspace of X. Let z be a coincidence point of T and S, that is $u = Sz \in Tz$. For any $x_0 \in Y$, let the sequence $\{Sx_n\}_{n=1}^{\infty}$ generated by $Sx_{n+1} \in Tx_n$ (9)

converges to u.

Let $\{Sy_n\}_{n=0}^{\infty} \subseteq X$ be an approximate sequence of $\{Sx_n\}_{n=1}^{\infty}$ and then define $\varepsilon_n = H(Sy_{n+1}, Ty_n), \quad n = 0,1,....$.

If the pair (S,T) satisfy condition (4), then

$$d(Sy_{n+1},u) \le d(u,Sx_{n+1}) + \alpha^{n+1}d(Sx_0,Sy_0) + \alpha\sum_{r=o}^{n}\alpha^{n-r}d(Sx_r,Tx_r) + \sum_{r=0}^{n}\alpha^{n-r}\varepsilon_r \tag{10}$$

Further, if Tz is a singleton that is $Tz = \{z\}$, then, the Picard iteration (9) is weakly (S,T) stable.

Proof :

Let $x,y \in X$. Then in view of (4), one of the following is true.

i. $H(Tx,Ty) \le qd(Sx,Sy)$;

ii. $H(Tx,Ty)\le qd(Sx,Tx)$

iii. $H(Tx,Ty)\le qd(Sy,Ty)$

$$\le q\left[d(Sy,Sx)+d(Sx,Tx)+H(Tx,Ty)\right]$$

$$\le \frac{q}{1-q}\left[d(Sx,Sy)+d(Sx,Tx)\right]$$

iv. $H(Tx,Ty)\le \frac{q}{2}\left[d(Sx,Ty)+d(Sy,Tx)\right]$

$$\le \frac{q}{2}\left[d(Sx,Sy)+d(Sy,Tx)+H(Tx,Ty)\right]$$

$$\le \frac{q}{2-q}\left[d(Sx,Sy)+d(Sx,Tx)\right]$$

Let $\alpha=\max\left\{q,\frac{q}{1-q},\frac{q}{2-q}\right\}$.

Therefore, in all cases,

$$H(Tx,Ty)\le \alpha\left[d(Sx,Sy)+d(Sx,Tx)\right] \tag{11}$$

Where $\alpha<1$ *and* $\alpha=\frac{q}{1-q}$

For any nonnegative integer n, we get

$$d(Sx_{n+1},Sy_{n+1})\le H(Tx_n,Sy_{n+1})$$

$$\le H(Tx_n,Ty_n)+H(Ty_n,Sy_{n+1})$$

$$\le \alpha\left[d(Sx_n,Sy_n)+d(Sx_n,Tx_n)\right]+\varepsilon_n$$

$$\le \alpha^2 d(Sx_{n-1},Sy_{n-1})+\alpha^2\left(d(Sx_n,Tx_{n-1})\right)+\alpha d(Sx_n,Tx_n)+\alpha\varepsilon_{n-1}+\varepsilon_n.$$

After($n-1$)steps, we obtain

$$d(Sx_{n+1},Sy_{n+1})\le \alpha^{n+1}d(Sx_0,Sy_o)+\alpha\sum_{r=o}^{n}\alpha^{n-r}d(Sx_r,Ty_r)+\sum_{r=0}^{n}\alpha^{n-r}\varepsilon_r.$$

Therefore, $d(u,Sy_{n+1})\le d(u,Sx_{n+1})+d(Sx_{n+1},Sy_{n+1})$

$$\le d(u,Sx_{n+1})+\alpha^{n+1}d(Sx_0,Sy_0)+\alpha\sum_{r=o}^{n}\alpha^{n-r}d(Sx_r,Tx_r)+\sum_{r=0}^{n}\alpha^{n-r}\varepsilon_r.$$

This yield the inequality (10).

Now, suppose that $\lim_{n\to\infty}Sy_n=u$ and Tz is a singleton and

$$\varepsilon_n = H(Sy_{n+1}, Ty_n) \le d(Sy_{n+1}, u) + d(u, Ty_n)$$
$$\le d(Sy_{n+1}, u) + H(Tz, Ty_n)$$
$$\le d(Sy_{n+1}, u) + \alpha\left[d(Sz, Sy_n) + d(Sz, Tz)\right],$$

Letting $n \to \infty$, we obtain $\varepsilon_n \to 0$.

Suppose that $\lim_{n\to\infty} \varepsilon_n = 0$.

Since $\alpha \in [0,1)$ and $\lim_{n\to\infty} Sx_n = u$, applying lemma (2.3) to the last term of the inequality (10) we obtain that $\sum_{r=o}^{n} \alpha^{n-r} \varepsilon_n = 0$.

Finally we show that $\lim_{n\to\infty}\left(\alpha \sum_{r=o}^{\infty} \alpha^{n-1} d(Sx_r, Tx_r)\right) = 0$ (11)

Let A denote the lower triangular matrix with entries $a_{nr} = \alpha^{n-1}$, then, $\lim_{n\to\infty} a_{nr} = 0$, for each r and $\lim_{n\to\infty} \sum_{r=0}^{\infty} a_{nr} = \frac{1}{1-\alpha}$, therefore, A is multiplicative i.e. for any convergent sequence $\{S_n\}$, we have $\lim_{n\to\infty} A(S_n) = \frac{1}{1-\alpha}$.

Thus the right side of (10) vanishes i.e. $\lim_{n\to\infty} y_n = u$.

Theorem 3.2

Let (X, d) be a metric space and $Y \subseteq X$. Let $T: Y \to CL(X)$, $S: Y \to X$ such that $TY \subseteq SY$ and one of SY or TY is a complete metric space of X. Let z be a coincidence point of T and S, that is $u = Sz \in Tz$.

For any $x_0 \in Y$, let the, sequence $\{Sx_n\}_{n=1}^{\infty}$ generated by $Sx_{n+1} \in Tx_n$ converges to u. let $\{Sy_n\}_{n=0}^{\infty} \subseteq X$ be an approximate sequence of $\{Sx_n\}_{n=1}^{\infty}$ and then define $\varepsilon_n = H(Sy_{n+1}, Ty_n)$, $n = 0,1,....$.

If the pair (S, T) satisfies condition (7) and Tz is a singleton, then, the Jungck Picard iteration is weakly (S, T)-stable.

Proof:

Suppose that $\lim_{n\to\infty} \varepsilon_n = 0$, then we shall establish that $\lim_{n\to\infty} Sy_n = u$.

$$d(Sy_{n+1},u) \le d(Sy_{n+1},Ty_n) + H(Ty_n,u)$$
$$\le \varepsilon_n + H(Ty_n,Tz)$$
$$\le \varepsilon_n + qd(Sz,Sy_n)\Phi(Sz,Tz)$$
$$\le \varepsilon_n + qd(Sz,Sy_n)\Phi(0)$$
$$= qd(Sz,Sy_n) + \varepsilon_n$$

Since $q \in [0,1)$, using lemma (2.3) yields $\lim_{n\to\infty} Sy_n = u = Sz$.

Conversely, suppose that $\lim_{n\to\infty} Sy_n = u$. then by(7) and triangular inequality, we have

$$\varepsilon_n = H(Sy_{n+1},Ty_n) \le d(Sy_{n+1},u) + d(u,Ty_n)$$
$$\le d(Sy_{n+1},u) + H(Tz,Ty_n)$$
$$\le d(Sy_{n+1},u) + qd(Sz,Sy_n)\Phi(Sz,Tz)$$
$$\le d(Sy_{n+1},u) + qd(Sy_n,Sz)\Phi(o)$$

Now taking limit, as $n \to \infty$.

This completes the proof of the theorem.

Theorem 3.3

Let (X,d) be a metric space and $Y \subseteq X$. let $T:Y \to CL(X)$, $S:Y \to X$ such that $TY \subseteq SY$ and one of SY or TY is a complete metric space of X. let z be a coincidence point of T and S,

For any $x_0 \in Y$, let the, sequence $\{Sx_n\}_{n=1}^{\infty}$ generated by $Sx_{n+1} \in Tx_n$ converges to u. let $\{Sy_n\}_{n=0}^{\infty} \subseteq X$ be an approximate sequence of $\{Sx_n\}_{n=1}^{\infty}$ and then define $\varepsilon_n = H(Sy_{n+1},Ty_n)$, $n = 0,1,....$.

If the pair (S,T) satisfies condition (8) and Tz is singleton. Then, the Jungck Picard iteration is weakly (S,T)-stable.

Proof:

Let $\varepsilon_n = H(Sy_{n+1},Ty_n)$, $n = 0,1,....$. and suppose that $\lim_{n\to\infty} \varepsilon_n = 0$. Then, we shall establish that $\lim_{n\to\infty} Sy_n = u$,

Using (8) and triangular inequality. Therefore,

$$d(Sy_{n+1},u) \le d(Sy_{n+1},Ty_n) + d(Ty_n,u)$$

$$\leq d(Sy_{n+1},Ty_n)+H(Ty_n,Tz)$$

$$=\varepsilon_n+H(Tz,Ty_n)$$

$$\leq \varepsilon_n+\psi\left(d(Sz,Sy_n)\right)e^{Ld(Sz,Tz)}$$

$$\leq \varepsilon_n+\psi\left(d(Sy_n,Sz)\right)e^{0}$$

$$\leq \varepsilon_n+\psi d(Sy_n,Sz). \qquad (12)$$

Using lemma (2.2).yields $\lim_{n\to\infty} Sy_n=u$

Conversely, suppose that $\lim_{n\to\infty} Sy_n=u$. Then by (8) and the triangular inequality, we obtain

$$\varepsilon_n=H(Sy_{n+1},Ty_n)\leq d(Sy_{n+1},u)+d(u,Ty_n)$$

$$=d(Sy_{n+1},u)+H(Tz,Ty_n)$$

$$\leq d(Sy_{n+1},u)+\psi\left(d(Sz,Sy_n)\right)e^{Ld(Sz,Tz)}$$

$$\leq d(Sy_{n+1},u)+\psi\left(d(Sy_n,Sz)\right)\to 0 \text{ as } n\to\infty.$$

Corollary 3.4 [Timis et al.2010]

Let (X,d) be a metric space and S,T maps on an arbitrary set Y with values in X such that $TY\subseteq SY$, and SY or TY is a complete subspace of X. let z be a coincidence point of T and S, that is $Tz=Sz=u$.

For any $x_0\in Y$, let the, sequence $\{Sx_n\}$ generated by the iterative procedure $Sx_{n+1}=Tx_n \quad n=0,1,2,...$ .converges to u. let $\{Sy_n\}_{n=0}^{\infty}\subset X$ be an approximate sequence of $\{Sx_n\}$ and then define $\varepsilon_n=H(Sy_{n+1},Ty_n), \quad n=0,1,....$.

If the pair (S,T) satisfy the following condition:

$$d(Tx,Ty)\leq q\max\{d(Sx,Sy),d(Sx,Tx),d(Sy,Ty),$$
$$\left[d(Sx,Ty)+d(Sy,Tx)/2\right]\};$$

For all $x,y\in Y$ and $q\in(0,1]$, then (10) holds and weakly (S,T)-stable.

Every stable iteration is weakly stable but the reverse may not be true. The following example of multi-valued map that satisfy contractive condition and for which the associated Picard iteration is not stable but weakly stable

Example 3.5

Let $X=[0,1]$ and $T:X\to CL(X), S:X\to X$ be given by

$$Tx=\begin{cases}\left[0,\frac{x+1}{2}\right] & , \ x\in\left[0,\frac{1}{2}\right]\\ \left\{\frac{1}{2}\right\} & , \ x\in\left(\frac{1}{2},1\right]\end{cases} \qquad \text{and} \quad Sx=x$$

Where $[0,1]$ is endowed with the usual metric. T and S have a unique fixed point at 0 i. e, $0=S(0)\in T(0)=\left[0,\frac{1}{2}\right]$

Let $x_0\in[0,1]$ and $x_{n+1}\in Tx_n$, for $n=1,2,...,$ $x_0\in\left[0,\frac{1}{2}\right]$, then $Sx_1=0\in Tx_0=\left[0,\frac{x_0+1}{2}\right]$ and if $x_0\in\left(\frac{1}{2},1\right]$, then $Sx_1=\frac{1}{2}\in Tx_0$ and $Sx_2=0\in Tx_1$, so $Sx_n=0$ for all $n\geq 2$,

Hence $\lim_{n\to\infty}Sx_n=0\in T(0)$

Let $\{Sy_n\}$ be an approximate sequence of $\{Sx_n\}$ there exist a decreasing sequence of positive number $\{\eta_n\}$ converging to some $n\geq 0$ such that

$|Sx_n-Sy_n|=|x_n-y_n|\leq\eta_n$, for all $\eta\geq 0$

Since $Sx_n=x_n=0$ for all $n\geq 2,$ we obtain $0\leq y_n\leq\eta_n$, $n\geq 0.$

Choose $\{\eta_n\}$ such that $\eta_n\leq\frac{1}{2}$ for all $n\geq 2.$

Then $Ty_n=0, n\geq 2$ and by $\lim_{n\to\infty}H(Sy_{n+1},Ty_n)=0,$ implies $\lim_{n\to\infty}Sy_n=0\in T(0).$

Hence the Picard iteration is weakly (S,T)-stable.

Now we have to show that the Picard iteration is not (S,T)-stable.

Let $Sy_n=y_n=\frac{n+1}{2n}$, $n\geq 1,$ then

$$\varepsilon_n=H(Sy_{n+1},Ty_n)=\left|\frac{(n+1)+1}{2(n+1)}-\frac{1}{2}\right|,$$

Because $Ty_n\geq\frac{1}{2},\quad n\geq 1$

Therefore $\lim_{n\to\infty}\varepsilon_n = 0$ but $\lim_{n\to\infty} Sy_n = y_n = \frac{1}{2}$,

So the Picard iteration is not (S,T)-stable.

Remark 3.6

Theorem 3.4 is a generalization of theorem 3.1 of Imoru and Olatinwa 2003 while Theorem 3.5 is a generalization of both Theorem P_1 and $P_2$of Imoru et al.2006

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حول نظريات الاستقرار الضعيف لعمليات
Jungck-Picard المكررة

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الملخص
قي هذا البحث حصلنا على بعض نتائج الاستقرار الضعيف لعمليات بيكارد المكررة في الفضاء المتري باستخدام شروط التقليص المختلفة نتائجنا هي تعميم الى بعض نتائج Harder and Hicks [4,5], Rhoades [15], Osilike [13], Berinde [2], Imoru and Olatinwa [7] Imoru et al. [11].