Irreducible modular spin characters of the symmetric group S_{19} modulo p = 11

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Abstract:

The main result of this paper is to find the irreducible modular spin characters of the symmetric groupS_{19}modulo p=11.

Section (1):

Introduction (1.1):

The symmetric group S_n has a representation group $\overline{S_n}$ of order$2(n!)$[I.Schur 1911], this group $\overline{S_n}$it has two kinds of characters, the first kind called ordinary characters which are indexed by the partitions of n. The second kind called spin (projective)characters of $S_n$which are indexed by the partitions of n with distinct parts which are called bar partitions of n [A.O.Morris and A.K.Yaseen 1988]. Finding irreducible modular spin characters equivalent to calculate the decomposition matrix[A.K.Yaseen 1987]. For p=11 Yaseen [A.K.Yaseen 1987] was found the modular spin characters of $S_n$for $11 \leq n \leq 14$ and for $n = 15,16$also calculated by Yaseen[A.K.Yaseen1995, see also Maas 2011],for n =17 and 18was calculate by Jassim and Taban[A.H. Jassimand S.A.Taban 2012].In thispaper we continue the works and we success to find the irreducible modular spin characters for$n = 19$ which we give in the appendix II.

Method for calculation(1.2):

There is no general method for a finding the Brauer characters for any non-abelian group, her we used several techniques as in [G.D. James and A. Kerber 1981],[A.H. Jassim and S.A.Taban 2012],[S.A.Taban1989],[A.K.Yaseen1987] and [A.K.Yaseen1995]. For the theory and back ground of a modular spin (projective) representation we refer to references [C.W. Curtis and I. Reiner 1966],[L. Dornhoff 1971,1972] and [B.M.Puttaswamaiah and J.D.Dixon 1977]. We give below some basic result which we used several times in calculations:

1. The degree of the spin characters $\langle\alpha\rangle = \langle\alpha_1, \dots, \alpha_m\rangle$is:

$deg\langle\alpha\rangle = 2^{\left[\frac{n-m}{2}\right]}\frac{n!}{\prod_{i=1}^{m}(\alpha_i!)}\prod_{1\le i<j\le m}\left(\alpha_i-\alpha_j\right)/\left(\alpha_i+\alpha_j\right)$[A.O.Morris 1962], [A.O.Morris and A.K.Yaseen 1988].

2. Let B be the block of defect one and let $b$be the number of$p$ −conjugate characters to the irreducible ordinary character $\chi$of G.Then:
   a) There exists a positive integer number N such that the irreducible ordinary characters of G are lying in the block B divided into two disjoint classes: B_1={$\chi \in B$ $|b \deg x \equiv N \; mod \;\; p^a$}, B_2={$\chi \in B$ $|b \deg x \equiv -N \; mod \;\; p^a$}
   b) Each coefficient of the decomposition matrix of the block B is 1 or 0.
   c) If $\alpha_1$and α_2 are not p −conjugate characters and are belong to the same class(B_1,B_2) above, then they have no irreducible modular character in common .
   d) For every irreducible ordinary character χin B_1, there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with one multiplicity [B.M.Puttaswamaiah and J.D.Dixon 1977].
3. If C is a principal character of G for an odd prime p and all the entries in C are divisible by a non-negative integer q, then $(1\backslash q)C$ is a principal character of G[G.D.James and A.Kerber 1981].
4. Let p be odd then:
   a) If n odd and$p \nmid n$then $\langle n\rangle$ is irreducible modular spin character.
   b) If n is odd and $p \nmid n$or $p \nmid (n-1)$, then $\langle n-1,1\rangle$ and $\langle n-1,1\rangle'$ are distinct irreducible modular spin characters [A.K.Yaseen 1987].
5. If C is a principal character of G for a prime p, then deg$C \equiv 0 \; mod \; p^A$, where o$(G) = p^A m$, $(p,m) = 1$[S.A.Taban 1989],[J.F.Humphreys 1977].
6. If the decomposition matrix $D_{n-1,p} = \left(d_{ij}\right)$ for S_{n-1} is known , then we can induced columns $\left(\psi_j \uparrow^{(r,\bar{r})} S_n\right)$ for S_n, these columns are a linear combination with non-negative coefficients from the columns of $D_{n,p}$ [G.D.James and A.Kerber 1981].

Notation(1.3):

p.s.	principle spin character.
p.i.s.	principle indecomposable spin character.
m.s.	modular spin character.
i.m.s.	irreducible modular spin character.
$(<\lambda>)^{no}$	(no) mean the number of i.m.s. in $<\lambda>$
$\equiv$	equivalence$mod 11$.

Section (2)Brauer trees to the symmetric group S_{19} , p=11:

The decomposition matrix for S_{19} modulo p=11 of degree (81,72) [A.O.Morris 1962],[A.O.Morris and A.K.Yaseen 1988].There are 33 blocks six of them B_1 ,B_2, … ,B_6, are of defect one and the others B_7,… ,$B_{33}$of defect zero.

Lemma (2.1)

The Brauer tree for the block B_1 is:

$$\langle 19\rangle^{*}\_\_\_\langle 11,8\rangle = \langle 11,8\rangle'\_\_\_\langle 10,8,1\rangle^{*}\_\_\_\langle 9,8,2\rangle^{*}\_\_\_\langle 8,7,4\rangle^{*}\_\_\_\langle 8,6,5\rangle^{*}$$

Proof:

On 11-regular classes we have $\langle 11,8\rangle = \langle 11,8\rangle'$

a) deg($\langle 11,8\rangle$ +$\langle 11,8\rangle'$) ≡deg $\langle 9,8,2\rangle^{*}$ ≡ deg$\langle 8,6,5\rangle^{*}$ ≡ 5 .

deg$\langle 19\rangle^{*}$ ≡deg $\langle 10,8,1\rangle^{*}$ ≡deg $\langle 8,7,4\rangle^{*}$ ≡ −5

b) By using (8,4)-inducing of p.i.s D_1, D_3, D_5, D_7 andD_9 for S_{18}(see appendix I) to S_{19} we have respectively :

1. $\langle 19\rangle + \langle 11,8\rangle + \langle 11,8\rangle'$
2. $\langle 11,8\rangle + \langle 11,8\rangle' + \langle 10,8,1\rangle^{*}$
3. $\langle 10,8,1\rangle^{*} + \langle 9,8,2\rangle^{*}$
4. $\langle 9,8,2\rangle^{*} + \langle 8,7,4\rangle^{*}$
5. $\langle 8,7,4\rangle^{*} + \langle 8,6,5\rangle^{*}$

we have the Braure tree for this blockB_1. ■

Lemma(2.2)

The Brauer tree for the block $B_2$is:

$$\begin{array}{ccccc} \langle 18,1\rangle - \langle 12,7\rangle & \backslash & & / & \langle 9,7,2,1\rangle - \langle 8,7,3,1\rangle - \langle 7,6,5,1\rangle \\ & & \langle 11,7,1\rangle^{*} & & \\ \langle 18,1\rangle' - \langle 12,7\rangle' & / & & \backslash & \langle 9,7,2,1\rangle' - \langle 8,7,3,1\rangle' - \langle 7,6,5,1\rangle' \end{array}$$

Proof:

a) deg{$\langle 18,1\rangle$,$\langle 18,1\rangle'$, $\langle 11,7,1\rangle^{*}$, $\langle 8,7,3,1\rangle$, $\langle 8,7,3,1\rangle'$}≡ 7

deg{$\langle 12,7\rangle$,$\langle 12,7\rangle'$, $\langle 9,7,2,1\rangle$, $\langle 9,5,2,1\rangle'$, $\langle 7,6,5,1\rangle$,$\langle 7,6,5,1\rangle'$}≡ −7.

b) By using (1,0)-inducing of p.i.s

D_5, D_6, D_7, D_8, D_9,D_{10}, D_{11}, D_1, D_2 and $D_3$for S_{18}(see appendix I) to S_{19} we have p.s.c_5, c_6, c_7, c_8 ,c_9, c_{10}, k_1,k_2, k_3, $k_4$respectively.

So we have the approximation matrix(Table (1))

(Table (1))

$\langle 18,1\rangle$	1	1								
$\langle 18,1\rangle'$	1		1							
$\langle 12,7\rangle$	1	1	1	1						
$\langle 12,7\rangle'$	1	1	1	1						
$\langle 11,7,1\rangle^{*}$		1	1	2	1	1				
$\langle 9,7,2,1\rangle$					1		1			
$\langle 9,7,2,1\rangle'$						1		1		
$\langle 8,7,3,1\rangle$							1		1	
$\langle 8,7,3,1\rangle'$								1		1
$\langle 7,6,5,1\rangle$									1	
$\langle 7,6,5,1\rangle'$										1
	k_1	k_2	k_3	k_4	c_5	c_6	c_7	c_8	c_9	c_{10}

$\langle 18,1\rangle \neq \langle 18,1\rangle'$ so k_1 splits to c_1 and c_2.

$k_4 = k_2 + k_3 - c_1 - c_2$, this give p.i.s. $k_2 - c_1$, $k_3 - c_2$.

Hence we have the Braure tree for this block B_2 ■.

Lemma(2.3)

The Brauer tree for the block B_5 is:

$\langle 16,2,1\rangle^{*}$_$\langle 13,5,1\rangle^{*}$_$\langle 12,5,2\rangle^{*}$_$\langle 11,5,2,1\rangle$=$\langle 11,5,2,1\rangle'$_$\langle 8,5,3,2,1\rangle^{*}$_$\langle 7,5,4,2,1\rangle^{*}$

Proof:

on11-regular classes we have $\langle 11,5,2,1\rangle = \langle 11,5,2,1\rangle'$

a) $\deg\langle 13,5,1\rangle^{*} \equiv \deg\,(\langle 11,5,2,1\rangle + \langle 11,5,2,1\rangle') \equiv \deg\,\langle 7,5,4,2,1\rangle^{*} \equiv 7$

$\deg\langle 16,2,1\rangle^{*} \equiv \deg\,\langle 12,5,2\rangle^{*} \equiv \deg\,\langle 8,5,3,2,1\rangle^{*} \equiv -7$

b) By using $(r,\bar{r})$-inducing ofp.i.s. D_{16},D_{18},D_{19},D_{20} andD_{28},for S_{18}(see appendix I) to S_{19} we get respectively .

1) $\langle 16,2,1\rangle^{*} + \langle 13,5,1\rangle^{*}$.
2) $2\langle 12,5,2\rangle^{*} + 2\,\langle 11,5,2,1\rangle + 2\langle 11,3,2,1\rangle' = 2k$ .so$k$ is p.i.
3) $\langle 11,5,2,1\rangle + \langle 11,5,2,1\rangle' + \langle 8,5,3,2,1\rangle^{*}$.
4) $\langle 8,5,3,2,1\rangle^{*}$+$\langle 7,5,4,2,1\rangle^{*}$.
5) $\langle 13,5,1\rangle^{*}$+ $\langle 12,5,2\rangle^{*}$.

So,we get the Brauer tree for the block B_5 ■.

<u>Lemma(2.4)</u>

The Brauer tree for the block B_6 is:

$\langle 15,3,1\rangle^*$ __ $\langle 14,4,1\rangle^*$ __ $\langle 12,4,3\rangle^*$ __ $\langle 11,4,3,1\rangle$=$\langle 11,4,3,1\rangle'$ __ $\langle 9,4,3,2,1\rangle^*$ __ $\langle 6,5,4,3,1\rangle^*$

Proof:

on11-regular classes we have $\langle 11,4,3,1\rangle = \langle 11,4,3,1\rangle'$

a) deg$\langle 14,4,1\rangle^*$ ≡deg $\langle 11,4,3,1\rangle + \langle 11,4,3,1\rangle')$ ≡deg $\langle 6,5,4,3,1\rangle^* \equiv 8$

deg$\langle 15,3,1\rangle^*$ ≡deg $\langle 12,4,3\rangle^*$ ≡deg $\langle 9,4,3,2,1\rangle^* \equiv -8$

b) By using $(r,\bar{r})$-inducing of p.i.s. D_{21},D_{23} ,D_{24} ,D_{25} ,D_{28},for S_{18} (see appendix I) to S_{19} we get respectively:

1) $\langle 15,3,1\rangle^* + \langle 14,4,1\rangle^*$.
2) $2\langle 12,4,3\rangle^* + 2\,\langle 11,4,3,1\rangle + 2\langle 11,4,3,1\rangle' = 2l$ .so$l$ is p.i.
3) $\langle 11,4,3,1\rangle + \langle 11,4,3,1\rangle' + \langle 9,4,3,2,1\rangle^*$.
4) $\langle 9,4,3,2,1\rangle^*$+$\langle 6,5,4,3,1\rangle^*$.
5) $\langle 14,4,1\rangle^*$+ $\langle 12,4,3\rangle^*$.

So, we get the Brauer tree for the block B_6■.

<u>Lemma(2.5)</u>

The Brauer tree for the block $\mathrm{B}_3$is:

$\langle 17,2\rangle - \langle 13,6\rangle$ \ $\qquad$ / $\langle 10,6,2,1\rangle - \langle 8,6,3,2\rangle - \langle 7,6,4,2\rangle$
$\qquad\qquad\qquad \langle 11,6,2\rangle^*$
$\langle 17,2\rangle' - \langle 13,6\rangle'$ / $\qquad$ \ $\langle 10,6,2,1\rangle' - \langle 8,6,3,2\rangle' - \langle 7,6,4,2\rangle'$

Proof:

a) deg$\{\langle 17,2\rangle, \langle 17,2\rangle', \langle 11,6,2\rangle^*, \langle 8,6,3,2\rangle, \langle 8,6,3,2\rangle'\} \equiv 9$

deg$\{\langle 13,6\rangle, \langle 13,6\rangle', \langle 10,6,2,1\rangle, \langle 10,6,2,1\rangle', \langle 7,6,4,2\rangle, \langle 7,6,4,2\rangle'\} \equiv -9$.

b) By using $(r,\bar{r})$-inducing we have (see appendix I):

$D_{11}\uparrow^{(2,10)} S_{19}$=$k_1$, $D_{12}\uparrow^{(2,10)} S_{19} = k_2$, $D_{14}\uparrow^{(2,10)} S_{19} = k_3$

$D_{15}\uparrow^{(2,10)} S_{19} = k_4$, $\langle 10,6,2\rangle\uparrow^{(0,1)} S_{19} = c_5$, $\langle 10,6,2\rangle'\uparrow^{(0,1)} S_{19} = c_6$.

Thus, we have the approximation matrix(Table (2))

Table(2)

	Ψ_1	Ψ_2	φ_5	φ_6	Ψ_3	Ψ_4	φ_1	φ_2
$\langle 17,2\rangle$	1						a	
$\langle 17,2\rangle'$	1							a
$\langle 13,6\rangle$	1	1					b	
$\langle 13,6\rangle'$	1	1						b
$\langle 11,6,2\rangle^*$		2	1	1			c	c
$\langle 10,6,2,1\rangle$			1		1		d	
$\langle 10,6,2,1\rangle'$				1	1			d
$\langle 8,6,3,2\rangle$					1	1	f	
$\langle 8,6,3,2\rangle'$					1	1		f
$\langle 7,6,4,2\rangle$						1	h	
$\langle 7,6,4,2\rangle'$						1		h
	k_1	k_2	c_5	c_6	k_3	k_4	Y_1	Y_2

Since $\langle 17,2\rangle \neq \langle 17,2\rangle'$ on $(11,\alpha)$-regular classes then either k_1 is split or there are another two columns. Suppose there are two columns such as Y_1 and Y_2

To describe columns Y_1 and Y_2

1. $\langle 17,2\rangle \downarrow S_{18} = (\langle 16,2\rangle^*)^1 + (\langle 17,1\rangle^*)^1$has 2 of i.m.s $(see\ appendix\ I)$ and form (Table(2))we have $a \in \{0,1\}$.
2. $\langle 13,6\rangle \downarrow S_{18} = (\langle 12,6\rangle^*)^2 + (\langle 13,5\rangle^*)^2$has 4 of i.m.s. we have $b \in \{0,1,2\}$.
3. $\langle 11,6,2\rangle^* \downarrow S_{18} = (\langle 10,6,2\rangle)^1 + (\langle 10,6,2\rangle')^1 + (\langle 11,5,2\rangle)^2 + (\langle 11,5,2\rangle')^2 + (\langle 11,6,1\rangle)^2 + (\langle 11,6,1\rangle')^2$has10 of i.m.s. we have $c \in \{0,1,2,3,4,5,6\}$.
4. $\langle 10,6,2,1\rangle \downarrow S_{18} = (\langle 9,6,2,1\rangle^*)^2 + (\langle 10,5,2,1\rangle^*)^2$+$(\langle 10,6,2\rangle)^1$ has 5 of i.m.s. we have $d \in \{0,1,2,3\}$.
5. $\langle 8,6,3,2\rangle \downarrow S_{18} = (\langle 7,6,3,2\rangle^*)^1 + (\langle 8,5,3,2\rangle^*)^2 + (\langle 8,6,3,1\rangle^*)^2$ has 5of i.m.s. we have $f \in \{0,1,2,3\}$
6. $\langle 7,6,4,2\rangle \downarrow S_{18} = (\langle 7,5,4,2\rangle^*)^1 + (\langle 7,6,3,2\rangle^*)^1 + (\langle 7,6,4,1\rangle^*)^1$has 3 of i.m.s. we have $h \in \{0,1,2\}$.

If $a = 0$ then k_1 splits to give $\langle 17,2\rangle +\ \langle 13,6\rangle$ and $\langle 17,2\rangle' +\ \langle 13,6\rangle'$

If $a = 1$:

1) Since $\langle 17,2\rangle \downarrow S_{18} \cap \langle 13,6\rangle \downarrow S_{18} =$ 2 of i.m.s for S_{18}, then
$\langle 17,2\rangle \cap \langle 13,6\rangle$ =$\Psi_1 + \varphi_1$ if $b \in \{1,2\}$
=φ_1 if b = 0 ;

2) There is no i.m.s. in $\langle 17,2\rangle \downarrow S_{18} \cap \langle 11,6,2\rangle^{*} \downarrow S_{18}$,so$c$ = 0 ;
3) There is no i.m.s. in $\langle 17,2\rangle \downarrow S_{18} \cap \langle 10,6,2,1\rangle \downarrow S_{18}$so d = 0 ;
4) There is no i.m.s. in $\langle 17,2\rangle \downarrow S_{18} \cap \langle 8,6,3,2\rangle \downarrow S_{18}$, so$f$ = 0 ;
5) There is no i.m.s. in$\langle 17,2\rangle \downarrow S_{18} \cap \langle 7,6,4,2\rangle \downarrow S_{18}$,so$h = 0$.

Wehave the possible columns

Y_1=$\langle 17,2\rangle + b\langle 13,6\rangle$,

Y_2 =$\langle 17,2\rangle' + b\langle 13,6\rangle'$;$b \in \{0,1,2\}$

deg$Y_1 \equiv 0$ and deg$Y_2 \equiv 0$ only when $b = 1$

So $k_1$splits to$\langle 17,2\rangle + \langle 13,6\rangle$, and$\langle 17,2\rangle' + \langle 13,6\rangle'$

Since $\langle 13,6\rangle \neq \langle 13,6\rangle'$on $(11,\alpha)$-regular classes then either k_2 is splits or there are two columns . If we suppose there are another two columns such as Y_1 and Y_2(as in Table (2)); to describe these two columns:

1) Since $\langle 13,6\rangle \downarrow S_{18} \cap \langle 11,6,2\rangle^{*} \downarrow S_{16}$has 2 of i.m.s for S_{18},
   $\langle 13,6\rangle \cap \langle 11,6,2\rangle^{*}$= $\Psi_2 + \varphi_3$ if $c \in \{1,2,3,4,5,6\}$ and$b = 1$ or $c = 1$ and$b \in \{1,2\}$
   =Ψ_2 +2φ_3 if $c \in \{2,3,4,5,6\}$ and$b = 2$
   =Ψ_2 if c =0 ;
2) There is no i.m.s. in $\langle 13,6\rangle \downarrow S_{18} \cap \langle 10,6,2,1\rangle \downarrow S_{18}$, so$d = 0$;
3) There is no i.m.s. in $\langle 13,6\rangle \downarrow S_{18} \cap \langle 8,6,3,2\rangle \downarrow S_{18}$, so$f = 0$;
4) There is no i.m.s. in $\langle 13,6\rangle \downarrow S_{18} \cap \langle 7,6,4,2\rangle \downarrow S_{18}$, so$h = 0$.

We get the possible columns

Y_1=$b\langle 13,6\rangle$ +c $\langle 11,6,2\rangle^{*}$,

Y_2 = $b\langle 13,6\rangle'$ + c $\langle 11,6,2\rangle^{*}$, $b \in \{1,2\}$, $c \in \{0,1,2,3,4,5,6\}$

deg$Y_1 \equiv 0$ and deg$Y_2 \equiv 0$ only when $b = c$.

So $k_2$splits to give $\langle 13,6\rangle$ +$\langle 11,6,2\rangle^{*}and\langle 13,6\rangle'$ +$\langle 11,6,2\rangle^{*}$ which is the same when $b = 0$.

Since $\langle 7,6,4,2\rangle \neq \langle 7,6,4,2\rangle'$ on $(11,\alpha)$-regular classesthen $k_4$splitsor there are two columns. If we suppose there are two columns such as Y_1 and Y_2 (as in Table (2)).To describe Y_1 and Y_2:

If $h \in \{1,2\}$:

1) There is no i.m.s. in $\langle 7,6,4,2\rangle \downarrow S_{18} \cap \langle 11,6,2\rangle^{*} \downarrow S_{18}$, so$c = 0$;
2) There is no i.m.s. in $\langle 7,6,4,2\rangle \downarrow S_{18} \cap \langle 10,6,2,1\rangle \downarrow S_{18}$, so$d = 0$;
3) Since $\langle 7,6,4,2\rangle \downarrow S_{18} \cap \langle 8,6,3,2\rangle \downarrow S_{18}$has 2 of i.m.s for S_{18}
   $\langle 7,6,4,2\rangle \cap \langle 8,4,3,2\rangle$= $\Psi_4 + \varphi_9$ if $f \in \{1,2,3\}$ and $h = 1$or $f = 1$ and $h \in \{1,2\}$
   =Ψ_4 + 2φ_9 if $f \in \{2,3\}$ and $h = 2$
   =Ψ_4 if f = 0 .

We get the possible columns

$Y_1 = f\,\langle 8,6,3,2\rangle + \langle 7,6,4,2\rangle$,

$Y_2 = f\,\langle 8,6,3,2\rangle' + \langle 7,6,4,2\rangle'$, $f \in \{0,1,2,3\}$, $h \in \{1,2\}$

$\deg Y_1 \equiv 0$ and $\deg Y_2 \equiv 0$ only when $f = h$.

So, k_4 splits to $\langle 8,6,3,2\rangle + \langle 7,6,4,2\rangle$ and $\langle 8,6,3,2\rangle' + \langle 7,6,4,2\rangle'$ which is the same when $h = 0$.

Now, since $\langle 8,6,3,2\rangle \neq \langle 8,6,3,2\rangle'$ on $(11,\alpha)$-regular classes, then k_3 splitsor there are two columns. If we suppose there are another two columns such as Y_1 and Y_2 (as in Table (2));to describe Y_1 and Y_2:

If $f \in \{1,2,3\}$:

1) There is no i.m.s. in $\langle 8,6,3,2\rangle \downarrow S_{18} \cap \langle 11,6,2\rangle^* \downarrow S_{18}$, so $c = 0$;
2) Since $\langle 8,6,3,2\rangle \downarrow S_{18} \cap \langle 10,6,3,1\rangle \downarrow S_{18}$ has 2 of i.m.s for S_{18}

$\langle 8,6,3,2\rangle \cap \langle 10,6,3,1\rangle = \Psi_3 + \varphi_7$ if $d \in \{1,2,3\}$ and $f = 1$ or $d = 1$ and $f \in \{1,2,3\}$

$= \Psi_3 + 2\varphi_7$ if $d \in \{2,3\}$ and $f = 2$

$= \Psi_3 + 3\varphi_7$ if $d = 3$ and $f = 3$

$= \Psi_3$ if $d = 0$.

We get the possible columns

$Y_1 = d\langle 10,6,2,1\rangle + f\,\langle 8,6,3,2\rangle$,

$Y_2 = d\langle 10,6,2,1\rangle' + f\,\langle 8,6,3,2\rangle'$, $d \in \{0,1,2,3\}$, $f \in \{1,2,3\}$

$\deg Y_1 \equiv 0$ and $\deg Y_2 \equiv 0$ only when $d = f$.

So, k_4 splits to $\langle 10,6,2,1\rangle + \langle 8,6,3,2\rangle$ and $\langle 10,6,2,1\rangle' + \langle 8,6,3,2\rangle'$ which is the same when $f = 0$.So we get the Brauer tree for the block B_3 ■.

<u>Theorem</u>

The decomposition matrix of the spin characters for S_{19} modulo $p = 11$ as given in appendix II

Proof:

We determine the decomposition matrices for all blocks of S_{19} except the block B_4 to complete the proof we calculate the decomposition matrix for the block B_4.

a) $\deg\{\langle 16,3\rangle, \langle 16,3\rangle', \langle 11,5,3\rangle^*, \langle 9,5,3,2\rangle, \langle 9,5,3,2\rangle'\} \equiv 9$

$\deg\{\langle 14,5\rangle, \langle 14,5\rangle', \langle 10,5,3,1\rangle, \langle 10,5,3,1\rangle', \langle 7,5,4,3\rangle, \langle 7,5,4,3\rangle'\} \equiv -9$.

b) By using $(r,\bar{r})$-inducing we get:

$D_{16} \uparrow^{(3,9)} S_{19} = k_1$, $D_{17} \uparrow^{(3,9)} S_{19} = k_2$, $D_{19} \uparrow^{(3,9)} S_{19} = k_3$

$D_{20} \uparrow^{(3,9)} S_{19} = k_4$, $\langle 10,5,3\rangle \uparrow^{(0,1)} S_{19} = c_5$, $\langle 10,5,3\rangle' \uparrow^{(0,1)} S_{19} = c_6$.

Thus, we have the approximation matrix (Table (3))

Table(3)

	Ψ_1	Ψ_2	φ_5	φ_6	Ψ_3	Ψ_4	φ_1	φ_2
$\langle 16,3\rangle$	1						a	
$\langle 16,3\rangle'$	1							a
$\langle 14,5\rangle$	1	1					b	
$\langle 14,5\rangle'$	1	1						b
$\langle 11,5,3\rangle^*$		2	1	1			c	c
$\langle 10,5,3,1\rangle$			1		1		d	
$\langle 10,5,3,1\rangle'$				1	1			d
$\langle 9,5,3,2\rangle$					1	1	f	
$\langle 9,5,3,2\rangle'$					1	1		f
$\langle 7,5,4,3\rangle$						1	h	
$\langle 7,5,4,3\rangle'$						1		h
	k_1	k_2	c_5	c_6	k_3	k_4	Y_1	Y_2

Since $\langle 16,3\rangle \neq \langle 16,3\rangle'$ on $(11,\alpha)$-regular classes then either k_1 is split or there are two columns. Suppose there are two columns such as $Y_1$and Y_2; to describe columns Y_1 and Y_2

1. $\langle 16,3\rangle \downarrow S_{18} = (\langle 15,3\rangle^*)^1 + (\langle 16,2\rangle^*)^1$has 2 of i.m.s $(see\ appendix\ I)$ and form (Table(2))we have $a \in \{0,1\}$.
2. $\langle 14,5\rangle \downarrow S_{18} = (\langle 13,5\rangle^*)^2 + (\langle 14,4\rangle^*)^2$has 4of i.m.s. we have $b \in \{0,1,2\}$.
3. $\langle 11,5,3\rangle^* \downarrow S_{18} = (\langle 10,5,3\rangle)^1 + (\langle 10,5,3\rangle')^1 + (\langle 11,4,3\rangle)^2 + (\langle 11,4,3\rangle')^2 + (\langle 11,5,2\rangle)^2 + (\langle 11,5,2\rangle')^2$has 10 of i.m.s. we have $c \in \{0,1,2,3,4,5,6\}$.
4. $\langle 10,5,3,1\rangle \downarrow S_{18} = (\langle 9,5,3,1\rangle^*)^1 + (\langle 10,4,3,1\rangle^*)^2 + (\langle 10,5,2,1\rangle)^2 + (\langle 10,5,3\rangle)^1$ has 6of i.m.s. we have $d \in \{0,1,2,3,4\}$.
5. $\langle 9,5,3,2\rangle \downarrow S_{18} = (\langle 8,5,3,2\rangle^*)^2 + (\langle 9,4,3,2\rangle^*)^2 + (\langle 9,5,3,1\rangle)^1$ has 5of i.m.s. we have $f \in \{0,1,2,3\}$
6. $\langle 7,5,4,3\rangle \downarrow S_{18} = (\langle 6,5,4,3\rangle^*)^1 + (\langle 7,5,4,2\rangle^*)^1$has 2of i.m.s. we have $h \in \{0,1\}$.

If $a = 0$ then k_1 splits to give $\langle 16,3\rangle + \langle 14,5\rangle$ and $\langle 16,3\rangle' + \langle 14,5\rangle'$

If $a = 1$:

1) Since $\langle 16,3\rangle \downarrow S_{18} \cap \langle 14,5\rangle \downarrow S_{18}$has 2 of i.m.s for S_{18}
   $\langle 16,3\rangle \cap \langle 14,5\rangle = \Psi_1 + \varphi_1$ if $b \in \{1,2\}$
   $= \varphi_1$ if $b = 0$;
2) There is no i.m.s. in $\langle 16,3\rangle \downarrow S_{18} \cap \langle 11,5,3\rangle^* \downarrow S_{18}$,so$c = 0$;

3) There is no i.m.s. in $\langle 16,3\rangle \downarrow S_{18} \cap \langle 10,5,3,1\rangle \downarrow S_{18}$so d = 0 ;
4) There is no i.m.s. in $\langle 16,3\rangle \downarrow S_{18} \cap \langle 9,5,3,2\rangle \downarrow S_{18}$, so$f$ = 0 ;
5) There is no i.m.s. in$\langle 16,3\rangle \downarrow S_{18} \cap \langle 7,5,4,3\rangle \downarrow S_{18}$,so$h = 0$.

Now,we get the possible columns

Y_1=$\langle 16,3\rangle +\ b\langle 14,5\rangle$,

Y_2 =$\langle 16,3\rangle' +\ b\langle 14,5\rangle'$,$b \in \{0,1,2\}$

deg$Y_1 \equiv 0$ and deg$Y_2 \equiv 0$ only when $b = 1$

So $k_1$splits to $\langle 16,3\rangle +\ \langle 14,5\rangle$, and $\langle 16,3\rangle' +\ \langle 14,5\rangle'$

Since $\langle 14,5\rangle \neq \langle 14,5\rangle'$ on $(11,\alpha)$-regular classes then either k_2 is splits or there are two columns . If we suppose there are two columns such as Y_1 and Y_2 (as in Table (3)); to describe these two columns:

If $b \in \{1,2\}$:

1) Since $\langle 14,5\rangle \downarrow S_{18} \cap \langle 11,5,3\rangle^* \downarrow S_{18}$has 2 of i.m.s for S_{18}
   $\langle 14,5\rangle \cap \langle 11,4,2\rangle^*$= $\Psi_2 + \varphi_3$ if $c \in \{1,2,3,4,5,6\}$ and $b = 1$
   or if $c = 1$ and $b \in \{1,2\}$
   = Ψ_2 +2φ_3 if $c \in \{2,3,4,5,6\}$ and $b = 2$
   =$\Psi_2$if $c = 0$;
2) There is no i.m.s. in $\langle 14,5\rangle \downarrow S_{18} \cap \langle 10,5,3,1\rangle \downarrow S_{18}$, so$d = 0$;
3) There is no i.m.s. in $\langle 14,5\rangle \downarrow S_{18} \cap \langle 9,5,3,2\rangle \downarrow S_{18}$, so$f = 0$;
4) There is no i.m.s. in $\langle 14,5\rangle \downarrow S_{18} \cap \langle 7,5,4,3\rangle \downarrow S_{18}$, so$h = 0$.

We get the possible columns

Y_1=$\langle 14,5\rangle$ +c $\langle 11,5,3\rangle^*$,

Y_2 = $\langle 14,5\rangle$ + c $\langle 11,5,3\rangle^*$, $c \in \{0,1,2,3,4,5,6\}$

deg$Y_1 \equiv 0$ and deg$Y_2 \equiv 0$ only when $b = c$

So $k_2$splits to give $\langle 14,5\rangle +\ \langle 11,5,3\rangle^*$ and $\langle 14,5\rangle' +\ \langle 11,5,3\rangle^*$ which is the same when $b = 0$.

Since $\langle 7,5,4,3\rangle \neq \langle 7,5,4,3\rangle'$ on $(11,\alpha)$-regular classesthen $k_4$splitsor there are two columns. If we suppose there are two columns such as Y_1 and Y_2 (as in Table (3)); to describe Y_1 and Y_2:

If $h = 1$:

1) There is no i.m.s. in $\langle 7,5,4,3\rangle \downarrow S_{18} \cap \langle 11,5,3\rangle^* \downarrow S_{18}$, so$c = 0$;
2) There is no i.m.s. in $\langle 7,5,4,3\rangle \downarrow S_{18} \cap \langle 10,5,3,1\rangle \downarrow S_{18}$, so$d = 0$;
3) Since $\langle 7,5,4,3\rangle \downarrow S_{18} \cap \langle 9,5,3,2\rangle \downarrow S_{18}$has 2 of i.m.s for S_{18}
   $\langle 7,5,4,3\rangle \cap \langle 9,5,3,2\rangle$= Ψ_4 + φ_9 if $f \in \{1,2,3\}$
   =Ψ_4 if f = 0 .

We get the possible columns

$Y_1 = f\ \langle 9,5,3,2\rangle + \langle 7,5,4,3\rangle$,

$Y_2 = f\ \langle 9,5,3,2\rangle' + \langle 7,5,4,3\rangle'$, $f \in \{0,1,2,3\}$

$\deg Y_1 \equiv 0$ and $\deg Y_2 \equiv 0$ only when $f = 1$

So, $k_4$splits to $\langle 9,5,3,2\rangle + \langle 7,5,4,3\rangle$ and $\langle 9,5,3,2\rangle' + \langle 7,5,4,3\rangle'$which is the same when $h = 0$.

Now, since $\langle 9,5,3,2\rangle \neq \langle 9,5,3,2\rangle'$on $(11, \alpha)$-regular classes and we have 9 columns, then k_3 must be a split to $\langle 10,5,3,1\rangle + \langle 9,5,3,2\rangle$ and$\langle 10,5,3,1\rangle' + \langle 9,5,3,2\rangle'$. So we get the decomposition matrix for the block B_4 ■.

From lemmas and theorem above we can find the 11-decomposition matrix for the spin characters of S_{19} .We write this decomposition matrix in appendix II

Appendix I

<u>The decomposition matrix for the spin characters of S_{18} , $p = 11$</u>[A. H. Jassim andS.A.Taban]

The spin characters	The decomposition matrix for the block B_1									
$\langle 18\rangle$	1									
$\langle 18\rangle'$		1								
$\langle 11,7\rangle^*$	1	1	1	1						
$\langle 10,7,1\rangle$			1		1					
$\langle 10,7,1\rangle'$				1		1				
$\langle 9,7,2\rangle$					1		1			
$\langle 9,7,2\rangle'$						1		1		
$\langle 8,7,3\rangle$							1		1	
$\langle 8,7,3\rangle'$								1		1
$\langle 7,6,5\rangle$									1	
$\langle 7,6,5\rangle'$										1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}

The spin characters	The decomposition matrix for the block B_2				
$\langle 17,1\rangle^*$	1				
$\langle 12,6\rangle^*$	1	1			
$\langle 11,6,1\rangle$		1	1		
$\langle 11,6,1\rangle'$		1	1		
$\langle 9,6,2,1\rangle^*$			1	1	
$\langle 8,6,3,1\rangle^*$				1	1
$\langle 7,6,4,1\rangle^*$					1
	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}

The spin characters	The decomposition matrix for the block B_3				
$\langle 16,2\rangle^*$	1				
$\langle 13,5\rangle^*$	1	1			
$\langle 11,5,2\rangle$		1	1		
$\langle 11,5,2\rangle'$		1	1		
$\langle 10,5,2,1\rangle^*$			1	1	
$\langle 8,5,3,2\rangle^*$				1	1
$\langle 7,5,4,2\rangle^*$					1
	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}

The spin characters	The decomposition matrix for the block B_4				
$\langle 15,3\rangle^*$	1				
$\langle 14,4\rangle^*$	1	1			
$\langle 11,4,3\rangle$		1	1		
$\langle 11,4,3\rangle'$		1	1		
$\langle 10,4,3,1\rangle^*$			1	1	
$\langle 9,4,3,2\rangle^*$				1	1
$\langle 6,5,4,3\rangle^*$					1
	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}

The spin characters	The decomposition matrix for the block B_5									
$\langle 15,2,1\rangle$	1									
$\langle 15,2,1\rangle'$		1								
$\langle 13,4,1\rangle$	1		1							
$\langle 13,4,1\rangle'$		1		1						
$\langle 12,4,2\rangle$			1		1					
$\langle 12,4,2\rangle'$				1		1				
$\langle 11,4,2,1\rangle^*$					1	1	1	1		
$\langle 8,4,3,21\rangle$							1		1	
$\langle 8,4,3,2,1\rangle'$								1		1
$\langle 6,5,4,2,1\rangle$									1	
$\langle 6,5,4,2,1\rangle'$										1
	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}

Appendix II

<u>The decomposition matrix for the spin characters of S_{19} , $p = 11$</u>

The spincharacters	The decomposition matrix for the block B_1				
$\langle 19\rangle^*$	1				
$\langle 11,8\rangle$	1	1			
$\langle 11,8\rangle'$	1	1			
$\langle 10,8,1\rangle^*$		1	1		
$\langle 9,8,2\rangle^*$			1	1	
$\langle 8,7,4\rangle^*$				1	1
$\langle 8,6,5\rangle^*$					1
	d_1	d_2	d_3	d_4	d_5

The spin characters	The decomposition matrix for the block B_2									
$\langle 18,1\rangle$	1									
$\langle 18,1\rangle'$		1								
$\langle 12,7\rangle$	1		1							
$\langle 12,7\rangle'$		1		1						
$\langle 11,7,1\rangle^*$			1	1	1	1				
$\langle 9,7,2,1\rangle$					1		1			
$\langle 9,7,2,1\rangle'$						1		1		
$\langle 8,7,3,1\rangle$							1		1	
$\langle 8,7,3,1\rangle'$								1		1
$\langle 7,6,5,1\rangle$									1	
$\langle 7,6,5,1\rangle'$										1
	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

The spin characters	The decomposition matrix for the block B_3									
$\langle 17,2\rangle$	1									
$\langle 17,2\rangle'$		1								
$\langle 13,6\rangle$	1		1							
$\langle 13,6\rangle'$		1		1						
$\langle 11,6,2\rangle^*$			1	1	1	1				
$\langle 10,6,2,1\rangle$					1		1			

$\langle 10,6,2,1\rangle'$						1		1		
$\langle 8,6,3,2\rangle$							1		1	
$\langle 8,6,3,2\rangle'$								1		1
$\langle 7,6,4,2\rangle$									1	
$\langle 7,6,4,2\rangle'$										1
	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

The spin characters	The decomposition matrix for the block B_4									
$\langle 16,3\rangle$	1									
$\langle 16,3\rangle'$		1								
$\langle 14,5\rangle$	1		1							
$\langle 14,5\rangle'$		1		1						
$\langle 11,5,3\rangle^*$			1	1	1	1				
$\langle 10,5,3,1\rangle$					1		1			
$\langle 10,5,3,1\rangle'$						1		1		
$\langle 9,5,3,2\rangle$							1		1	
$\langle 9,5,3,2\rangle'$								1		1
$\langle 7,5,4,3\rangle$									1	
$\langle 7,5,4,3\rangle'$										1
	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}

The spin characters	The decomposition matrix for the block B_5				
$\langle 16,2,1\rangle^*$	1				
$\langle 13,5,1\rangle^*$	1	1			
$\langle 12,5,2\rangle^*$		1	1		
$\langle 11,5,2,1\rangle$			1	1	
$\langle 11,5,2,1\rangle'$			1	1	
$\langle 8,5,3,2,1\rangle^*$				1	1
$\langle 7,5,4,2,1\rangle^*$					1
	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

The spin characters	The decomposition matrix for the block B_6				
$\langle 15,3,1\rangle^*$	1				
$\langle 14,4,1\rangle^*$	1	1			
$\langle 12,4,3\rangle^*$		1	1		
$\langle 11,4,3,1\rangle$			1	1	
$\langle 11,4,3,1\rangle'$			1	1	
$\langle 9,4,3,2,1\rangle^*$				1	1
$\langle 6,5,4,3,1\rangle^*$					1
	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}

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