

The Brauer trees of the symmetric groups S_{17} and S_{18} modulo $p = 11$

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Abstract:

In this paper we find the Brauer trees of the symmetric groups S_{17} and S_{18} modulo $p=11$ which can give the irreducible modular spin characters of S_{17} and S_{18} modulo $p=11$, also we give the 11 –decomposition matrix of S_{17}, S_{18} .

Introduction:

Schur showed that the symmetric group S_n has a representation group $\overline{S_n}$ which is of order $2(n!)$, and it has a central subgroup $Z = \{1, -1\}$ such that $\overline{S_n}/Z \cong S_n$ [I.Schur 1911]. The representations of $\overline{S_n}$ fall into two classes [A.O.Morris 1962], [I.Schur 1911]:

- 1) Those which have Z in their kernel; these are called the ordinary representations of S_n , the irreducible representations and characters of S_n are indexed by the partition of n .
- 2) The representations which do not have Z in their kernel; these representation are called spin(projective) representations of S_n the irreducible spin representations are indexed by the partitions of n with distinct parts which are called bar partitions of n [A.O.Morris and A.K.Yassen 1988].

For $p = 11$ Yaseen [A.K.Yassen 1987] was found the modular irreducible spin character of S_n for $11 \leq n \leq 14$ and for $n = 15, 16$ also was found by Yaseen [A.K.Yassen 1995], we use the techniques as given in [G.D.James and A.Kerber 1981], [Lukas Maas 2010], to find the modular irreducible spin character modulo 11 for S_{17} and S_{18} .

The main aim of this paper is to calculate the decomposition matrix for the spin characters for $S_{17}, S_{18}, p = 11$.

Preliminaries: Any spin character of S_n can be written as a linear combination, with non-negative integer coefficients, of the irreducible spin character [L.Dornhoff 1972].

1. The degree of the spin characters $\langle \alpha \rangle = \langle \alpha_1, \dots, \alpha_m \rangle$ is:

$\deg\langle\alpha\rangle = 2^{\lfloor \frac{n-m}{2} \rfloor} \frac{n!}{\prod_{i=1}^m (\alpha_i!)} \prod_{1 \leq i < j \leq m} (\alpha_i - \alpha_j) / (\alpha_i + \alpha_j)$ [A.O.Morris 1962], [A.O.Morris and A.K.Yassen 1988].

2. Let B be the block of defect one and let b the number of P –conjugate characters to the irreducible ordinary character χ of G .

Then [B.M.Puttaswamaiah and J.D.Dixon 1977]:

- a) There exists a positive integer number N such that the irreducible ordinary characters of G are lying in the block B divided into two disjoint classes : $B_1 = \{\chi \in B \mid b \deg \chi \equiv N \pmod{p^a}\}$
 $B_2 = \{\chi \in B \mid b \deg \chi \equiv -N \pmod{p^a}\}$
 - b) Each coefficient of the decomposition matrix of the block B is 1 or 0.
 - c) If α_1 and α_2 are not p –conjugate characters and are belong to the same class (B_1, B_2) above , then they have no irreducible modular character in common .
 - d) For every irreducible ordinary character χ in B_1 , there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with one multiplicity .
3. If C is a principal character of G for an odd prime p and all the entries in C are divisible by a non-negative integer q , then $(1/q)C$ is a principal character of G [G.D.James and A.Kerber 1981].
4. Let p be odd then [A.O.Morris and A.K.Yassen 1988] :
- a) $\langle n \rangle$ and $\langle n \rangle'$ are irreducible modular spin characters of degree $2^{\lfloor n-1/2 \rfloor}$ which are denoted by $\varphi\langle n \rangle$ and $\varphi\langle n \rangle'$ respectively, and $\varphi\langle n \rangle \neq \varphi\langle n \rangle'$. If $p \nmid n$ and n is even, except when n is odd and $p \mid n$ in which case $\langle n \rangle = \varphi\langle n \rangle + \varphi\langle n \rangle'$, where $\varphi\langle n \rangle$ and $\varphi\langle n \rangle'$ are distinct irreducible modular spin characters of degree $2^{\lfloor (n-3)/2 \rfloor}$.
 - b) If n is even and $p \nmid n$ or $p \nmid (n-1)$, then $\langle n-1, 1 \rangle$ is an irreducible modular spin character of degree $2^{\lfloor (n-2)/2 \rfloor} \times (n-2)$ which is denoted by $\varphi\langle n-1, 1 \rangle^*$.
 - c) If n is odd and $p \nmid n$ or $p \nmid n-1$, then $\langle n-1, 1 \rangle$ and $\langle n-1, 1 \rangle'$ are distinct irreducible modular spin characters of degree $2^{\lfloor (n-3)/2 \rfloor} \times (n-2)$ which are denoted by $\varphi\langle n-1, 1 \rangle$ and $\varphi\langle n-1, 1 \rangle'$ respectively.
5. If C is a principal character of G for a prime p , then $\deg C \equiv 0 \pmod{p^A}$, where $o(G) = p^A m$, $(p, m) = 1$ [S.A.Taban 1989], [J.F.Humphreys 1977].
6. Let $\beta_1^*, \beta_2, \beta_2', \beta_3, \beta_3'$ be modular spin characters where β_1^* is a double character, $\beta_2 \neq \beta_2'$ are associate modular spin characters (real), and $\beta_3 \neq \beta_3'$ are associate modular spin characters (complex). Let $\varphi_1^*, \varphi_2, \varphi_2', \varphi_3, \varphi_3'$ be irreducible modular spin characters, where

φ_1^* is a double character, $\varphi_2 \neq \varphi_2'$ and $\varphi_3 \neq \varphi_3'$ are associate irreducible modular spin characters (*real*), (*complex*) respectively then [A.K.Yassen 1987]:

- a) $\beta_1^*, \beta_2, \beta_2'$ contains φ_3 and φ_3' with the same multiplicity, β_1^* which contains φ_2 and φ_2' with the same multiplicity.
- b) β_3 and β_3' contains $\varphi_1^*, \varphi_2, \varphi_2'$ with the same multiplicity.
- c) φ_3 is a constituent of β_3 with the same multiplicity as that of φ_3' in β_3' .

7. If the decomposition matrix $D_{n-1,p} = (d_{ij})$ for S_{n-1} is known, then we can induce columns $(\psi_j \uparrow^{(r,\bar{r})} S_n)$ for $S_n[4]$, these columns are a linear combination with non-negative coefficients from the columns of $D_{n,p}$ [G.D.James and A.Kerber 1981].

Notation:

p.s.	principle spin character.
p.i.s.	principle indecomposable spin character.
m.s.	modular spin character.
i.m.s.	irreducible modular spin character.
$\langle \lambda \rangle^{no}$	(no) mean the number of i.m.s. in $\langle \lambda \rangle$
$\equiv$	Equivalence mod 11.

Section (1)

The decomposition matrix for S_{17} modulo $p=11$ of degree (57, 51) [A.O.Morris 1962], [A.O.Morris and A.K.Yassen 1988]. There are 25 blocks four of them B_1, B_2, B_3, B_4 , are of defect one, the others $B_5, \dots, B_{25}$ of defect zero.

Lemma (1.1) The Brauer tree for the block B_1 is:

$$\langle 17 \rangle^* \text{---} \langle 11,6 \rangle = \langle 11,6 \rangle' \text{---} \langle 10,6,1 \rangle^* \text{---} \langle 9,6,2 \rangle^* \text{---} \langle 8,6,3 \rangle^* \text{---} \langle 7,6,4 \rangle^*$$

Proof:

$$a) \deg \langle 9,6,2 \rangle^* \equiv \deg \langle 7,6,4 \rangle^* \equiv \deg (\langle 11,6 \rangle' + \langle 11,6 \rangle) \equiv 8.$$

$$\deg \langle 17 \rangle^* \equiv \deg \langle 10,6,1 \rangle^* \equiv \deg \langle 8,6,3 \rangle^* \equiv -8$$

b) By using (6,6)-inducing of p.i.s for S_{16} (see appendix I) to S_{17} we get on:

$$D_1 \uparrow^{(6,6)} S_{17} = \langle 17 \rangle + \langle 11,6 \rangle + \langle 11,6 \rangle'$$

$$D_3 \uparrow^{(6,6)} S_{17} = \langle 11,6 \rangle + \langle 11,6 \rangle' + \langle 10,6,1 \rangle^*$$

$$D_5 \uparrow^{(6,6)} S_{17} = \langle 10,6,1 \rangle^* + \langle 9,6,2 \rangle^*$$

$$D_7 \uparrow^{(6,6)} S_{17} = \langle 9,6,2 \rangle^* + \langle 8,6,3 \rangle^*$$

$$D_9 \uparrow^{(6,6)} S_{17} = \langle 8,6,3 \rangle^* + \langle 7,6,4 \rangle^*$$

on 11-regular classes we have

$$1. \langle 11,6 \rangle = \langle 11,6 \rangle'$$

2. $\langle 11,6,1 \rangle = \langle 17 \rangle^* + \langle 10,6,1 \rangle^* + \langle 8,6,3 \rangle^* - \langle 9,6,2 \rangle^* - \langle 7,6,4 \rangle^*$. Hence, we have the Brauer tree for this block B_1 . ■

Lemma(1.2)

The Brauer tree for the block B_2 is:

$$\begin{array}{ccc} \langle 16,1 \rangle - \langle 12,5 \rangle & \backslash & \langle 11,5,1 \rangle^* / \langle 9,5,2,1 \rangle - \langle 8,5,3,1 \rangle - \langle 7,5,4,1 \rangle \\ \langle 16,1 \rangle' - \langle 12,5 \rangle' & / & \langle 9,5,2,1 \rangle' - \langle 8,5,3,1 \rangle' - \langle 7,5,4,1 \rangle' \end{array}$$

Proof:

- a) $\deg\{\langle 16,1 \rangle, \langle 16,1 \rangle', \langle 11,5,1 \rangle^*, \langle 8,5,3,1 \rangle, \langle 8,5,3,1 \rangle'\} \equiv 6$
 $\deg\{\langle 12,5 \rangle, \langle 12,5 \rangle', \langle 9,5,2,1 \rangle, \langle 9,5,2,1 \rangle', \langle 7,5,4,1 \rangle, \langle 7,5,4,1 \rangle'\} \equiv -6.$
 b) By using (1,0)-inducing of p.i.s for S_{16} (see appendix I) to S_{17} we get p.i.s.
 $D_5 \uparrow^{(1,0)} S_{17}, D_6 \uparrow^{(1,0)} S_{17}, D_7 \uparrow^{(1,0)} S_{17}, D_8 \uparrow^{(1,0)} S_{17}, D_9 \uparrow^{(1,0)} S_{17}, D_{10} \uparrow^{(1,0)} S_{17}$, and
 $D_{11} \uparrow^{(1,0)} S_{17} = k_1$, p.s
 $D_1 \uparrow^{(1,0)} S_{17} = k_2, D_2 \uparrow^{(1,0)} S_{17} = k_3, D_3 \uparrow^{(1,0)} S_{17} = k_4.$

So we have the approximation matrix (Table(1))

Table (1)

$\langle 16,1 \rangle$	1	1								
$\langle 16,1 \rangle'$	1		1							
$\langle 12,5 \rangle$	1	1	1	1						
$\langle 12,5 \rangle'$	1	1	1	1						
$\langle 11,5,1 \rangle^*$		1	1	2	1	1				
$\langle 9,5,2,1 \rangle$					1		1			
$\langle 9,5,2,1 \rangle'$						1		1		
$\langle 8,5,3,1 \rangle$							1		1	
$\langle 8,5,3,1 \rangle'$								1		1
$\langle 7,5,4,1 \rangle$									1	
$\langle 7,5,4,1 \rangle'$										1
	k_1	k_2	k_3	k_4	c_5	c_6	c_7	c_8	c_9	c_{10}

$\langle 16,1 \rangle \neq \langle 16,1 \rangle'$ so k_1 splits to c_1 and c_2 .

$k_4 = k_2 + k_3 - c_1 - c_2$, this give p.i.s. $k_2 - c_1, k_3 - c_2$.

Hence we have the Brauer tree for this block B_2 . ■

Lemma(1.3)

The Brauer tree for the block B_4 is:

$$\langle 14,2,1 \rangle^* _ \langle 13,3,1 \rangle^* _ \langle 12,3,2 \rangle^* _ \langle 11,3,2,1 \rangle = \langle 11,3,2,1 \rangle' _ \langle 7,4,3,2,1 \rangle^* _ \langle 6,5,3,2,1 \rangle^*$$

Proof:

$$a) \deg\langle 13,3,1 \rangle^* \equiv \deg\langle 6,5,3,2,1 \rangle^* \equiv \deg\langle 11,3,2,1 \rangle + \langle 11,3,2,1 \rangle' \equiv 8$$

$$\deg\langle 14,2,1 \rangle^* \equiv \deg\langle 12,3,2 \rangle^* \equiv \deg\langle 7,4,3,2,1 \rangle^* \equiv -8$$

b) By using $(r, \bar{r})$ -inducing of p.i.s. $D_{16}, D_{18}, D_{19}, D_{20}, \langle 12,3,1 \rangle$, for S_{16} (see appendix I) to S_{17} we get on.

$$1) \langle 14,2,1 \rangle^* + \langle 13,3,1 \rangle^*.$$

$$2) 2\langle 12,3,2 \rangle^* + 2\langle 11,3,2,1 \rangle + 2\langle 11,3,2,1 \rangle' = 2k \text{ so } k \text{ is p.i.}$$

$$3) \langle 11,3,2,1 \rangle + \langle 11,3,2,1 \rangle' + \langle 7,4,3,2,1 \rangle^*.$$

$$4) \langle 7,4,3,2,1 \rangle^* + \langle 6,5,3,2,1 \rangle^*.$$

$$5) \langle 13,3,1 \rangle^* + \langle 12,3,2 \rangle^*.$$

on 11-regular classes we have

$$1. \langle 11,3,2,1 \rangle = \langle 11,3,2,1 \rangle'$$

$$2. \langle 11,3,2,1 \rangle = \langle 14,2,1 \rangle^* + \langle 12,3,2 \rangle^* + \langle 7,4,3,2,1 \rangle^* - \langle 13,3,1 \rangle^* - \langle 6,5,3,2,1 \rangle^*$$

Then, we get the Brauer tree for the block B_4 ■.

Lemma(1.4)

The Brauer tree for the block B_3 is:

$$\begin{array}{ccc} \langle 15,2 \rangle - \langle 13,4 \rangle & \backslash & \langle 11,4,2 \rangle^* \\ \langle 15,2 \rangle' - \langle 13,4 \rangle' & / & \end{array} \begin{array}{c} \langle 10,4,2,1 \rangle - \langle 8,4,3,2 \rangle - \langle 6,5,4,2 \rangle \\ \langle 10,4,2,1 \rangle' - \langle 8,4,3,2 \rangle' - \langle 6,5,4,2 \rangle' \end{array}$$

Proof:

$$a) \deg\{\langle 13,4 \rangle, \langle 13,4 \rangle', \langle 10,4,2,1 \rangle, \langle 10,4,2,1 \rangle', \langle 6,5,4,2 \rangle, \langle 6,5,4,2 \rangle'\} \equiv 9$$

$$\deg\{\langle 15,2 \rangle, \langle 15,2 \rangle', \langle 11,4,2 \rangle^*, \langle 8,4,3,2 \rangle, \langle 8,4,3,2 \rangle'\} \equiv -9.$$

b) By using $(r, \bar{r})$ -inducing of p.i.s for S_{16} (see appendix I) to S_{17} we get on:

$$D_{11} \uparrow^{(2,10)} S_{17} = k_1, D_{12} \uparrow^{(2,10)} S_{17} = k_2, D_{14} \uparrow^{(2,10)} S_{17} = k_3$$

$$D_{15} \uparrow^{(2,10)} S_{17} = k_4, \langle 10,4,2 \rangle \uparrow^{(0,1)} S_{17} = c_5, \langle 10,4,2 \rangle' \uparrow^{(0,1)} S_{17} = c_6.$$

Thus, we have the approximation matrix (Table (2))

Table(2)

	Ψ_1	Ψ_2	φ_5	φ_6	Ψ_3	Ψ_4	φ_1	φ_2
$\langle 15,2 \rangle$	1						a	
$\langle 15,2 \rangle'$	1							a
$\langle 13,4 \rangle$	1	1					b	
$\langle 13,4 \rangle'$	1	1						b
$\langle 11,4,2 \rangle^*$		2	1	1			c	c
$\langle 10,4,2,1 \rangle$			1		1		d	

$\langle 10,4,2,1 \rangle'$				1	1			d
$\langle 8,4,3,2 \rangle$					1	1	f	
$\langle 8,4,3,2 \rangle'$					1	1		f
$\langle 6,5,4,2 \rangle$						1	h	
$\langle 6,5,4,2 \rangle'$						1		h
	k_1	k_2	c_5	c_6	k_3	k_4	Y_1	Y_2

Since $\langle 15,2 \rangle \neq \langle 15,2 \rangle'$ on $(11, \alpha)$ -regular classes then either k_1 is split or there are another two columns. Suppose there are two columns such as Y_1 and Y_2

To describe columns Y_1 and Y_2

1. $\langle 15,2 \rangle \downarrow S_{16} = (\langle 14,2 \rangle^*)^1 + (\langle 15,1 \rangle^*)^1 = 2$ of i.m.s (see appendix I) and form (Table(2)) we have $a \in \{0,1\}$.
2. $\langle 13,4 \rangle \downarrow S_{16} = (\langle 12,4 \rangle^*)^2 + (\langle 13,3 \rangle^*)^2 = 4$ of i.m.s. we have $b \in \{0,1,2\}$.
3. $\langle 11,4,2 \rangle^* \downarrow S_{16} = (\langle 10,4,2 \rangle)^1 + (\langle 10,4,2 \rangle')^1 + (\langle 11,3,2 \rangle)^2 + (\langle 11,3,2 \rangle')^2 + (\langle 11,4,1 \rangle)^2 + (\langle 11,4,1 \rangle')^2 = 10$ of i.m.s. we have $c \in \{0,1,2,3,4,5,6\}$.
4. $\langle 10,4,2,1 \rangle \downarrow S_{16} = (\langle 9,4,2,1 \rangle^*)^2 + (\langle 10,3,2,1 \rangle^*)^2 + (\langle 10,4,2 \rangle)^1 = 5$ of i.m.s. we have $d \in \{0,1,2,3\}$.
5. $\langle 8,4,3,2 \rangle \downarrow S_{16} = (\langle 7,4,3,2 \rangle^*)^2 + (\langle 8,4,3,1 \rangle^*)^2 = 4$ of i.m.s. we have $f \in \{0,1,2\}$
6. $\langle 6,5,4,2 \rangle \downarrow S_{16} = (\langle 6,5,3,2 \rangle^*)^1 + (\langle 6,5,4,1 \rangle^*)^1 = 2$ of i.m.s. we have $h \in \{0,1\}$.

If $a = 0$ then k_1 splits to give $\langle 15,2 \rangle + \langle 13,4 \rangle$ and $\langle 15,2 \rangle' + \langle 13,4 \rangle'$

If $a = 1$:

- 1) Since $\langle 15,2 \rangle \downarrow S_{16} \cap \langle 13,4 \rangle \downarrow S_{16} = 2$ of i.m.s for S_{16}
 $\langle 15,2 \rangle \cap \langle 13,4 \rangle = \Psi_1 + \varphi_1$ if $b \in \{1,2\}$
 $= \varphi_1$ if $b = 0$;
- 2) There is no i.m.s. in $\langle 15,2 \rangle \downarrow S_{16} \cap \langle 11,4,2 \rangle^* \downarrow S_{16}$, then $c = 0$;
- 3) There is no i.m.s. in $\langle 15,2 \rangle \downarrow S_{16} \cap \langle 10,4,2,1 \rangle \downarrow S_{16}$, then $d = 0$;
- 4) There is no i.m.s. in $\langle 15,2 \rangle \downarrow S_{16} \cap \langle 8,4,3,2 \rangle \downarrow S_{16}$, then $f = 0$;
- 5) There is no i.m.s. in $\langle 15,2 \rangle \downarrow S_{16} \cap \langle 6,5,4,2 \rangle \downarrow S_{16}$, then $h = 0$.

We, now, get the possible columns

$$Y_1 = \langle 15,2 \rangle + b \langle 13,4 \rangle,$$

$$Y_2 = \langle 15,2 \rangle' + b \langle 13,4 \rangle', \quad b \in \{0,1,2\}$$

$$\deg Y_1 \equiv 0 \quad \text{and} \quad \deg Y_2 \equiv 0 \quad \text{only when } b = 1$$

then k_1 splits to $\langle 15,2 \rangle + \langle 13,4 \rangle$, and $\langle 15,2 \rangle' + \langle 13,4 \rangle'$

Since $\langle 13,4 \rangle \neq \langle 13,4 \rangle'$ on $(11, \alpha)$ then either k_2 is splits or there are two columns . If we suppose there are another two columns such as Y_1 and Y_2 (as in Table (2) with $a = 0$)

To describe these two columns:

Since $\langle 15,2 \rangle \downarrow$ is two i.m.s. then $b \in \{0,1\}$, now if $b = 1$ we have :

- 1) Since $\langle 13,4 \rangle \downarrow S_{16} \cap \langle 11,4,2 \rangle^* \downarrow S_{16} = 2$ of i.m.s for S_{16}
 $\langle 13,4 \rangle \cap \langle 11,4,2 \rangle^* = \Psi_2 + \varphi_3$ if $c \in \{1,2,3,4,5,6\}$
 $= \Psi_2$ if $c = 0$;
- 2) There is no i.m.s. in $\langle 13,4 \rangle \downarrow S_{16} \cap \langle 10,4,2,1 \rangle \downarrow S_{16}$, then $d = 0$;
- 3) There is no i.m.s. in $\langle 13,4 \rangle \downarrow S_{16} \cap \langle 8,4,3,2 \rangle \downarrow S_{16}$, then $f = 0$;
- 4) There is no i.m.s. in $\langle 13,4 \rangle \downarrow S_{16} \cap \langle 6,5,4,2 \rangle \downarrow S_{16}$, then $h = 0$.

We, get the possible columns

$$Y_1 = \langle 13,4 \rangle + c \langle 11,4,2 \rangle^* ,$$

$$Y_2 = \langle 13,4 \rangle' + c \langle 11,4,2 \rangle^* , c \in \{0,1,2,3,4,5,6\}$$

$$\deg Y_1 \equiv 0 \text{ and } \deg Y_2 \equiv 0 \text{ only when } c = 1$$

So k_2 splits to give $\langle 13,4 \rangle + \langle 11,4,2 \rangle^*$ and $\langle 13,4 \rangle' + \langle 11,4,2 \rangle^*$ which is the same when $b = 0$.

Since $\langle 6,5,4,2 \rangle \neq \langle 6,5,4,2 \rangle'$ on $(11, \alpha)$ then k_4 splits or there are two columns. If we suppose there are another two columns such as Y_1 and Y_2 (as in Table (2) with $= 0, b = 0$).

To describe Y_1 and Y_2 :

If $h = 1$:

- 1) There is no i.m.s. in $\langle 6,5,4,2 \rangle \downarrow S_{16} \cap \langle 11,4,2 \rangle^* \downarrow S_{16}$, then $c = 0$;
- 2) There is no i.m.s. in $\langle 6,5,4,2 \rangle \downarrow S_{16} \cap \langle 10,4,2,1 \rangle \downarrow S_{16}$, then $d = 0$;
- 3) Since $\langle 6,5,4,2 \rangle \downarrow S_{16} \cap \langle 8,4,3,2 \rangle \downarrow S_{16} = 2$ of i.m.s for S_{16}
 $\langle 6,5,4,2 \rangle \cap \langle 8,4,3,2 \rangle = \Psi_4 + \varphi_9$ if $f \in \{1,2\}$
 $= \Psi_4$ if $f = 0$.

We, get the possible columns

$$Y_1 = f \langle 8,4,3,2 \rangle + \langle 6,5,4,2 \rangle ,$$

$$Y_2 = f \langle 8,4,3,2 \rangle' + \langle 6,5,4,2 \rangle' , f \in \{0,1,2\}$$

$$\deg Y_1 \equiv 0 \text{ and } \deg Y_2 \equiv 0 \text{ only when } f = 1$$

So, k_4 splits to $\langle 8,4,3,2 \rangle + \langle 6,5,4,2 \rangle$ and $\langle 8,4,3,2 \rangle' + \langle 6,5,4,2 \rangle'$ which is the same when $h = 0$.

Now, since $\langle 8,4,3,2 \rangle \neq \langle 8,4,3,2 \rangle'$ on $(11, \alpha)$ -regular classes and we have 9 columns, then k_3 must be a split to $\langle 10,4,2,1 \rangle + \langle 8,4,3,2 \rangle$ and $\langle 10,4,2,1 \rangle' + \langle 8,4,3,2 \rangle'$.

So we get the Brauer tree for the block B_3 ■.

From lemmas above we can find the 11-decomposition matrix for the spin characters of S_{17} . We write this decomposition matrix in appendix II

Section(2)

The decomposition matrix for S_{18} modulo $p=11$ of degree (69, 61) [A.O.Morris 1962],[A.O.Morris and A.K.Yassen 1988].

There are 31 blocks $B_1, \dots, B_5$, of defect one and the others blocks of defect zero.

Lemma(2.1)

The Brauer tree for the block B_2 is:

$$\langle 17,1 \rangle^* _ \langle 12,6 \rangle^* _ \langle 11,6,1 \rangle = \langle 11,6,1 \rangle' _ \langle 9,6,2,1 \rangle^* _ \langle 8,6,3,1 \rangle^* _ \langle 7,6,4,1 \rangle^*$$

Proof:

$$a) \deg \langle 17,1 \rangle^* \equiv \deg \langle 8,6,3,1 \rangle^* \equiv \deg(\langle 11,6,1 \rangle + \langle 11,6,1 \rangle') \equiv 4$$

$$\deg \langle 12,6 \rangle^* \equiv \deg \langle 9,6,2,1 \rangle^* \equiv \deg \langle 7,6,4,1 \rangle^* \equiv -4$$

b) The p.i.s. for S_{18} :

$$d_2 \uparrow^{(1,0)} S_{18}, d_3 \uparrow^{(1,0)} S_{18}, d_4 \uparrow^{(1,0)} S_{18}, d_5 \uparrow^{(1,0)} S_{18}, d_6 \uparrow^{(6,6)} S_{18}$$

With the relations on 11-regular classes

$$1. \langle 11,6,1 \rangle = \langle 11,6,1 \rangle'$$

$$2. \langle 11,6,1 \rangle = \langle 12,6 \rangle^* + \langle 9,6,2,1 \rangle^* + \langle 7,6,4,1 \rangle^* - \langle 17,1 \rangle^* - \langle 8,6,3,1 \rangle^*$$

We have the Brauer tree for this block B_2 ■ .

Lemma(2.2)

The Brauer tree for the block B_3 is:

$$\langle 16,2 \rangle^* _ \langle 13,5 \rangle^* _ \langle 11,5,2 \rangle = \langle 11,5,2 \rangle' _ \langle 10,5,2,1 \rangle^* _ \langle 8,5,3,2 \rangle^* _ \langle 7,5,4,2 \rangle^*$$

Proof:

$$a) \deg \langle 13,5 \rangle^* \equiv \deg \langle 10,5,2,1 \rangle^* \equiv \deg \langle 7,5,4,2 \rangle^* \equiv 6$$

$$\deg \langle 16,2 \rangle^* \equiv \deg \langle 8,5,3,2 \rangle^* \equiv \deg(\langle 11,5,2 \rangle + \langle 11,5,2 \rangle') \equiv -6$$

b) The p.i.s. for S_{18} :

$$d_6 \uparrow^{(2,10)} S_{18}, d_8 \uparrow^{(2,10)} S_{18}, d_{10} \uparrow^{(2,10)} S_{18},$$

$$d_{12} \uparrow^{(2,10)} S_{18}, d_{14} \uparrow^{(2,10)} S_{18},$$

with the relations on 11-regular classes

$$1. \langle 11,5,2 \rangle = \langle 11,5,2 \rangle'$$

$$2. \langle 11,5,2 \rangle = \langle 13,5 \rangle^* + \langle 10,5,2,1 \rangle^* + \langle 7,5,4,2 \rangle^* - \langle 16,2 \rangle^* - \langle 8,5,3,2 \rangle^*$$

We have the Brauer tree for this block B_3 ■ .

Lemma(2.3)

The Brauer tree for the block B_4 is :

$$\langle 15,3 \rangle^* _ \langle 14,4 \rangle^* _ \langle 11,4,3 \rangle = \langle 11,4,3 \rangle' _ \langle 10,4,3,1 \rangle^* _ \langle 9,4,3,2 \rangle^* _ \langle 6,5,4,3 \rangle^*$$

Proof:

$$a) \deg \langle 15,3 \rangle^* \equiv \deg(\langle 11,4,3 \rangle + \langle 11,4,3 \rangle') \equiv \deg \langle 9,4,3,2 \rangle^* \equiv 4$$

$$\deg \langle 14,4 \rangle^* \equiv \deg \langle 10,4,3,1 \rangle^* \equiv \deg \langle 6,5,4,3 \rangle^* \equiv -4$$

b) The p.i.s. for S_{18} :

$$d_{16} \uparrow^{(3,9)} S_{18}, d_{18} \uparrow^{(3,9)} S_{18}, d_{20} \uparrow^{(3,9)} S_{18}, d_{22} \uparrow^{(3,9)} S_{18}, d_{24} \uparrow^{(3,9)} S_{18}$$

With the relations on 11-regular classes

1. $\langle 11,4,3 \rangle = \langle 11,4,3 \rangle'$
2. $\langle 11,4,3 \rangle = \langle 14,4 \rangle^* + \langle 10,4,3,1 \rangle^* + \langle 6,5,4,3 \rangle^* - \langle 15,3 \rangle^* - \langle 9,4,3,2 \rangle^*$

We have the Brauer tree for this block B_4 ■ .

Lemma(2.4)

The Brauer tree for the block B_5 is :

$$\begin{array}{ccc} \langle 15,2,1 \rangle - \langle 13,4,1 \rangle - \langle 12,4,2 \rangle & \searrow & \langle 11,4,2,1 \rangle^* \\ \langle 15,2,1 \rangle' - \langle 13,4,1 \rangle' - \langle 12,4,2 \rangle' & \swarrow & \end{array} \begin{array}{c} \langle 8,4,3,2,1 \rangle - \langle 6,5,4,2,1 \rangle \\ \langle 8,4,3,2,1 \rangle' - \langle 6,5,4,2,1 \rangle' \end{array}$$

Proof:

- a) $\deg\{\langle 13,4,1 \rangle, \langle 13,4,1 \rangle', \langle 11,4,2,1 \rangle^*, \langle 6,5,4,2,1 \rangle, \langle 6,5,4,2,1 \rangle'\} \equiv 6$
 $\deg\{\langle 15,2,1 \rangle, \langle 15,2,1 \rangle', \langle 12,4,2 \rangle, \langle 12,4,2 \rangle', \langle 8,4,3,2,1 \rangle, \langle 8,4,3,2,1 \rangle'\} \equiv -6$

b) By using $(r, \bar{r})$ -inducing of p.i.s for S_{17} to S_{18} we get on p.i.s:

$$d_{16} \uparrow^{(1,0)} S_{18}, d_{17} \uparrow^{(1,0)} S_{18}, d_{22} \uparrow^{(1,0)} S_{18},$$

$$d_{23} \uparrow^{(1,0)} S_{18}, d_{24} \uparrow^{(1,0)} S_{18}, d_{25} \uparrow^{(1,0)} S_{18}$$

and p.s.

$$d_{27} \uparrow^{(4,8)} S_{18} = k_1, d_{18} \uparrow^{(1,0)} S_{18} = k_2, d_{19} \uparrow^{(1,0)} S_{18} = k_3,$$

Since $\langle 12,4,2,1 \rangle$ and $\langle 12,4,2,1 \rangle'$ are p.i.s. of S_{19} (of defect 0 in $S_{19}, p = 11$) and:

$$\langle 12,4,2,1 \rangle \downarrow_{(1,0)} S_{18} = \langle 12,4,2 \rangle + \langle 11,4,2,1 \rangle^* = m_1$$

$$\langle 12,4,2,1 \rangle' \downarrow_{(1,0)} S_{18} = \langle 12,4,2 \rangle' + \langle 11,4,2,1 \rangle^* = m_2$$

Then, k_4 must be a split to m_1 and m_2

Now since $k_1 = k_2 + k_3 - m_1 - m_2$, either $(k_2 - m_2 \text{ and } k_3 - m_1)$ or

$(k_3 - m_2 \text{ and } k_2 - m_1)$ are p.s.

Let $c_3 = k_2 - m_2, c_4 = k_3 - m_1$

Hence, we have the Brauer tree for this block B_5 ■ .

Lemma(2.5)

The Brauer tree for the block B_1 is:

$$\begin{array}{ccc} \langle 18 \rangle & \searrow & \langle 10,7,1 \rangle - \langle 9,7,2 \rangle - \langle 8,7,3 \rangle - \langle 7,6,5 \rangle \\ \langle 18 \rangle' & \swarrow & \end{array} \begin{array}{c} \langle 11,7 \rangle^* \\ \langle 10,7,1 \rangle' - \langle 9,7,2 \rangle' - \langle 8,7,3 \rangle' - \langle 7,6,5 \rangle' \end{array}$$

Proof:

- a) $\deg\{\langle 11,7 \rangle^*, \langle 9,7,2 \rangle, \langle 9,7,2 \rangle', \langle 7,6,5 \rangle, \langle 7,6,5 \rangle'\} \equiv 8$

$$\deg\{\langle 18 \rangle, \langle 18 \rangle', \langle 10, 7, 1 \rangle, \langle 10, 7, 1 \rangle', \langle 8, 7, 3 \rangle, \langle 8, 7, 3 \rangle'\} \equiv -8.$$

b) By using $(r, \bar{r})$ -inducing of p.i.s for S_{17} (appendix II) to S_{18} :

$$d_1 \uparrow^{(7,5)} S_{18} = k_1, d_3 \uparrow^{(7,5)} S_{18} = k_2, d_4 \uparrow^{(7,5)} S_{18} = k_3, d_5 \uparrow^{(7,5)} S_{18} = k_4,$$

$$\langle 10, 7 \rangle \uparrow^{(1,0)} S_{18} = c_3, \langle 10, 7 \rangle' \uparrow^{(1,0)} S_{18} = c_4$$

k_1 must be split to c_1 and c_2 . Since $\langle 7, 6, 5 \rangle \neq \langle 7, 6, 5 \rangle'$ on $(11, \alpha)$ and $\langle 7, 6, 5 \rangle \downarrow S_{17} = (\langle 7, 6, 4 \rangle^*)^1$ is i.m.s in S_{17} then k_4 splits to c_9 and c_{10} .

we get the matrix (Table (3))

Table (3)

	φ_1	φ_2	φ_3	φ_4	Ψ_1	Ψ_2	φ_9	φ_{10}	φ_7	φ_8
$\langle 18 \rangle$	1									
$\langle 18 \rangle'$		1								
$\langle 11, 7 \rangle^*$	1	1	1	1					b	b
$\langle 10, 7, 1 \rangle$			1		1				c	
$\langle 10, 7, 1 \rangle'$				1	1					c
$\langle 9, 7, 2 \rangle$					1	1			d	
$\langle 9, 7, 2 \rangle'$					1	1				d
$\langle 8, 7, 3 \rangle$						1	1		f	
$\langle 8, 7, 3 \rangle'$						1		1		f
$\langle 7, 6, 5 \rangle$							1			
$\langle 7, 6, 5 \rangle'$								1		
	c_1	c_2	c_3	c_4	k_2	k_3	c_9	c_{10}	Y_1	Y_2

Since $\langle 8, 7, 3 \rangle \neq \langle 8, 7, 3 \rangle'$ on $(11, \alpha)$ -regular classes, then either k_3 is splits or there are two columns. If we suppose there are two columns Y_1 and Y_2 (as in Table (3)).

We, now, describe the columns Y_1 and Y_2

1. $\langle 11, 7 \rangle^* \downarrow S_{17} = (\langle 10, 7 \rangle)^1 + (\langle 10, 7 \rangle')^1 + (\langle 11, 6 \rangle)^2 + (\langle 11, 6 \rangle')^2 = 6$ of i.m.s.
(see appendix II) and from (Table(3)) we have $b \in \{0, 1, 2\}$
2. $\langle 10, 7, 1 \rangle \downarrow S_{17} = (\langle 9, 7, 1 \rangle^*)^1 + (\langle 10, 6, 1 \rangle^*)^2 + (\langle 10, 7 \rangle)^1 = 4$ of i.m.s. so $c \in \{0, 1, 2\}$.
3. $\langle 9, 7, 2 \rangle \downarrow S_{17} = (\langle 8, 7, 2 \rangle^*)^1 + (\langle 9, 6, 2 \rangle^*)^2 + (\langle 9, 7, 1 \rangle^*)^1 = 4$ of i.m.s. so $d \in \{0, 1, 2\}$.
4. $\langle 8, 7, 3 \rangle \downarrow S_{17} = (\langle 8, 6, 3 \rangle^*)^2 + (\langle 8, 7, 2 \rangle^*)^1 = 3$ of i.m.s.
so $f \in \{0, 1\}$

If $f = 1$:

- 1) There is no i.m.s. in $\langle 8, 7, 3 \rangle \downarrow S_{17} \cap \langle 11, 7 \rangle^* \downarrow S_{17}$, then $b = 0$.

2) There is no i.m.s. in $\langle 8,7,3 \rangle \downarrow S_{17} \cap \langle 10,7,1 \rangle \downarrow S_{17}$, then $c = 0$

3) Since $\langle 8,7,3 \rangle \downarrow S_{17} \cap \langle 9,7,2 \rangle \downarrow S_{17} = 2$ of i.m.s. and

$$\langle 8,7,3 \rangle \cap \langle 9,7,2 \rangle = \Psi_2 + \varphi_7 \text{ if } d \in \{1,2\}$$

$$= \varphi_7 \text{ if } d = 0$$

So the possible columns

$$Y_1 = d \langle 9,7,2 \rangle + \langle 8,7,3 \rangle,$$

$$Y_2 = d \langle 9,7,2 \rangle' + \langle 8,7,3 \rangle', d \in \{0,1,2\}$$

$\deg Y_1 \equiv 0$ and $\deg Y_2 \equiv 0$ only when $d = 1$, so k_3 splits to give $\langle 9,7,2 \rangle + \langle 8,7,3 \rangle$, and $\langle 9,7,2 \rangle' + \langle 8,7,3 \rangle'$, which is the same when $f = 0$.

Now, since $\langle 9,7,2 \rangle \neq \langle 9,7,2 \rangle'$ on $(11, \alpha)$ -regular classes and we have 9 columns; then, k_2 must be a splits to $\langle 10,7,1 \rangle + \langle 9,7,2 \rangle$, and $\langle 10,7,1 \rangle' + \langle 9,7,2 \rangle'$. Hence, we have the Brauer tree for this block B_1 ■.

From lemmas above, we can find the 11-decomposition matrix for the spin characters of S_{18} . We write this decomposition matrix in appendix III

Appendix I

The decomposition matrix for the spin characters of S_{16} , $p = 11$

The spin characters	The decomposition matrix for the block B_1									
$\langle 16 \rangle$	1									
$\langle 16 \rangle'$		1								
$\langle 11,5 \rangle^*$	1	1	1	1						
$\langle 10,5,1 \rangle$			1		1					
$\langle 10,5,1 \rangle'$				1		1				
$\langle 9,5,2 \rangle$					1		1			
$\langle 9,5,2 \rangle'$						1		1		
$\langle 8,5,3 \rangle$							1		1	
$\langle 8,5,3 \rangle'$								1		1
$\langle 7,5,4 \rangle$									1	
$\langle 7,5,4 \rangle'$										1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}

The spin characters	The decomposition matrix for the block B_2				
$\langle 15,1 \rangle^*$	1				
$\langle 12,4 \rangle^*$	1	1			
$\langle 11,4,1 \rangle$		1	1		
$\langle 11,4,1 \rangle'$		1	1		
$\langle 9,4,2,1 \rangle^*$			1	1	
$\langle 8,4,3,1 \rangle^*$				1	1
$\langle 6,5,4,1 \rangle^*$					1
	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}

The spin characters	The decomposition matrix for the block B_3				
$\langle 14,2 \rangle^*$	1				
$\langle 13,3 \rangle^*$	1	1			
$\langle 11,3,2 \rangle$		1	1		
$\langle 11,3,2 \rangle'$		1	1		
$\langle 10,3,2,1 \rangle^*$			1	1	
$\langle 7,4,3,2 \rangle^*$				1	1
$\langle 6,5,3,2 \rangle^*$					1
	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}

Appendix II

The decomposition matrix for the spin characters of S_{17} , $p = 11$

The spin characters	The decomposition matrix for the block B_1				
	1				
	1	1			
	1	1			
		1	1		
			1	1	
				1	1
					1
	d_1	d_2	d_3	d_4	d_5

The spin characters	The decomposition matrix for the block B_2									
$\langle 16,1 \rangle$	1									
$\langle 16,1 \rangle'$		1								
$\langle 12,5 \rangle$	1		1							
$\langle 12,5 \rangle'$		1		1						
$\langle 11,5,1 \rangle^*$			1	1	1	1				
$\langle 9,5,2,1 \rangle$					1		1			
$\langle 9,5,2,1 \rangle'$						1		1		
$\langle 8,5,3,1 \rangle$							1		1	
$\langle 8,5,3,1 \rangle'$								1		1
$\langle 7,5,4,1 \rangle$									1	
$\langle 7,5,4,1 \rangle'$										1
	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

The spin characters	The decomposition matrix for the block B_3									
$\langle 15,2 \rangle$	1									
$\langle 15,2 \rangle'$		1								
$\langle 13,4 \rangle$	1		1							
$\langle 13,4 \rangle'$		1		1						
$\langle 11,4,2 \rangle^*$			1	1	1	1				
$\langle 10,4,2,1 \rangle$					1		1			
$\langle 10,4,2,1 \rangle'$						1		1		
$\langle 8,4,3,2 \rangle$							1		1	
$\langle 8,4,3,2 \rangle'$								1		1
$\langle 6,5,4,2 \rangle$									1	
$\langle 6,5,4,2 \rangle'$										1
	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

The spin characters	The decomposition matrix for the block B_4				
$\langle 14,2,1 \rangle^*$	1				
$\langle 13,3,1 \rangle^*$	1	1			
$\langle 12,3,2 \rangle^*$		1	1		
$\langle 11,3,2,1 \rangle$			1	1	
$\langle 11,3,2,1 \rangle'$			1	1	
$\langle 7,4,3,2,1 \rangle^*$				1	1
$\langle 6,5,3,2,1 \rangle^*$					1
	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

Appendix III

The decomposition matrix for the spin characters of $S_{18}, p = 11$

The spin characters	The decomposition matrix for the block B_1									
$\langle 18 \rangle$	1									
$\langle 18 \rangle'$		1								
$\langle 11,7 \rangle^*$	1	1	1	1						
$\langle 10,7,1 \rangle$			1		1					
$\langle 10,7,1 \rangle'$				1		1				
$\langle 9,7,2 \rangle$					1		1			
$\langle 9,7,2 \rangle'$						1		1		
$\langle 8,7,3 \rangle$							1		1	
$\langle 8,7,3 \rangle'$								1		1
$\langle 7,6,5 \rangle$									1	
$\langle 7,6,5 \rangle'$										1
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

The spin characters	The decomposition matrix for the block B_2				
$\langle 17,1 \rangle^*$	1				
$\langle 12,6 \rangle^*$	1	1			
$\langle 11,6,1 \rangle$		1	1		
$\langle 11,6,1 \rangle'$		1	1		
			1	1	
$\langle 8,6,3,1 \rangle^*$				1	1
$\langle 7,6,4,1 \rangle^*$					1
	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

The spin characters	The decomposition matrix for the block B_3				
$\langle 16,2 \rangle^*$	1				
$\langle 13,5 \rangle^*$	1	1			
$\langle 11,5,2 \rangle$		1	1		
$\langle 11,5,2 \rangle'$		1	1		
$\langle 10,5,2,1 \rangle^*$			1	1	
$\langle 8,5,3,2 \rangle^*$				1	1
$\langle 7,5,4,2 \rangle^*$					1
	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}

The spin characters	The decomposition matrix for the block B_4				
$\langle 15,3 \rangle^*$	1				
$\langle 14,4 \rangle^*$	1	1			
$\langle 11,4,3 \rangle$		1	1		
$\langle 11,4,3 \rangle'$		1	1		
$\langle 10,4,3,1 \rangle^*$			1	1	
$\langle 9,4,3,2 \rangle^*$				1	1
$\langle 6,5,4,3 \rangle^*$					1
	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

The spin characters	The decomposition matrix for the block B_5									
$\langle 15,2,1 \rangle$	1									
$\langle 15,2,1 \rangle'$		1								
$\langle 13,4,1 \rangle$	1		1							
$\langle 13,4,1 \rangle'$		1		1						
$\langle 12,4,2 \rangle$			1		1					
$\langle 12,4,2 \rangle'$				1		1				
$\langle 11,4,2,1 \rangle^*$					1	1	1	1		
$\langle 8,4,3,2,1 \rangle$							1		1	
$\langle 8,4,3,2,1 \rangle'$								1		1
$\langle 6,5,4,2,1 \rangle$									1	
$\langle 6,5,4,2,1 \rangle'$										1
	d_{26}	a	a	a	a	a	a	a	a	a

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أشجار برور للزمرتين التناظريتين S_{17} و S_{18} معيار $p=11$

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الملخص

في هذا البحث تم ايجاد أشجار برور للزمرتين التناظريتين S_{17} و S_{18} معيار $p=11$ والتي تعطي الشخصيات الأسقاطية المعيارية غير القابلة للتحليل لـ S_{17} و S_{18} معيار $p=11$ ، كذلك وجدنا مصفوفتي التجزئة للزمرتين S_{17} و S_{18} معيار $p=11$.