On approximation f by

(α, β, γ)-Baskakov Operators

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Abstract:

In the present paper, we study some application properties of the approximation for the sequences $M_{n,\gamma}^{\alpha,\beta}(f;x)$ and $B_{n,\gamma}^{\alpha,\beta}(f;\ x)$. These sequences depend on the arbitrary (but fixed) parameters α, β and γ. Here, we study the effect of these parameters on tends speed of the two families of operators $M_{n,\gamma}^{\alpha,\beta}(f;x)$ and $B_{n,\gamma}^{\alpha,\beta}(f;\ x)$ and the CPU times which are occurring on the approximation by a choosing fixed n.

Key word: Korovkins' conditions, (α, β, γ)-Baskakov Operators, (α, β, γ)- Baskakov Kantorovich operators.

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1- Introduction

The classical Baskakov operators (L_n) of bounded continuous functions $f(x)$ on the interval $[0,\infty)$, which defined as: [3]

Suppose that

$$p_{n,k}(x) = (-1)^k \frac{x^k}{k!}\ \varphi_n^{(k)}(x),$$

The n-th order of classical Baskakov is defined as:

$$(L_n f)(x) = \sum_{k=0}^{\infty} p_{n,k}(x)\, f(\tfrac{k}{n}), \qquad (1.1)$$

where $n \in N, x \in [0,b], b > 0$.

The article proved the Korovkins' conditions for the convergence of Baskakov operators. [4]

Berens and Suzuki were studied the classes for continuous functions with compact support and getting some results concerning bounded continuous functions. [8], [9]

Bernstein polynomials and Szasz-Mirakian operators are the especial cases of Baskakov operators considered by May. [7]

In recent years, some applications had been done for sequences of linear positive operators by use Maple programs.

Sharma was studied the rate of convergence of q-Durrmeyer operators and he used maple programming to describe the approximation for two sequences of operators. [5]

Mursaleen and Asif khan, they studied approximation properties of q-Bernstein–Shurer operators and they found the error estimate. In addition, they proved graphically the convergence for f by these operators. [6]

Gupta introduced and studied a generalization of the Baskakov –Durrmeyer operators. This generalization are defined as:

For x∈ [0, ∞), γ =1,

$$B_{n,\gamma}(f;x) = \sum_{k=0}^{\infty} P_{n,k,\gamma}(x) \int_0^{\infty} b_{n,k,\gamma}(t)\, f(t)\, dt$$

$$+ P_{n,0,\gamma}(x) f(0)$$

where $P_{n,k,\gamma}(x)$ and $b_{n,k,\gamma}(t)$ as defined as:

$$P_{n,k,\gamma}(x) = \frac{\Gamma(\frac{n}{\gamma+k})}{\Gamma(k+1)\Gamma(\frac{n}{\gamma})} \cdot \frac{(\gamma x)^k}{(1+\gamma x)^{(\frac{n}{\gamma})+k}}$$

$$b_{n,k,\gamma}(t) = \frac{\gamma\, \Gamma(\frac{n}{\gamma+k+1})}{\Gamma(k)\,\Gamma(\frac{n}{\gamma+k})} \cdot \frac{(\gamma t)^{k-1}}{(1+\gamma x)^{(\frac{n}{\gamma})+k+1}} \qquad (1.2)$$

Then, he introduced modification of Baskakov operators using weight functions of Bate base functions depend of parameter γ, and getting some results concerning Baskakov operators from them approximation theorem, rate of convergence, weighted approximation theorem. [1], [2]

We define (α, β, γ)- Baskakov operators $M_{n,\gamma}^{\alpha,\beta}(f;x)$ in this research, we prove the Korovkin conditions for the operators $M_{n,\gamma}^{\alpha,\beta}(f;x)$ and $B_{n,\gamma}^{\alpha,\beta}(f;\ x)$.

Theorem (2-1) (Korovkin Theorem):

For x∈ [0, ∞), $f \in [0,1]$and by applying Korovkin Theorem on the operator $M_{n,\gamma}^{\alpha,\beta}(f;x)$, we have:

1. $M_{n,\gamma}^{\alpha,\beta}$ $(1;x)$=1
2. $M_{n,\gamma}^{\alpha,\beta}$ $(t;x)$=$\frac{nx}{n+\beta}+\frac{\propto}{n+\beta}$
3. $M_{n,\gamma}^{\alpha,\beta}$ $(t^2;x)$=$\frac{n^2x^2}{(n+\beta)^2}+\frac{1+2\propto}{(n+\beta)^2}\{nx\}+\frac{\propto^2}{(n+\beta)^2}$
4. $M_{n,\gamma}^{\alpha,\beta}$ $(t^m;x)$ =$\frac{n^m x^m x}{(n+\beta)^m}+$ $\frac{m(m-1)+2\propto m}{2(n+\beta)^m}\{n^{m-1}x^{m-1}\}+T.L.P.(x)+\frac{\propto^m}{(n+\beta)^m}$

Proof:

The operators $M_{n,\gamma}^{\alpha,\beta}$ are well define on the function $1,t,t^2,t^m$ we obtain.

1. $M_{n,\gamma}^{\alpha,\beta}$ $(1;x)$=$\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}$ =1
2. $B_{n,\gamma}^{\alpha,\beta}$ $(t;x)$ =$\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}\cdot\frac{k+\propto}{n+\beta}$

$=\frac{1}{n+\beta}\{\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}.k+\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}.\propto\}$

$=\frac{nx}{n+\beta}+\frac{\propto}{n+\beta}\to x$ as n → ∞

3. $M_{n,\gamma}^{\alpha,\beta}$ $(t^2;x)$ =$\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}f(\frac{k+\propto}{n+\beta})^2$

$=\frac{1}{(n+\beta)^2}\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}.(k^2+2\propto k+\propto^2)$

$=\frac{1}{(n+\beta)^2}\{\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}\ k^2+\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}(2\propto k)+\propto^2\}$

$=\frac{1}{(n+\beta)^2}\{n^2x^2+\gamma x^2+nx\}+\frac{2\propto}{(n+\beta)^2}\{nx\}$
$+\frac{\propto^2}{(n+\beta)^2}$

$=\frac{n^2x^2}{(n+\beta)^2}+\frac{1+2\propto}{(n+\beta)^2}\{nx\}+\frac{\propto^2}{(n+\beta)^2}\to x^2$

as n → ∞

4. $M_{n,\gamma}^{\alpha,\beta}$ $(t^m;x)$= $\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}f(\frac{k+\propto}{n+\beta})^m$

$=\frac{1}{(n+\beta)^m}\sum_{k=0}^{\infty}\mathrm{P}_{n,k,\gamma(x)}(k+\propto)^m$

In this paper is an application study to the sequences $M_{n,\gamma}^{\alpha,\beta}(.;x)$, $B_{n,\gamma}^{\alpha,\beta}(.;x)$ and $L_n(f,x)$ on the two test function $f(x)=\frac{x^3}{3}-\frac{x^2}{2}+\frac{3}{16}x$, $f(t)=sin(10t)\ exp(-3t)+0.3$ to show that the effect of the parameters (α,β,γ) in the sequences $M_{n,\gamma}^{\alpha,\beta}(.;x)$, $B_{n,\gamma}^{\alpha,\beta}(.;x)$ on the tends speed of approximation .The results which are done are describe by the graphs of the test function and the approximations of the sequences $M_{n,\gamma}^{\alpha,\beta}(.;x)$, $B_{n,\gamma}^{\alpha,\beta}(.;x)$ and $L_n(f,x)$. In addition, we give some tables of the CPU time which are occurring on the approximation of the test function by a choosing fixed n.

2- Construction of the Operators {$M_{n,\gamma}^{\alpha,\beta}$ (f,x) }

In this part, we introduce the operators $M_{n,\gamma}^{\alpha,\beta}$ (f,x) and state some of their properties.

Definition 2-1

Let$f\in[0,1], x\in[0,\infty), k\in N^0=\{0,1,2,\ldots\}$ for some $0\le\alpha\le\beta$, and $n\in N=\{1,2,\ldots\}$.The(α,β,γ)- Baskakov Operators in special case i.e. $\gamma=1,\alpha=\beta=0$ is reduce to the operators (1.1).

The will-known (α,β,γ)- Baskakov operators $M_{n,\gamma}^{\alpha,\beta}$, (α,β,γ)- Baskakov Kantorovich operators $B_{n,\gamma}^{\alpha,\beta}$ with two parameters α and β with $0\le\alpha\le\beta$ on two test function $f(x)$ and investigated convergence and approximation properties of these operators, such as defined:

$$M_{n,\gamma}^{\alpha,\beta}(f(t),x)=\sum_{k=0}^{\infty}P_{n,k,\gamma}\ (x)\,f\left(\frac{k+\alpha}{n+\beta}\right)\quad(2.1)$$

$$B_{n,\gamma}^{\alpha,\beta}(f(t);x)=n\sum_{k=0}^{\infty}P_{n,k,\gamma}\int_{\frac{k}{n}}^{\frac{k+1}{n}}f(t)dt\quad(2.2)$$

Where

$$P_{n,k,\gamma}\ (x)=\frac{\Gamma(\frac{n}{\gamma}+k)}{\Gamma(k+1)\,\Gamma(\frac{n}{\gamma})}\cdot\frac{(\gamma x)^k}{(1+\gamma x)^{(\frac{n}{\gamma})+k}},$$

$$f(x)=\frac{x^3}{3}-\frac{x^2}{2}+\frac{3}{16}x\quad(2.3)$$

$$f(t)=sin(10t)\ exp(-3t)+0.3\quad(2.4)$$

The following theorem help us to study the Korovkin conditions for convergence for two operators $M_{n,\gamma}^{\alpha,\beta}$, $B_{n,\gamma}^{\alpha,\beta}$.

3. $B_{n,\gamma}^{\alpha,\beta}\ (t^2,x)$ = n$\sum_{k=0}^{\infty}$ $\mathrm{P}_{n,k,\gamma^{(x)}}$ $\int_{\frac{k}{n}}^{\frac{k+1}{n}} t^2.dt$

$= \frac{n}{3n^3}\sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\ \{(\mathrm{k}+1)^3 - k^3\}$

$= \frac{1}{3n^2}\ \sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\ \{3\mathrm{k}^2 + 3k + 1\}$

$= \frac{1}{n^2}\{n^2x^2+, y\, x^2 + nx\} + \frac{1}{n^2}\{nx\} + \frac{1}{3n^2} \rightarrow x^2 \quad as\ n \rightarrow \infty$

4. $B_{n,\gamma}^{\alpha,\beta}\ (t^m,x)$ = n $\sum_{k=0}^{\infty}$ $\mathrm{P}_{n,k,\gamma^{(x)}}$ $\int_{\frac{k}{n}}^{\frac{k+1}{n}} t^m.dt$

$= \frac{n}{n^{m+1}(m+1)}\sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\ \{(k+1)^{m+1} - k^{m+1}\}$

$= \frac{1}{n^m(m+1)}\sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\{k^{m+1} + (m+1)k^m + \frac{m(m+1)}{2}k^{m-1} + \cdots + (m+1)k + 1 - k^{m+1}\}$

$= \frac{1}{n^m}\ \sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\, k^m + \frac{m}{2n^m}\ \sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\, k^{m-1} + \cdots + \frac{1}{n^m}\sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\ k + \frac{1}{n^m(m+1)}$

$B_{n,\gamma}^{\alpha,\beta}\ (t^m,x) = x^m + \frac{m^2}{2n}x^{m-1} + \ T.L.P.(x) + \frac{1}{(m+1)n^m}$ □

3- Numerical Example

Here, we give a numerical example for the approximation of operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$ for different values of the parameters α,β,γ by take the two test functions on [0, 1].

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} + \frac{3}{16}x. \qquad (2.3)$$

$$f(t) = sin(10t)\ exp(-3t) + 0.3 \qquad (2.4)$$

$= \frac{1}{(n+\beta)^m}\{\sum_{k=0}^{\infty} \mathrm{P}_{n,k,\gamma^{(x)}}\ \ k^m + \frac{\propto m}{(n+\beta)^m}\sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\ \ k^{m-1}$

$+T.L.P(x)\} + \frac{\propto^m}{(n+\beta)^m}$

$M_{n,\gamma}^{\alpha,\beta}\ (t^m;x) = \frac{n^m x^m x}{(n+\beta)^m} + \frac{m(m-1)+2\propto m}{2(n+\beta)^m}$

$\{n^{m-1}x^{m-1}\} + T.L.P.(x) + \frac{\propto^m}{(n+\beta)^m} \rightarrow x^m$ as n $\rightarrow \infty$ □

Theorem (2-2)
((α,β,γ) −Baskakov Kantorovich operators)

The following equation hold:

$B_{n,\gamma}^{\alpha,\beta}\ (f(t);x)$ = n $\sum_{k=0}^{\infty}$ $\mathrm{P}_{n,k,\gamma^{(x)}}$ $\int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)dt$

1. $B_{n,\gamma}^{\alpha,\beta}\ (1,x)=1$
2. $B_{n,\gamma}^{\alpha,\beta}\ (t,x)=\mathrm{x}+\frac{1}{2n}$
3. $B_{n,\gamma}^{\alpha,\beta}\ (t^2,x)=x^2 + \frac{2}{n^2}x + \frac{1}{3n^2}$
4. $B_{n,\gamma}^{\alpha,\beta}\ (t^m,x)=x^m + \frac{m^2}{2n}x^{m-1} + T.L.P(x) + \frac{1}{(m+1)n^m}$

Proof:

1. $B_{n,\gamma}^{\alpha,\beta}\ (1,x)$ = n $\sum_{k=0}^{\infty}$ $\mathrm{P}_{n,k,\gamma^{(x)}}$ $\int_{\frac{k}{n}}^{\frac{k+1}{n}} dt$

   = n $\sum_{k=0}^{\infty}$ $\mathrm{P}_{n,k,\gamma^{(x)}}\{\frac{1}{n}\} = 1$

2. $B_{n,\gamma}^{\alpha,\beta}\ (t,x)$ = n $\sum_{k=0}^{\infty}$ $\mathrm{P}_{n,k,\gamma^{(x)}}$ $\int_{\frac{k}{n}}^{\frac{k+1}{n}} t.dt$

   $=n\ \sum_{k=0}^{\infty} P_{n,k,\gamma^{(x)}}\ \{\frac{2k+1}{n^2}\}$

   $= \frac{2}{2n}\sum_{k=0}^{\infty} \mathrm{P}_{n,k,\gamma^{(x)}}\ .\mathrm{k} + \frac{1}{2\mathrm{n}}$

   $= \frac{2nx}{2n} + \frac{1}{2\mathrm{n}} \ \rightarrow x\ as\ n \rightarrow \infty$

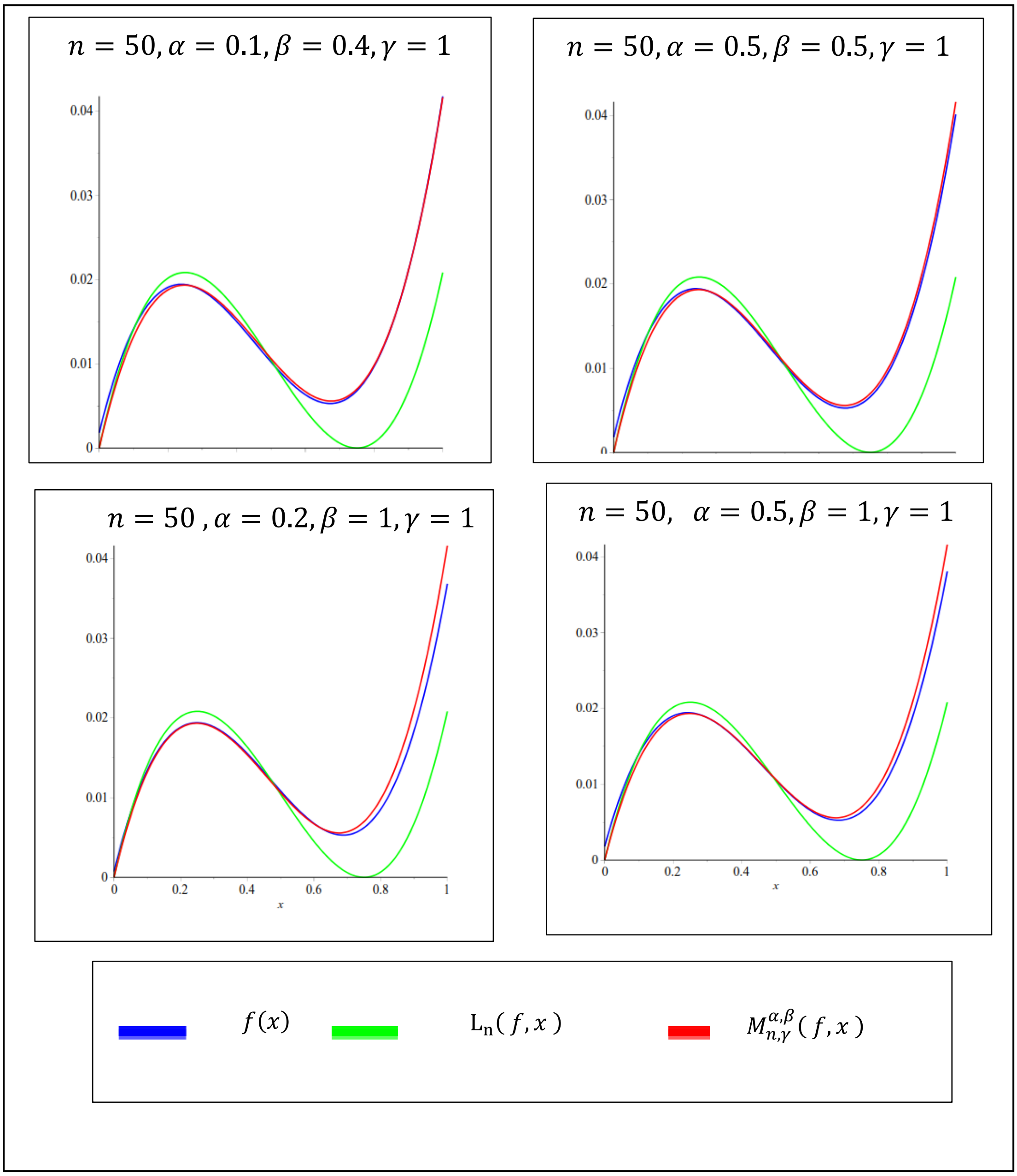

Figure (3.1)

Approximation test function $f(x)$ by $M_{n,\gamma}^{\alpha,\beta}(f,x)$ for $n = 50$

3-1The CPU time

The following table is explain the CPU time for the operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$, $L_n(f,x)$ by test function (2.3), where n=50. We found the best CPU time introduced by $L_n(f,x)$ by using the same test function f.

Figure 3.1, explains the tends speed of the operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$ by first test function (2.3), when the values n=50, $\gamma = 1$ fixed, such as if n increases tends speed of $M_{n,\gamma}^{\alpha,\beta}(f,x)$ will fail in application, and take variance values of the α, β, such that $0 \le \alpha \le \beta$ we get the best tends speed by $M_{n,\gamma}^{\alpha,\beta}(f,x)$ to approximating the test function when $\alpha = 0.5$, $\beta = 1 and\ \gamma = 1$.In addition, the $M_{n,\gamma}^{\alpha,\beta}(f,x)$ operators is returns to the classical operators $\mathrm{L_n}(f,x)$ when $\gamma = 1, \alpha = 0, \beta = 0$.

Table (3.1)

Explains the CPU time for $n = 50$

The sequence	γ	α	β	CPU time
$M_{n,\gamma}^{\alpha,\beta}(f,x)$	1	0.5	1	12.12s
$\mathrm{L_n}(f,x)$	1	0	0	11.07s

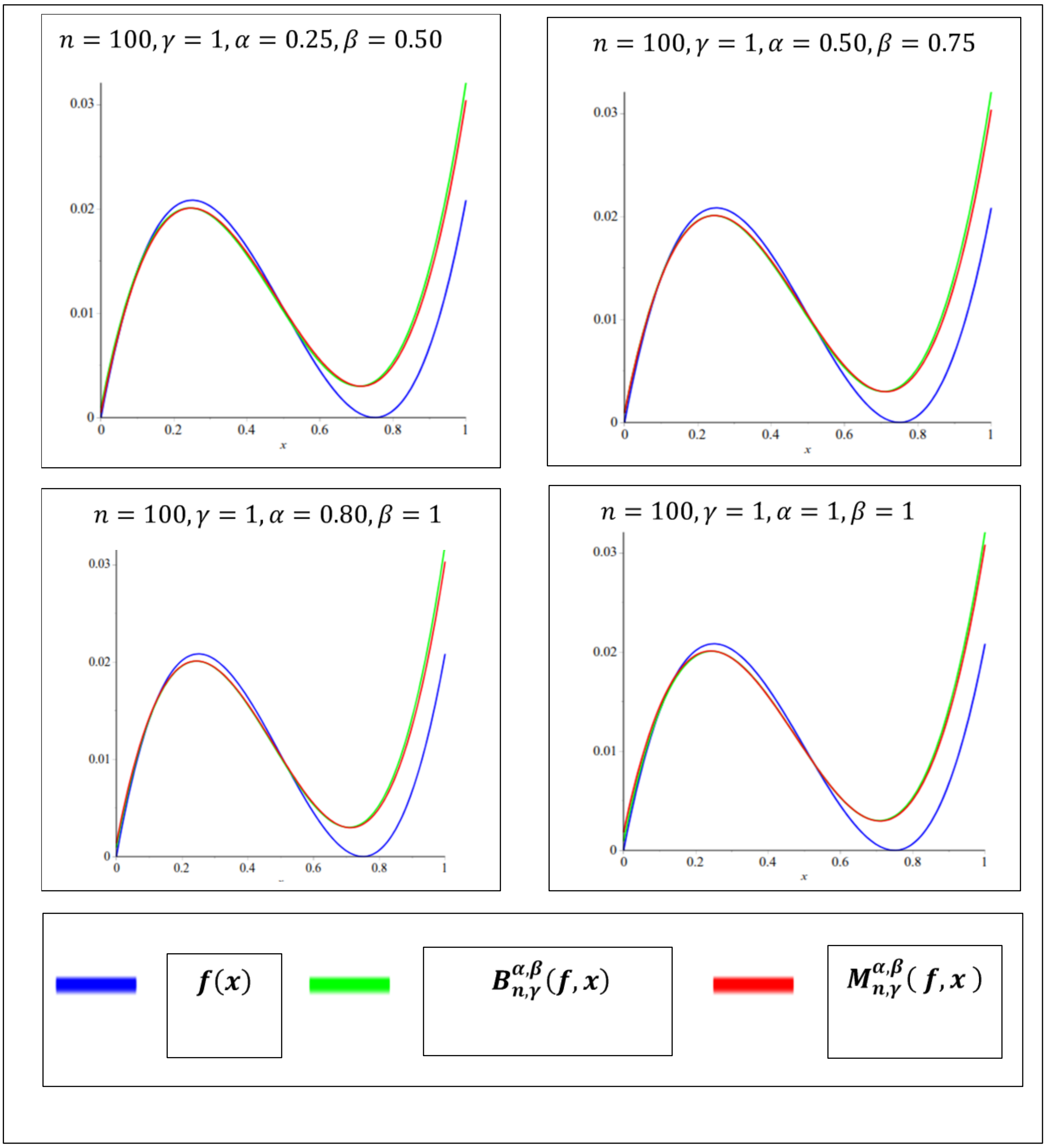

Figure 3.2

Approximation test function $f(x)$ by $M_{n,\gamma}^{\alpha,\beta}(f,x)$ and $B_{n,\gamma}^{\alpha,\beta}(f,x)$ for $n = 100$

3-2 The CPU time

The following table is explain the CPU time for the operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$, $B_{n,\gamma}^{\alpha,\beta}(f,x)$ where $n = 100$. We found the best CPU time introduced by $B_{n,\gamma}^{\alpha,\beta}(f,x)$ by using the same test function f .

Figure 3.2 explains the tends speed of (α,β,γ)- Baskakov operators $M_{n,\gamma}^{\alpha,\beta}$ with(α,β,γ)-Baskakov Kantorovich operators $B_{n,\gamma}^{\alpha,\beta}$ by first test function (2.3),when take the values $n = 100, \gamma = 1$ and take variance values of the α, β, such that $0 \leq \alpha \leq \beta$ we get the best case is $\alpha = 1$ and $\beta = 1$.

Table (3.2)

Explains the CPU time for $n = 100$

The sequence	γ	A	B	CPU time
$M_{n,\gamma}^{\alpha,\beta}(f,x)$	1	1	1	31.26S
$B_{n,\gamma}^{\alpha,\beta}(f,x)$	1	1	1	28.48S

Now we will test the second function (2.4) on the same two sequence of operators with the same steps as above.

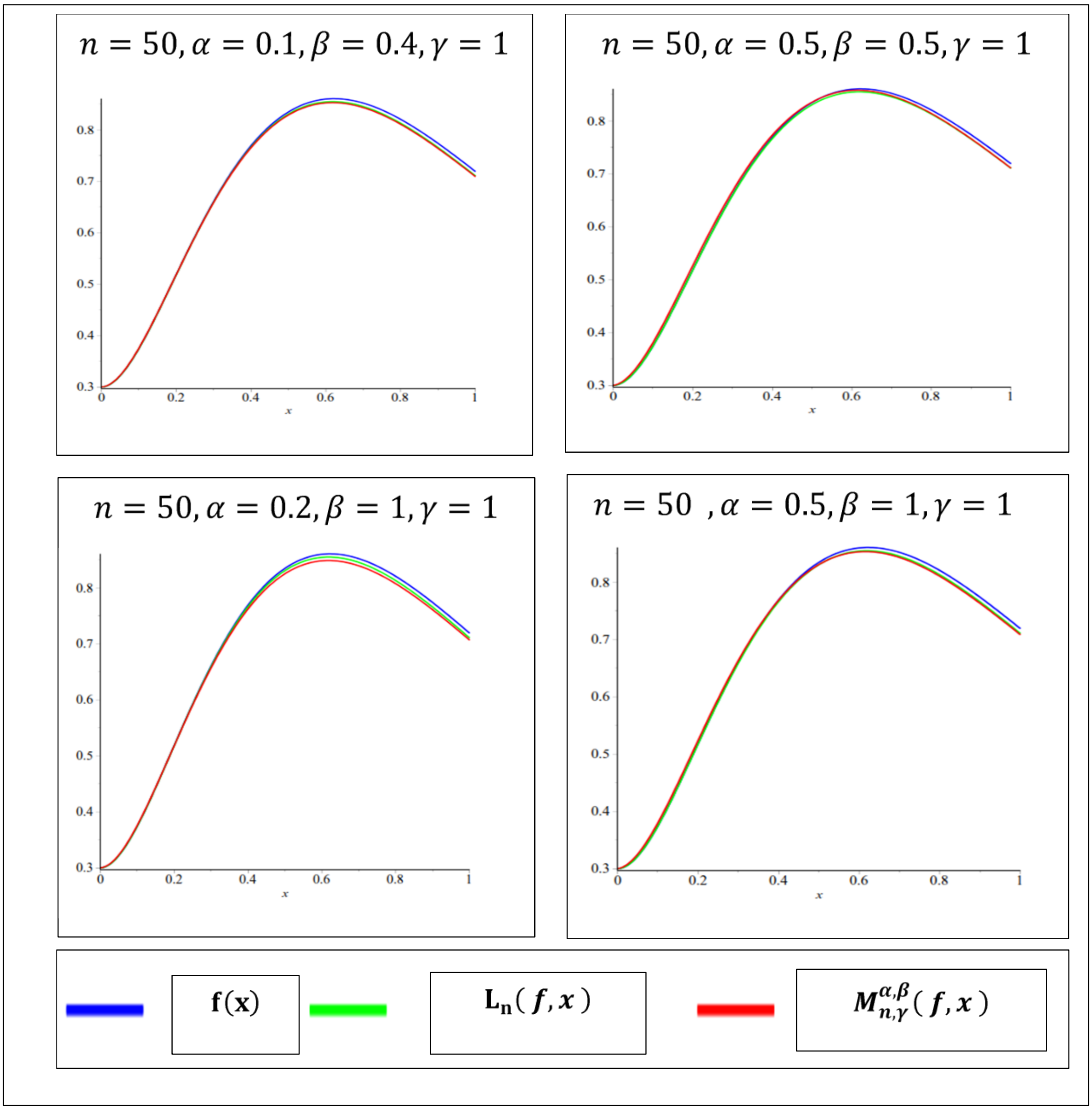

Figure (3.3)

Approximation $f(x)$ by $M_{n,\gamma}^{\alpha,\beta}(f, x)$ for $n = 50$

3-3 The CPU time: The following table is explain the CPU time for the operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$, $L_n(f,x)$ by test function (2.4), where n=50.

Table (3.3)

Explains the CPU time for $n = 50$

The sequence	γ	α	β	CPU time
$M_{n,\gamma}^{\alpha,\beta}(f,x)$	1	0.5	1	4.71s
$L_n(f,x)$	1	0	0	4.78s

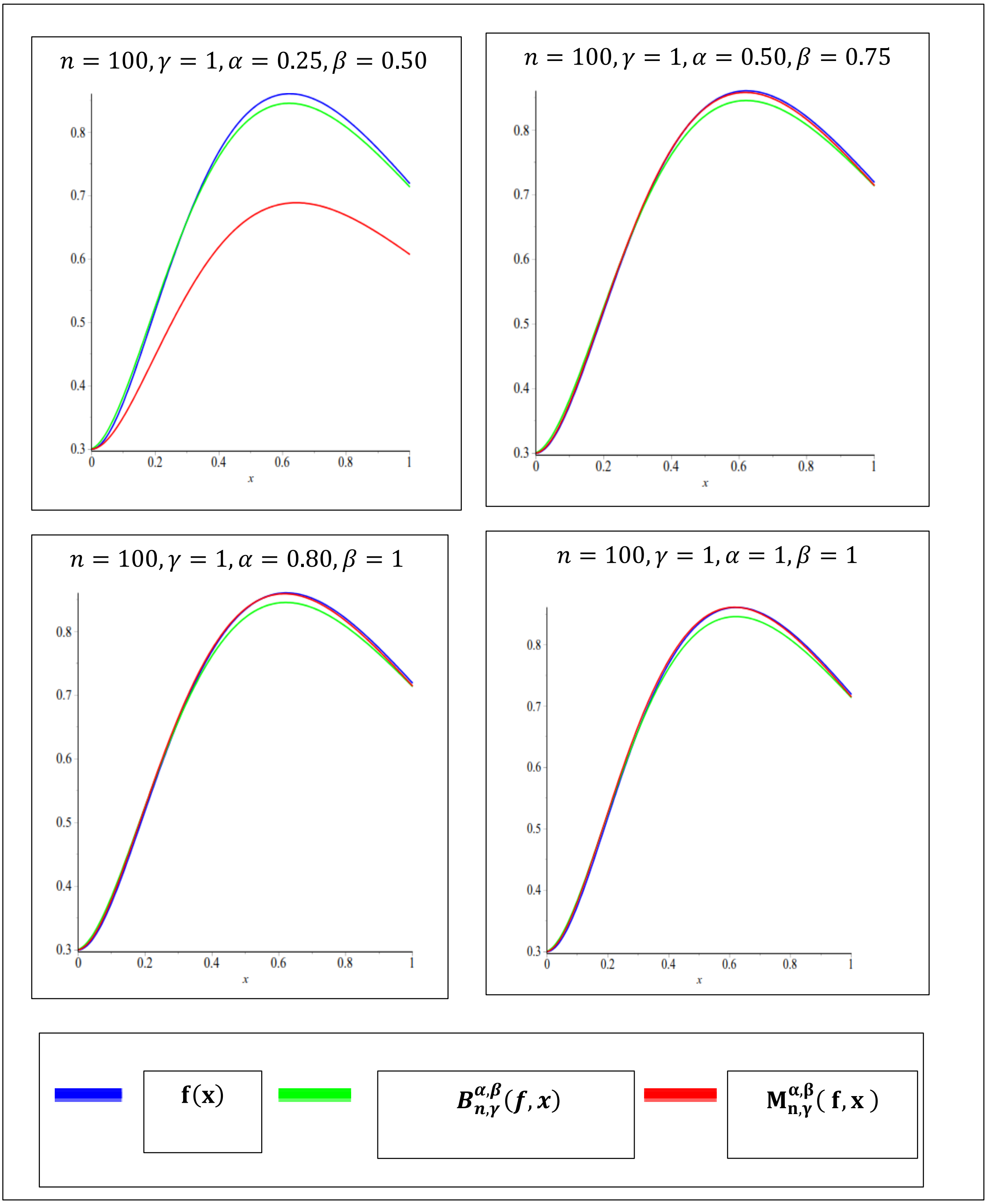

Figure 3.4

Approximation test function $f(x)$ by $M_{n,\gamma}^{\alpha,\beta}(f,x)$ and $B_{n,\gamma}^{\alpha,\beta}(f,x)$ for $n = 100$

function(2.4),where $n = 100$. We found the best CPU time introduced by $M_{n,\gamma}^{\alpha,\beta}(f, x)$ by using the same test function f .

3-4 The CPU time

The following table is explain the CPU time for the operators $M_{n,\gamma}^{\alpha,\beta}(f, x)$, $B_{n,\gamma}^{\alpha,\beta}(f, x)$ by test

Table (3.4)

Explains the CPU time for $n = 100$

The sequence	γ	α	B	CPU time
$M_{n,\gamma}^{\alpha,\beta}(f, x)$	1	1	1	4.45S
$B_{n,\gamma}^{\alpha,\beta}(f, x)$	1	1	1	19.01S

4- Comparing Between Test Functions

Test function	The operaters
Test function (2.3)	$M_{n,\gamma}^{\alpha,\beta}(f(t), x) = \sum_{k=0}^{\infty} P_{n,k,\gamma}(x) f\left(\frac{k+\alpha}{n+\beta}\right)$
Test function (2.4)	$M_{n,\gamma}^{\alpha,\beta}(f(t), x) = \sum_{k=0}^{900} P_{n,k,\gamma}(x) f\left(\frac{k+\alpha}{n+\beta}\right)$
Test function (2.3)	$B_{n,\gamma}^{\alpha,\beta}(f(t); x) = n \sum_{k=0}^{\infty} P_{n,k,\gamma} \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t) dt$
Test function (2.4)	$B_{n,\gamma}^{\alpha,\beta}(f(t); x) = n \sum_{k=0}^{900} P_{n,k,\gamma} \int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t) dt$
Test function (2.4)	The best tends speed of $M_{n,\gamma}^{\alpha,\beta}(f(t), x)$
Test function (2.4)	The best CUP time for $M_{n,\gamma}^{\alpha,\beta}(f(t), x)$, where $n = 100$

5- Conclusions

In this paper, we defined the sequence of a linear positive operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$ depends on the parameters α,β,γ and give some of its properties. In addition, we made an application of the sequences$M_{n,\gamma}^{\alpha,\beta}(f,x)$, $B_{n,\gamma}^{\alpha,\beta}(f,x)$ to show the effect of these parameters α,β,γ on tends speed occurs by these operators are betters than all tends speed of the sequence $L_n(f,x)$, where f is the test function. We also find a better effect of the parameters when $0 \leq \alpha \leq \beta$ betters than previous cases of parameters α,β,γ. Finally, by the applying the two operators $M_{n,\gamma}^{\alpha,\beta}(f,x)$, $B_{n,\gamma}^{\alpha,\beta}(f,x)$ we get the best CPU time introduced by $M_{n,\gamma}^{\alpha,\beta}(f,x)$ by using the second test function.

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حول تقريب الدالة الاختبارية f للمؤثرات الخطية باسكوف -(α, β, γ)

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المستخلص

في بحثنا هذا درسنا بعض الخواص التطبيقية لتقريب المتتابعات ضمن المؤثرين $M_{n,\gamma}^{\alpha,\beta}(f;x)$, $B_{n,\gamma}^{\alpha,\beta}(f;\ x)$. تلك المتتابعات تعتمد على تأثير البارامترات β ،α، γ وعليه قمنا بدراسة تأثيرها من ناحية سرعة الوصول لكلا المؤثرين وحساب الوقت اللازم للتقريب بواسطة اختيار قيمه ثابتة ل n .