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DETERMINATION OF STRESS INTENSITY FACTORS FOR BLUNTED CRACK PLATES USING (XFEM) WITH LEVEL SET AND ENRICHMENT FUNCTIONS

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ABSTRACT :

An extended finite element method (**XFEM**) is developed to determine the stress intensity factors (**SIF**) for plates with blunted cracks. A program was developed and written in matlab code uses a level set function and enrichment functions to analyze blunted crack problems. XFEM is based on the local-PU (Partition of Unity) and applied in the commentary discontinuous regions (crack and blunt). Two edge cracked plates in two loading conditions were analyze using the developed program. This study verifies the benefit of using blunting crack technique to eliminate the singularity at the tips, especially at small radii of the blunt. An excellent agreement between SIF calculated from interaction integral (J-integral) and closed form solution were found for tensile and shear loading conditions. Displacement, stress, and crack opening displacement distributions were drawn as a function of blunting radii.

Keywords: XFEM, SIF, Blunted Crack, level-set, Enrichment functions, M-integral, Crack tip opening displacement.

الخلاصه :

طريقة العناصر المحددة الممتدة طورت لحساب معاملات شدة الاجهاد في الصفائح المحتوية على شقوق مدورة. تم نطوير برنامج كتب بلغة الماتلاب يستخدم طريقة ضبط المستويات ودوال الاغناء في المناطق الغير مستمرة مثل الشقوق المدورة. حالتين دراسة تم اختارها مع حالتي تحميل مختلفة لغرض التحليل باستخدام البرنامج المطور. هذه الدراسة اثبتت الفائدة من استخدام التدوير في رأس الشق لازالة الاحادية في رؤوس الشق خاصة عند الاقطار الصغيرة من التدوير. تطابق ممتاز تم الحصول عليه بين معامل شدة الاجهاد المحسوب من التكامل التداخلي (تكامل-J) و الحل التحليلي وجد لحالات تحميل الشد والقص. توزيع الازاحات, الاجهادات, وازاحات فتح الشق تم رسمها كدالة لانصاف اقطار التدوير.

INTRODUCTION

Several methods for the analysis of cracks without re-meshing have been developed in recent years, both in the area of mesh less methods and in the framework of extended finite *element* method (XFEM) (**Moes**, **etal. 1999**). In these methods, the standard basis is enriched to describe the singular near tip solution and a step function is used for the displacement discontinuity along the crack edges. Even if these methods do not require a mesh in the XFEM, or the nodal arrangement in the Element Free Galerkin (EFG) method, or other mesh less methods, it is still necessary to describe the geometry of the crack by lines in 2D and 3D surfaces to locate the discontinuity and to build the discontinuous approximation (**Ventura, etal. 2002**). The XFEM and Meshless methods have been developed to facilitate the modeling of features and arbitrary discontinuities. In contrast to the standard FEM, these methods do not need fine nor special mesh structure close to the discontinuous surfaces and other features, this is due to the use of enrichment functions capable of representing these features (**Belytschko, etal. 1999**,**Duarte, etal. 2000**). The first application of XFEM was in the area of elastic fracture mechanics. More recent applications include cracks in orthotropic solids, dynamic cracks, elastic-plastic media, and dislocation problems (**Ventyral, etal. 2009**). The XFEM and its coupling with level set functions was studied and analyzed to model arbitrary discontinuities. The level set function allows for the treatment of internal boundaries and interfaces without any explicit treatment of the interface geometry (**Ventyral, etal. 2009**). The level set functions provide a convenient and an appealing means for tracking moving interfaces, their merging and their interaction with boundaries, modeling and defining internal boundaries and voids with greater flexibility and computational efficiency (**Ahmed 2009**). In the XFEM the displacement field is decomposed into two parts, a continuous and discontinuous part. The continuous part is the ordinary displacement field corresponding to the situation without any crack while the enrichment with the discontinuous displacement field enables the element to include a discontinuity, such as a cohesive crack, The enrichment of the displacement field is based on a local partition of unity (**Asferg, etal. 2004**).

Crack Tip Blunting

Crack tip blunting always used in practice. Usually, the crack tip can be practically stopped from extension by setting a hole at the tip of the crack. The major advantages of the drilling a hole at the tip are to prevent the propagation of the crack, and to reduce the deformation in the crack domain. The mathematical model of an ideal sharp crack is not physically admissible and the model of a blunt crack with a small but finite crack tip radius seems to be more realistic [41]. The equations of stresses in the blunt crack under mode I (opening mode) loading condition can be written for a tip radius **ρ** as follows (**Ewalds and Wanhill 1986**, **Mikheevskiy 2009**)

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)-\frac{K_I}{\sqrt{2\pi r}}\frac{\rho}{2r}\cos\frac{3\theta}{2}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+\frac{K_I}{\sqrt{2\pi r}}\frac{\rho}{2r}\cos\frac{3\theta}{2} \qquad (1)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}-\frac{K_I}{\sqrt{2\pi r}}\frac{\rho}{2r}\sin\frac{3\theta}{2}$$

Blunting has a minor effect to the size and shape of the plastic zone, a blunted crack tip acts as a free surface and locally reduce the triaxiality of the state of stress (at **$r = \rho/2$ $\sigma_x = 0$ for $\Theta = 0$**), thus increased blunting will reduce the maximum stress and move its location away from the crack tip. Under liner elastic fracture mechanics (**LEFM**), the crack tip opening displacement (**CTOD**) is clearly zero. However, when the material is allowed to yield, the crack tip will blunt resulting in a non-zero crack tip opening displacement (**Saouma 2000**). The difference between blunting of an initially crack under LEFM assumptions and CTOD without blunting at sharp crack is shown in Fig.1.

Extended Finite Element Method (XFEM)

The extended finite element method was developed to ease the difficulties in solving problems with localized features that are not efficiently resolved by mesh refinement. One of the initial applications was the modeling of cracks in materials. In this implementation, discontinuous basis functions are added to the standard polynomial basis functions of FEM for nodes that belonged to elements intersected by a crack to provide a basis function that include crack opening displacements. The major advantage of XFEM is that, in such problems, the finite element mesh does not need to be updated to track the crack path. Subsequent research has illustrated the more general use of the method for problems involving singularities, material interfaces, regular meshing of microstructures such as voids, and other problems where a localized fracture can be described by an appropriate set of basis functions. Enrichment can be used as the principle of increasing the order of completeness in XFEM, therefore, it may clearly more higher accuracy of approximation of displacement can be achieved by including the determined form of the analytical solution.

The enriched functions can be selected depends on the priori solution of the problem. To use enrichment in a crack analysis, this is adequate to an increase in the accuracy of the approximation, if analytical near crack tip solutions are someway entered in the enrichment terms.

Modeling of Blunt Crack with Level Set Method

The level set method is a universal implementation for the description of the discontinuous region such as crack and blunt. It characterizes the surface of these discontinuities by a signed distance function and models the growth of the surface by a suitable progressing equation. This function is defined by a set of nodal values.

In this work, two level set functions are used to represent the crack problems, the first level set **ϕ** is not generally sufficient to describe the crack, a second set **ψ** at the crack tip is required.

For the general definition of level set method, consider a domain **Ω** divided into two non-overlapping sub domains Ω_1 and Ω_2, sharing an interface **Γ**, as illustrated in Fig.2, the level set function **$\phi(x)$** is defined as:

$$\phi(x) = \begin{cases} > 0 & X \in \Omega_1 \\ = 0 & X \in \boldsymbol{\Gamma} \\ < 0 & X \in \Omega_2 \end{cases} \qquad (2)$$

One of the common choices for the level set function(Fig.3):

$$\phi(x) = \begin{cases} d & X \in \ \Omega_1 \\ -d & X \in \ \Omega_2 \end{cases} \qquad (3)$$

where ***d*** is the normal distance from **x** to the interface **Γ**. This can be written as:

$$\phi(x) = min_{xi \in \boldsymbol{\Gamma} c} \|x - xi\| \ sign(\boldsymbol{n} \, . \, [x - xi]) \qquad (4)$$

Discretization of the level set allows for the evaluation of the element level based on the nodal level set values $\boldsymbol{\phi_i = \phi(x_i)}$ and known finite element shape functions $\boldsymbol{N_i(x)}$:

$$\phi(\boldsymbol{x}) = \sum_{i=1}^{n} N_i(x) \ \phi_i \qquad (5)$$

where $\boldsymbol{\phi_i}$ is the level set value on the node ***i***.
Mathematically, using the level set function for blunting, the enrichment function can be evaluated from the second set as follows **(Ahmed 2009)**:

$$V\ (X) = \begin{cases} +1 & \psi\ (x)\ > 0 \\ 0 & \psi(x)\ <\ 0 \end{cases} \qquad (6)$$

The above equation is practically an important concept for implicitly defining the level set function for describing the crack, in general, as line shape. Also, it can be used for the representation of the blunt in the tip of the crack, by using a mathematical description of a circle in two dimensions, such as:

$$\psi(x,y) = \sqrt{x^2 + y^2} - rr \tag{7}$$

where ***rr*** is the radius of the hole represent crack tip blunting.

Results and Discussions

In order to explain the effect of crack blunting in two-dimensional LEFM problems, the XFEM was used to analyze the stress field near the crack tip. A hole (blunt) is set at the crack tip to avoid crack propagation. Level set approach , described above, is used to represent the crack and the tip blunt. The SIF is calculated by using the J-integral technique, the numerical solution of SIF is compared with analytical solutions available in the literatures. Two case studies have been analyzed. The results have been shown for different number of elements and different crack tip blunting sizes, with plane strain condition for all cases, using programs developed in this work using Matlab.

Case Study (1): An Edge Crack with Blunted Tip under Tensile Stress

A finite plate under tension load contained crack at the edge is considered as shown in Fig.4. XFEM is used as numerical method to analyze the crack tip stresses, strains and displacements fields. The analysis performed using a uniform mesh of 4-noded quadrilateral elements. Signed distance function and near-tip enrichment function are used to enrich the field near the crack tip. A plane strain condition was considered with tensile stress equal to 1 Mpa. The mesh and the dimension of plate, and the crack are shown in Fig.5. The stress intensity factor (SIF) is calculated and compared with exact solution given in **Ewalds and Wanhill**. The stress intensity factor K_I is plotted against the radius of the blunt ***rr*** and applied stress **σ,** as shown in Fig.6 and Fig.7 respectively. The results shows an excellent agreement especially at small radii. To increase the accuracy of the analysis, the number of the elements, in the mesh, can be increased as shown in Fig.8. The crack opening displacement **δ** is plotted against the radius of blunt, applied stress, and width of the plate, as shown in Figures 9-11, respectively. Fig. 9 shows clearly that **δ** increases with the increase of the blunt radius. Also it shows that the best accuracy of results observed at large number of elements, that's due to the increase of the degrees of freedom which lead to an accurate calculation of the displacement field near the crack tip. Similar conclusion can be drawn for Fig.10 and Fig.11, respectively. The behavior of the stress singularity at crack tip in the y-direction (direction of opening the crack) is plotted in Fig,(12). It is clear, from this figure, that the stress singularity in the crack tip blunt decreases compared with the stress singularity in the case of crack without blunt, that's because blunting the crack will prevent crack propagation to take place. Also increasing the number of elements leads to decrease the stress singularity. Large number of elements decrease the size of elements at the crack tip and that leads to increase the effect of the enrichment function which leads to better accuracy.

Case Study (2): An Edge Crack with Blunted Tip under shear Stress

The same finite plate in case study (1), subjected to shear loading condition as shown in Fig.13. Considering shear loading, means the present of mode II (shearing mode) stress intensity factor. K_{II} as a function of blunt radius ***rr*** and applied shear stress **τ** are plotted in Fig.14 and Fig.15 respectively. Fig.14, represent the relationship between K_{II} and the applied shear stress, and shows the good agreement between the XFEM results and exact solution. While Fig.15, explain the connection between K_{II} and the crack tip blunting radius. The figure shows a little difference between the XFEM numerical solution and the theoretical exact solution at small radii of blunt, and a relatively diverge results for large blunt radii. The crack opening displacement behavior in mode II against applied shear stress is plotted in Fig.16. It's clear, from this figure, that's $\boldsymbol{\delta}$ increase when $\boldsymbol{\tau}$ is increased, as increment of the shear stress tends to extend the distance of the initial crack. The results of $\boldsymbol{\delta}$ for three radii of blunting illustrate that, the decrease of the radius of the blunt leads to shift the curve of the crack opening displacement, as shown in Fig.17. The blunting shear stress for mode II at different polar radii is shown in Fig.18. Which explain that, the increment of the polar radius leads to increase the shear stress $\boldsymbol{\tau_{xy}}$ for blunting of crack.

Conclusions

The XFEM program developed in this work for blunted cracks gives reasonable accurate results compared to closed form solutions for the two selected loading cases. Also, the coupling of the level set method with XFEM seems to give a very efficient tool for tackling crack tip blunting problems. Finally, for tensile load case, the best agreement between calculated SIF and closed form solution was found at blunt radius equals to **0.05 m,** while for the shear case at blunt radius **0.13 m.**

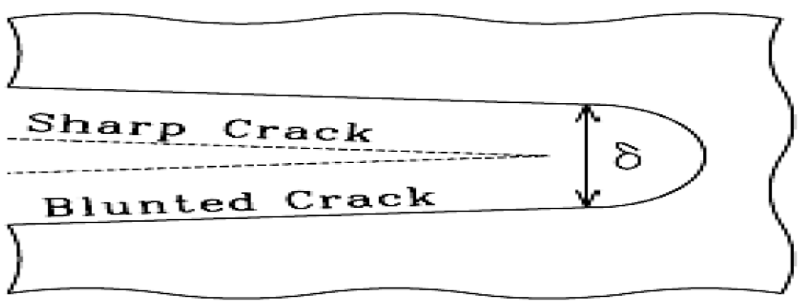


Fig.1 Crack tip opening displacement at blunted crack [17].

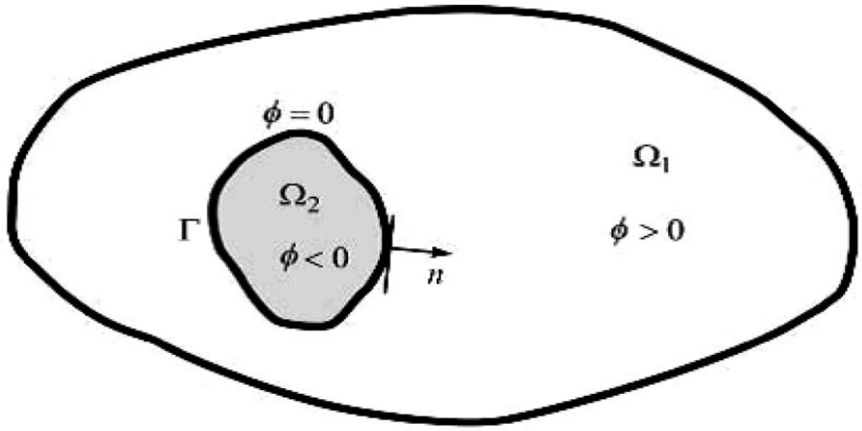


Fig.2 Definition of the level set function [1].

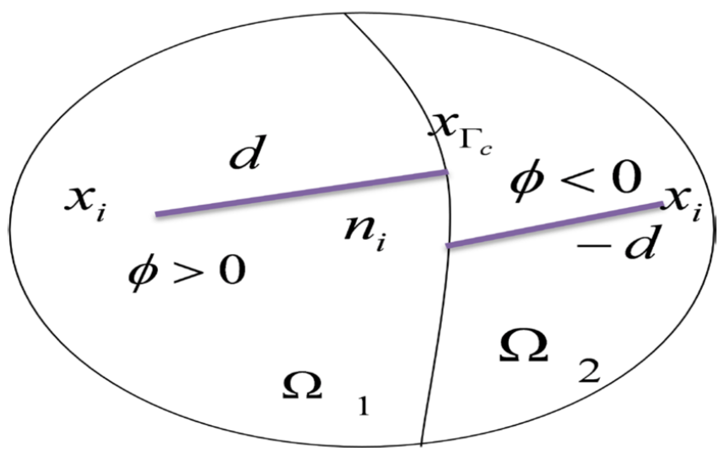


Fig.3 Definition of Signed distance function on discontinuous domain.

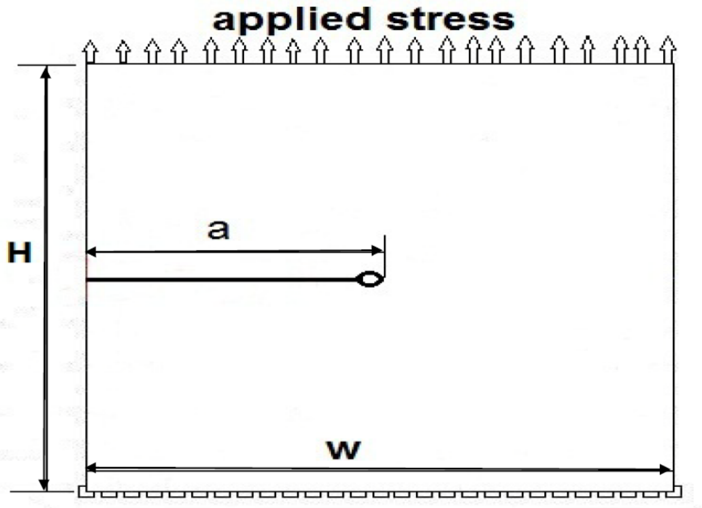


Fig.4 An edge crack with blunted tip under tensile stress.

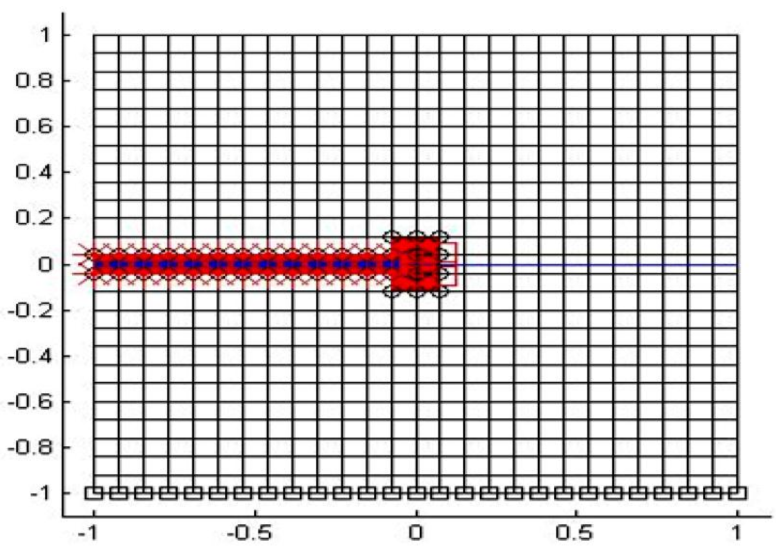


Fig. 5 Mesh and dimensions of case study (1).

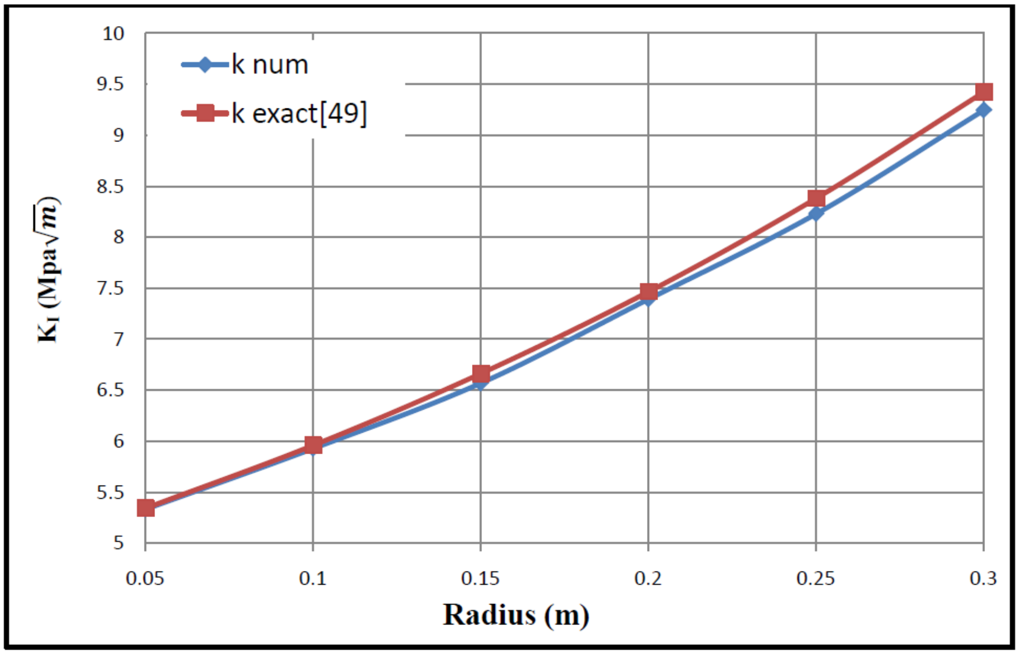


Fig. 6 Comparison of K_I values with exact solutions for different crack tip blunting.

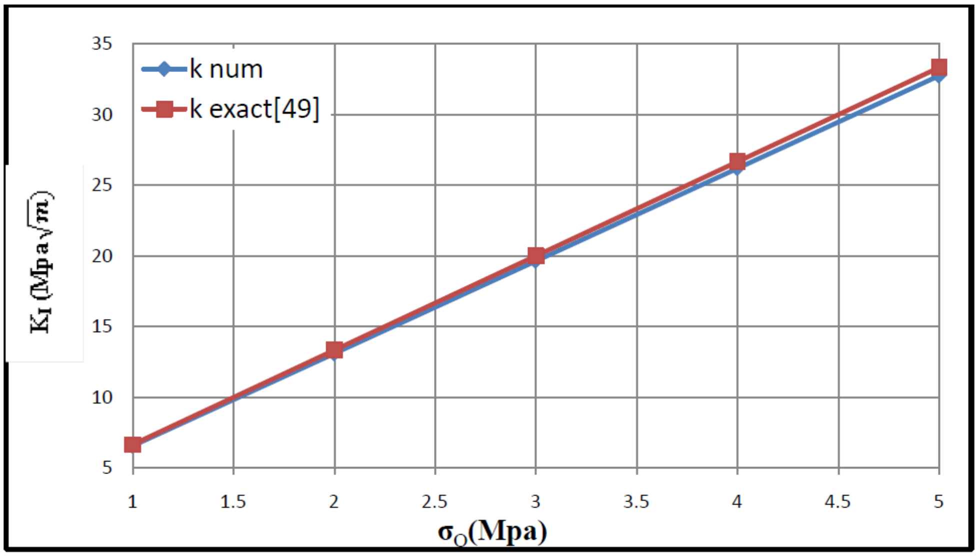


Fig. 7 Comparison of K_I values with exact solutions for different applied stresses.

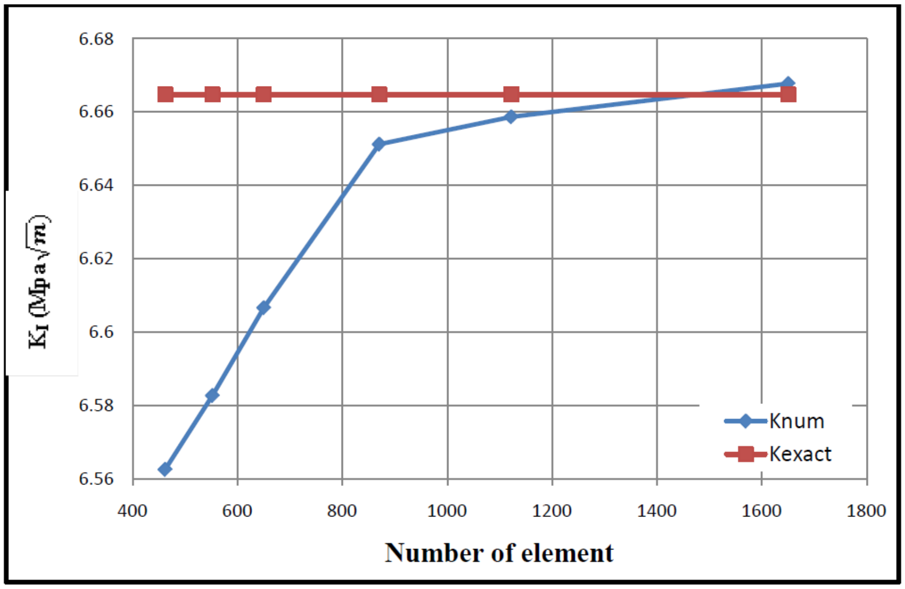


Fig. 8 Comparison of K_I values with exact solutions for different number of elements.

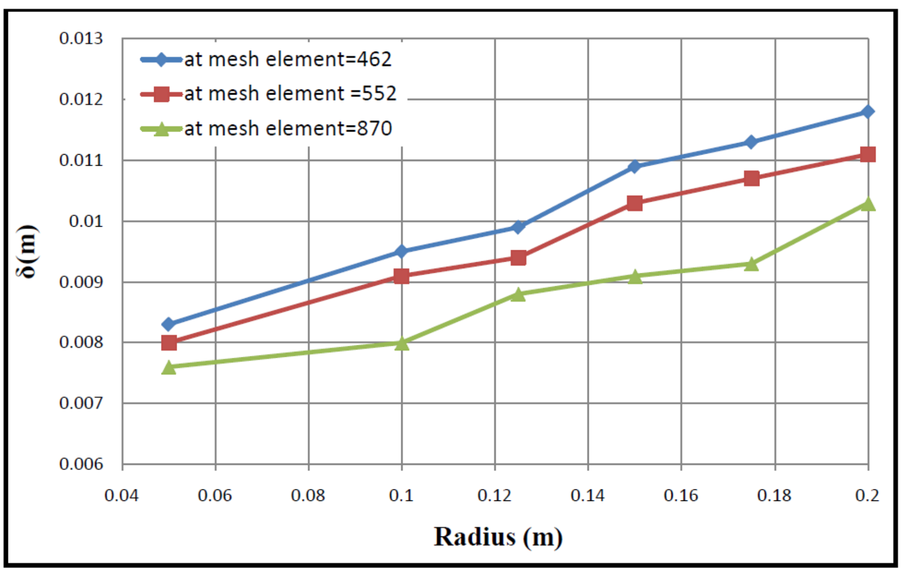


Fig. 9 Crack opening displacement for different Blunt radius at three mesh sizes.

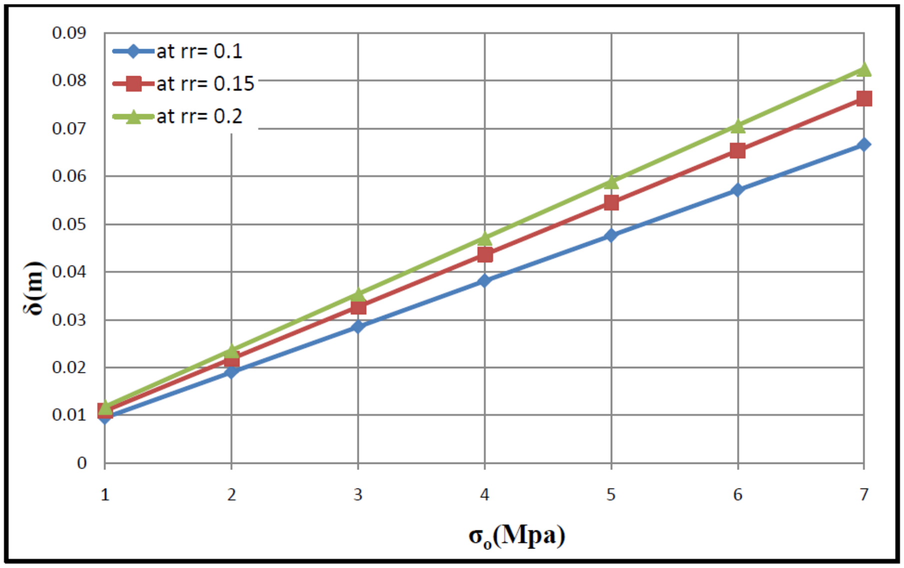


Fig. 10 Crack opening displacement for different applied stress at three blunt radii.

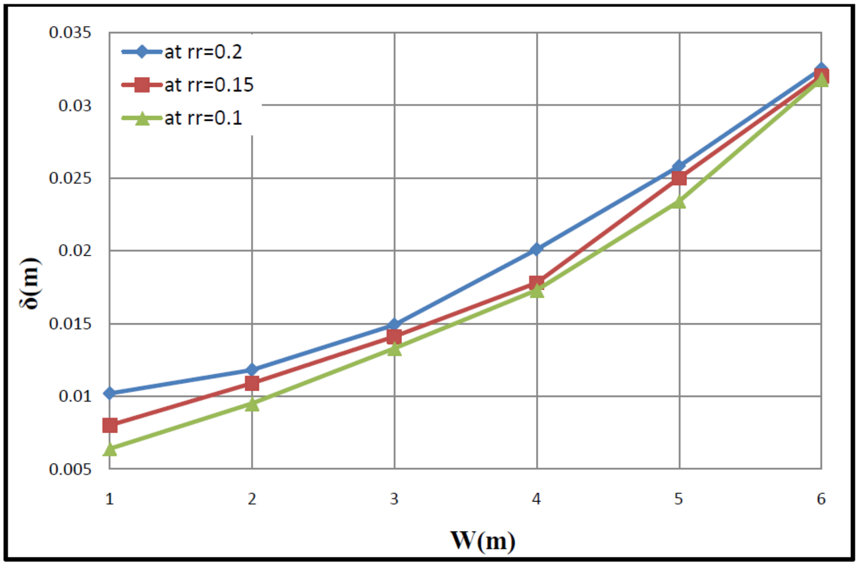
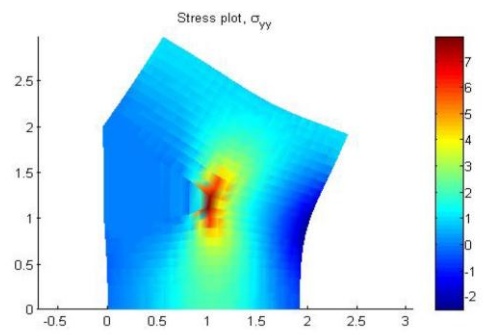
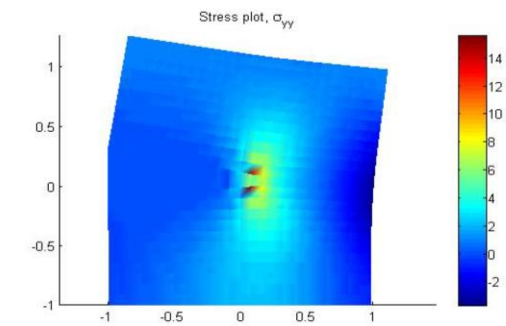


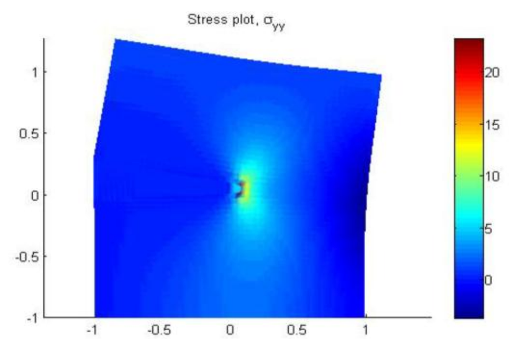
Fig.11 Crack opening displacement for different plate width at three blunt radii.



A- Stress distrbution In Y-Direction Fo Crack Tip With Out Blunting.



B- Stress Distribution In Y-Direction Fo Crack Tip Blunting At Mesh 552.



C- Stress Distribution In Y-Direction Fo Crack Tip Blunting At Mesh 2545.

Fig. (12) The behavior of singularity at the crack tip in the y-direction with and without Crack tip blunting.

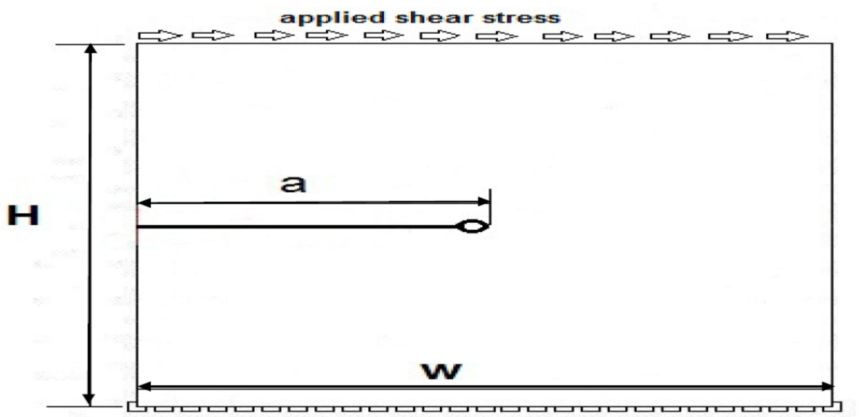


Fig. 13 An edge crack with blunted tip under shear stress.

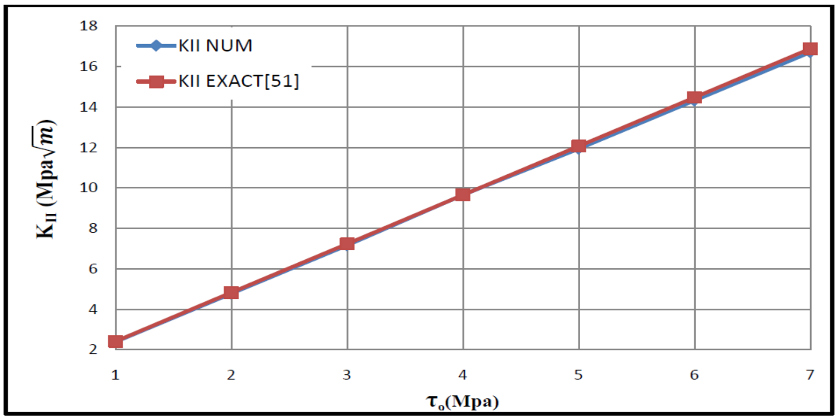


Fig. 14 Comparison of K_{II} values with exact solutions for different shear stress values.

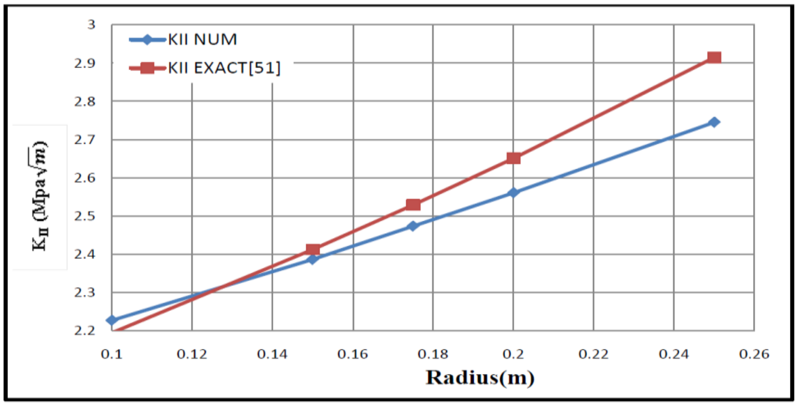


Fig. 15 Comparison of K_{II} values with exact solutions for different crack blunt radii.

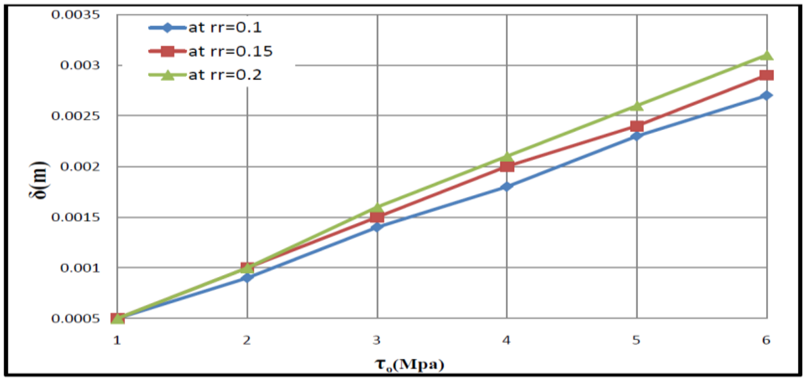


Fig. 16 Crack opening displacement for different applied shear stress for three radii of crack blunting.

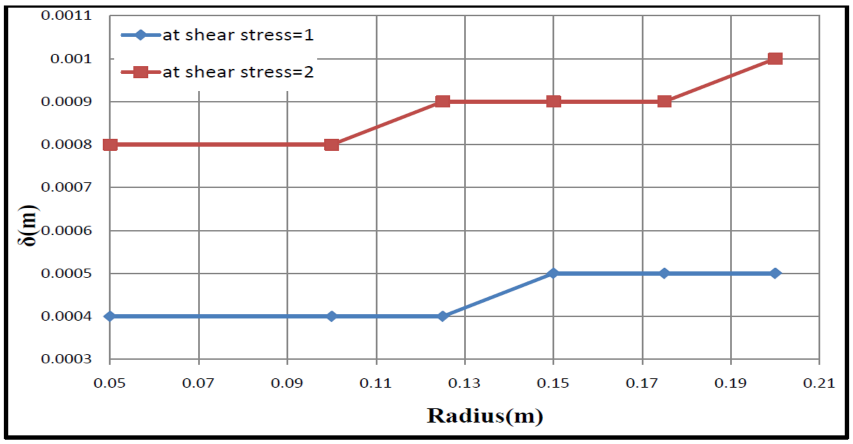


Fig. 17 Crack opening displacement for different crack blunting radii for two shear stress values.

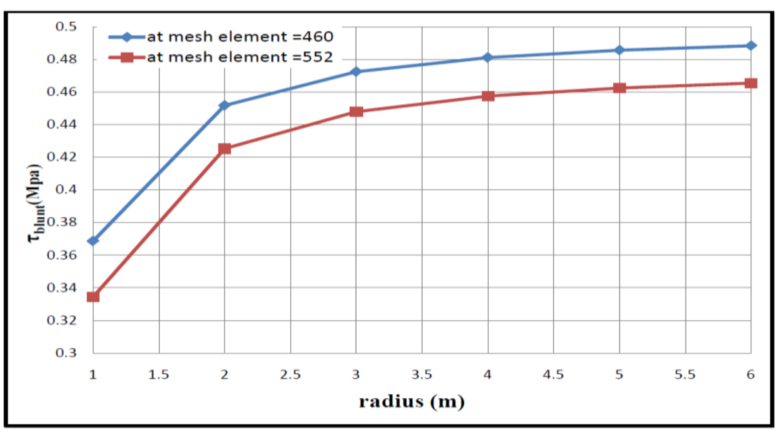


Fig. 18 Blunting Shear stress for Mode II at different radii from the crack tip.

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