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A new approach for solving compressible Navier-Stokes equations

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ABSTRACT

In this paper, we introduce the analytical approximate solutions for one and two-dimension compressible Navier-Stokes equations by applying a relatively new method named splitting decomposition homotopy perturbation method. The new methodology depends on combining Adomian decomposition and Homotopy perturbation methods with the splitting time scheme for differential operators. The numerical results which we obtained from the solutions of the two problems, show that the new method is efficient with good converge and high accuracy compared with the two standard methods i.e. Adomian decomposition method and Homotopy perturbation method.

1. Introduction

Navier-Stokes equations are non-linear partial differential equations which are called compressible if the density of fluid is changed. The names of these equations are taken from the two physicists Claude-Louis Navier and George Gabriel Stokes in the nineteenth century. Also, these equations are considered as the most important physical equations which describe a large number of phenomena of different applications in many research fields that may be used in modeling weather, liquid flow in channels and pipes, gas flow around flying bodies, and movement of stars in the galaxy. Many scientists and researchers attempted to find solutions for these equations by different acting methods; For example, Fonseca (2016) used Tanh-method to find analytic solution of one and two-dimensional compressible Navier-Stokes equations. Perron et al. (2004) applied finite volume method to solve three-dimensional Navier-Stokes equation. Wahab et al. (2015) presented analytical approximate solutions for Navier-Stokes equation by using homotopy perturbation method. Shahmohamadi and Mohammadpour (2014) suggested analytic solution for three-dimensional Navier-Stokes equation by using homotopy analysis method and Pade'-approximate method. Al-Saif (2015) proposed Adomian decomposition methods to introduce analytical approximate solutions for two-dimensional Navier-Stokes equations. Where, Adomian decomposition method (ADM) and homotopy perturbation method (HPM) are active and strong in finding solutions for mathematical model in physics and engraining problems ones, so we can apply them to solve partial (ordinary) differential equations either linear or non-linear for initial-boundary value problems. The name of the first method is taken from the scientist who discovered it; namely, Adomian (1988), and the second was found for the first time by the Chinese Mathematician; He (1999). There are several researchers who attempted to develop and improve these two methods through the past few years; for example, Luo et al. (2006) revised ADM cases involving inhomogeneous boundary conditions using a suitable transformation. They solved inhomogeneous heat and wave equations. Zhu et al. (2005) present a new algorithm for calculating Adomian polynomials. The algorithm requires less formula than the previous method developed by Adomian. Luo (2005) suggested active methods for ADM which is a two-steps Adomian decomposition method (TSADM) to reach the solution. TSADM reduces the repetitive mathematical processes that are applied to find the solution and also he makes comparison for the results. The results showed that TSADM is an active and efficient method which has high accuracy in finding solutions. Also, in many works (Zhang and Lu, 2011; Inc, 2004; Ali and Al-Saif, 2008), the authors use ADM to find analytic and approximate solutions for different problems. In the same direction of modification, the HPM is active to find solutions for non-linear equations (Jin, 2008; Hemeda, 2012; Ganji et al., 2007). Recently in Al-Saif and Al-Griffi (2017); we follow the example of researchers in the development of these two methods. We combine Adomian decomposition and

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Homotopy perturbation methods with the splitting time scheme for differential operators to discover a new methodology namely splitting decomposition homotopy perturbation method (SDHPM), which is applied to solve unsteady one-dimensional Navier-Stokes equation. The numerical results that are obtained by SDHPM showed that it is quite accurate, reasonable convergent and easily implemented. From the literature review and depending on our humble knowledge, we observed that the ADM and HPM are not yet used to study the current problems. This matter was the motive for us to use them in the present study with the application of both developed methods that are presented previously in Al-Saif and Al-Griffi (2017), to examine their validity.

The aim of this paper is to extend the application of our proposed method SDHPM (Al-Saif and Al-Griffi, 2017) to solve unsteady state one and two-dimensional compressible Navier-Stokes equations. The numerical results which we obtained showed the efficiency and activity of a relative new method to solve one and two-dimension compressible Navier-Stokes equations, and compare its reliability, efficiency and accuracy with standard ADM and HPM.

2. The main idea of the SDHPM method

In this section, the basic idea of SDHPM will be discussed. It depends on the algorithms of ADM and HPM. To illustrate the main idea of the Adomian decomposition method and Homotopy perturbation method, we consider the general equation as in the differential operators form:

$$Lu + Ru + Nu = g \tag{1a}$$

with the initial condition;

$$u_0 = u(x,0), \tag{1b}$$

where L is an easily invertible linear differential operator, R is the remaining linear part, Nu is the nonlinear term, $u = u(x,t)$ is exact solution of Eq. 1, and $g = g(x,t)$ is known analytic function.

Algorithm of ADM: The application of the Adomian decomposition method (Adomian, 1988) on Eq. 1a, is as the following:

$$u = L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu) \tag{2}$$

where $L^{-1}(.) = \int_0^t (.)dt$ is the inverse operator of $L = \frac{\partial}{\partial t}$.

The decomposition method represents the solution of Eq. 2 as the following infinite series:

$$u = \sum_{n=0}^{\infty} u_n \tag{3}$$

The nonlinear operator $Nu = \Psi(u)$ is decomposed as:

$$Nu = \sum_{n=0}^{\infty} A_n \tag{4}$$

where A_n are Adomian 's polynomials (Seng et al., 1996), which are define as:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\Psi\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0} \quad n = 0,1,2,\ldots \tag{5}$$

substituting Eqs. 3 and 4 into Eq. 2, we have

$$u = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n\right)\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \tag{6}$$

consequently, it can be written as:

$$\left.\begin{aligned} u_0 &= \emptyset + L^{-1}(g) \\ u_1 &= -L^{-1}(R(u_0) - L^{-1}(A_0)) \\ u_2 &= -L^{-1}(R(u_1) - L^{-1}(A_1)) \\ &\vdots \\ u_n &= -L^{-1}(R(u_{n-1}) - L^{-1}(A_{n-1})) \end{aligned}\right\}, \tag{7}$$

where $\emptyset = u(x,0)$ is the initial condition.

Hence all the terms of u are calculated and the general solution is obtained according to ADM as $u = \sum_{n=0}^{\infty} u_n$. The convergence of this series has been proved in Seng et al. 1996. However, for some problems this series cannot be determined (Çelik et al., 2006), so we use an approximation of the solution from truncated series

$$U_M = \sum_{n=0}^{M} u_n$$

with

$$\lim_{M\to\infty} U_M = u. \tag{8}$$

Algorithm of HPM: To illustrate the basic idea of the homotopy technique (Liao, 1995; Liao, 1997) for Eq. 1, with the boundary condition:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0\,, \tag{9}$$

where, B is a boundary operator, we construct a homotopy $v(r,p): \Omega \times [0,1] \to R$ which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[L(v) + R(v) + N(v) - g] = 0, \;\; p \in [0,1] \tag{10a}$$

or

$$H(v,p) = L(v) - L(u_0) + p\,L(u_0) + p[R(v) + N(v) - g] = 0, \tag{10b}$$

where, $p \in [0,1]$is an embedding parameter,$u_0$is an initial approximate of Eq. 1, which satisfies the boundary conditions. Obviously, from Eq. 10 we have;

$$H(v,0) = L(v) - L(u_0) = 0, \tag{11}$$
$$H(v,1) = L(v) + R(v) + N(v) - g = 0 \tag{12}$$

Which the latter is actually, Eq. 1 with solution $u(r)$ and Eq. 11 has $u_0(r)$ its solution, so we have $v(r,0) = u_0(r)$, $v(r,1) = u(r)$, where $r = x \in \Omega$, is spatial independent variable. The changing process of p from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation, and $(v) - L(u_0)$, and $L(v) + R(v) + N(v) - g$ are called homotopic.

Assume that the solution of Eq. 10 can be written as a power series in$p$:

$$v = v_0 + p\, v_1 + p^2 v_2 + \cdots, \tag{13}$$

setting $p = 1$ results in the approximate solution of Eq. 1:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots. \tag{14}$$

Algorithm of SDHPM: Now, from above algorithms we can construct the basic idea of algorithm of SDHPM as follows: we decomposed the linear differential operator L in Eq. 1a into two parts of differential operators:

$$L(u) = \alpha L(w) + \beta L(h), \tag{15}$$

where $\alpha + \beta = 1,\ \alpha, \beta \in [0,1]$. By this definition, we can split Eq. 1a into two types of differential operator equations; one is linear and another is non-linear as:

$$L(w) + R(w) = 0, \tag{16}$$
$$L(h) + N(h) - g = 0. \tag{17}$$

We apply ADM as explained above on Eq. 16 to find the solution as series $w_n,\ n = 1,2,\ldots$ depending on the initial condition u_0, then using the result as an initial condition for the series solution $h_n,\ n = 1,2,\ldots$ that is obtained by using algorithm of HPM for Eq. 17 respectively. Repeating this iterative procedure between Eq. 16 and Eq. 17 by exchange, in order to reach to the original series solution $u_n,\ n = 1,2,\ldots$, then use (8) to obtain on the solution u.

2.1. Algorithm analysis of SDHPM for one-dimensional CNSE

Consider the unsteady state one-dimensional compressible Navier-Stokes equations as the form:

$$\rho_t + \rho\, u_x + u\, \rho_x = 0, \tag{18}$$
$$\rho\,(u_t + u u_x) - \mu_1 u_{xx} - \tfrac{\mu_2}{3} u_{xx} + nk\rho^{n-1}\rho_x = 0. \tag{19}$$

with the initial conditions:

$$u_0 = u(x,0),\ \rho_0 = \rho(x,0) \tag{20}$$

where u represent the velocity component of the fluid, ρ its density and the parameters μ_1 and μ_2 are the kinematic viscosities of the fluid.

Now, we start applying the ADM algorithm for Eqs. 18-19 with the initial conditions (20), $n = 2$, and divided Eq. 19 onρ. If $L_t = \frac{\partial}{\partial t}$, $L_x = \frac{\partial}{\partial x}$, $L_{xx} = \frac{\partial^2}{\partial x^2}$ then the Eqs. 18-19 can rewrite with operator form as:

$$L_t\rho + \rho\, L_x u + u\, L_x\rho = 0, \tag{21}$$
$$L_t u + u L_x u - \frac{\mu_1}{\rho} L_{xx} u - \frac{\mu_2}{3\rho} L_{xx} u + 2k L_x \rho = 0. \tag{22}$$

By taking the inverse operator L_t^{-1}, the Eqs. 21-22 are given by;

$$\rho(x,t) = \rho(x,0) - L_t^{-1}(\rho\, L_x u) - L_t^{-1}(u\, L_x\rho) = 0, \tag{23}$$
$$u(x,t) = u(x,0) - L_t^{-1}(u L_x u) + L_t^{-1}\left(\frac{\mu_1}{\rho} L_{xx} u\right) + L_t^{-1}\left(\frac{\mu_2}{3\rho} L_{xx} u\right) - L_t^{-1}(2k L_x \rho) = 0. \tag{24}$$

The components solutions can be written as; $u(x,t) = \sum_{n=0}^{\infty} u_n$, and $\rho(x,t) = \sum_{n=0}^{\infty} \rho_n$ with the nonlinear operator $Nu = \Psi(u) = u L_x u$.

The associated decomposition method is given by:

$$u_0 = u(x,0),\ \rho_0 = \rho(x,0) \tag{25a}$$

$$\left.\begin{array}{c} \rho_{n+1} = -L_t^{-1}(\rho_n L_x u_n) - L_t^{-1}(u_n L_x \rho_n) \\ u_{n+1} = -L_t^{-1}(\Psi(u_n)) + L_t^{-1}\left(\frac{\mu_1}{\rho_{n+1}} L_{xx} u_n\right) + \\ L_t^{-1}\left(\frac{\mu_2}{3\rho_{n+1}} L_{xx} u_n\right) - L_t^{-1}(2k L_x \rho_{n+1}) \end{array}\right\}. \tag{25b}$$

We decomposed Ψ according to the series $\sum_{n=0}^{\infty} A_n$, where A_n is calculated by the Adomian polynomial, then we obtain:

$$\left.\begin{array}{c} A_0 = u_0 \frac{\partial u_0}{\partial x} \\ A_1 = u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x} \\ A_2 = u_0 \frac{\partial u_2}{\partial x} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_0}{\partial x} \\ \vdots \end{array}\right\} \tag{26}$$

and so on. Consequently the iterative solutions are;

$$\left.\begin{array}{c} \rho_0 = \rho(x,0)\ , u_0 = u(x,0) \\ \rho_1 = -L_t^{-1}(\rho_0 L_x u_0) - L_t^{-1}(u_0 L_x \rho_0) \\ u_1 = -L_t^{-1}(A_0) + L_t^{-1}\left(\frac{\mu_1}{\rho_1} L_{xx} u_0\right) + L_t^{-1}\left(\frac{\mu_2}{3\rho_1} L_{xx} u_0\right) - L_t^{-1}(2k L_x \rho_1) \\ \rho_2 = -L_t^{-1}(\rho_1 L_x u_1) - L_t^{-1}(u_1 L_x \rho_1) \\ u_2 = -L_t^{-1}(A_1) + L_t^{-1}\left(\frac{\mu_1}{\rho_2} L_{xx} u_1\right) + L_t^{-1}\left(\frac{\mu_2}{3\rho_2} L_{xx} u_1\right) - L_t^{-1}(2k L_x \rho_2) \\ \vdots \end{array}\right\} \tag{27}$$

and so on.

Now, by using HPM algorithm to Eqs. 18-19, we have:

$$H(f,p) = (1-p)\left[\frac{\partial f}{\partial t} - \frac{\partial \rho_0}{\partial t}\right] + p\ \left[\frac{\partial f}{\partial t} + f\frac{\partial v}{\partial x} + v\frac{\partial f}{\partial x}\right] = 0$$

$$H(v,p) = (1-p)\left[\frac{\partial v}{\partial t} - \frac{\partial u_0}{\partial t}\right] + p\ \left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + 2k\frac{\partial f}{\partial x} - \frac{\mu_1}{f}\frac{\partial^2 v}{\partial x^2} - \frac{\mu_2}{3f}\frac{\partial^2 v}{\partial x^2}\right] = 0$$

or

$$\frac{\partial f}{\partial t} - \frac{\partial \rho_0}{\partial t} = p\ \left[-f\frac{\partial v}{\partial x} - v\frac{\partial f}{\partial x} - \frac{\partial \rho_0}{\partial t}\right] = 0$$

$$\frac{\partial v}{\partial t}-\frac{\partial u_0}{\partial t}=p\left[-v\frac{\partial v}{\partial x}-2k\frac{\partial f}{\partial x}+\frac{\mu_1}{f}\frac{\partial^2 v}{\partial x^2}+\frac{\mu_2}{3f}\frac{\partial^2 v}{\partial x^2}\right]=0.$$

By assuming the solution as a power series in p, we have:

$$\frac{\partial}{\partial t}(f_0+pf_1+p^2f_2+\cdots)-\frac{\partial\rho_0}{\partial t}=p\left[-(f_0+pf_1+p^2f_2+\cdots)\frac{\partial}{\partial x}(v_0+pv_1+p^2v_2+\cdots)-(v_0+pv_1+p^2v_2+\cdots)\frac{\partial}{\partial x}(f_0+pf_1+p^2f_2+\cdots)-\frac{\partial\rho_0}{\partial t}\right]$$

$$\frac{\partial}{\partial t}(v_0+pv_1+p^2v_2+\cdots)-\frac{\partial u_0}{\partial t}=p\left[-(v_0+pv_1+p^2v_2+\cdots)\frac{\partial}{\partial x}(v_0+pv_1+p^2v_2+\cdots)-2k\frac{\partial}{\partial x}(f_0+pf_1+p^2f_2+\cdots)+\frac{\mu_1}{(f_0+pf_1+p^2f_2+\cdots)}\frac{\partial^2}{\partial x^2}(v_0+pv_1+p^2v_2+\cdots)+\frac{\mu_2}{3(f_0+pf_1+p^2f_2+\cdots)}\frac{\partial^2}{\partial x^2}(v_0+pv_1+p^2v_2+\cdots)-\frac{\partial u_0}{\partial t}\right]$$

By equating the terms which have the same powers of p; we get

$$\left.\begin{array}{ll}
p^0: \frac{\partial f_0}{\partial t}-\frac{\partial\rho_0}{\partial t}=0 & and,\quad \frac{\partial v_0}{\partial t}-\frac{\partial u_0}{\partial t}=0\\
p^1: \frac{\partial f_1}{\partial t}+\frac{\partial\rho_0}{\partial t}+v_0\frac{\partial f_0}{\partial x}+f_0\frac{\partial v_0}{\partial x}=0 & and,\quad \frac{\partial v_1}{\partial t}+\frac{\partial u_0}{\partial t}+v_0\frac{\partial v_0}{\partial x}+\mu_1(2-f_0)\frac{\partial^2 v_0}{\partial x^2}+\frac{\mu_2}{3}(2-f_0)\frac{\partial^2 v_0}{\partial x^2}=0\\
p^2: \frac{\partial f_2}{\partial t}+v_0\frac{\partial f_1}{\partial x}+v_1\frac{\partial f_0}{\partial x}+f_0\frac{\partial v_1}{\partial x}+f_1\frac{\partial v_0}{\partial x}=0 & and,\quad \frac{\partial v_2}{\partial t}+v_0\frac{\partial v_1}{\partial x}+v_1\frac{\partial v_0}{\partial x}+\mu_1(2-f_0)\frac{\partial^2 v_1}{\partial x^2}-\mu_1 f_1\frac{\partial^2 v_0}{\partial x^2}+\frac{\mu_2}{3}(2-f_0)\frac{\partial^2 v_1}{\partial x^2}-\frac{\mu_2}{3}f_1\frac{\partial^2 v_0}{\partial x^2}=0\\
\vdots &
\end{array}\right\}\quad (28)$$

and so on.

Then the analytical approximate solution can be found by setting $p=1$ as;

$$\rho=\lim_{p\to 1}f=f_0+f_1+f_2+\cdots$$

and

$$u=\lim_{p\to 1}v=v_0+v_1+v_2+\cdots \quad (29)$$

now, we are applying the SDHPM algorithm for Eq. 18 as the form:

By applying (16) and (17) with$\alpha=\beta=0.5$ on (18), we obtain:

$$L(w)=\rho=-2L_t^{-1}(u\,L_x\rho) \quad (30)$$
$$L(h)=\rho=-2L_t^{-1}(\rho\,L_x u) \quad (31)$$

applying ADM for (30) with initial conditions $u_0=u(x,0)$, $\rho_0=\rho(x,0)$ to obtain;

$$w_1=w_0+L_t^{-1}\left(-2\,u_0\frac{\partial\rho_0}{\partial x}\right)\ ,\ where\ w_0=\rho_0, \quad (32)$$

applying HPM for (31) with the result of (32) to obtain;

$$h_1=h_0+L_t^{-1}\left(-2\left(w_1\frac{\partial u_0}{\partial x}\right)-\frac{\partial w_1}{\partial t}\right)\ ,\ where\ h_0=w_1 \quad (33)$$

then

$$\rho_1=\alpha\,w_1+\beta\,h_1\,. \quad (34)$$

Using the same procedure on Eq. 19 to obtain:

$$w_1^*=w_0^*+L_t^{-1}\left(2\left(\mu_1+\frac{\mu_2}{3}\right)\frac{\partial^2 u_0}{\partial x^2}\right)\ ,\ where\ w_0^*=u_0, \quad (35)$$

$$h_1^*=h_0^*+L_t^{-1}\left(-2\left(w_1^*\frac{\partial w_1^*}{\partial x}\right)-4k\frac{\partial w_1}{\partial x}-\frac{\partial w_1^*}{\partial t}\right)\ ,\ where\ h_0^*=w_1^* \quad (36)$$

then

$$u_1=\alpha\,w_1^*+\beta\,h_1^* \quad (37)$$

After repeating this procedure between two schemes (ADM &HPM) by exchange, we have:

$$w_2=w_1+L_t^{-1}\left(-2\,u_1\frac{\partial\rho_1}{\partial x}\right)\ , \quad (38)$$

$$h_2=h_1+L_t^{-1}\left(-2\left(w_1\frac{\partial u_1}{\partial x}\right)-2\left(\rho_1\frac{\partial w_1^*}{\partial x}\right)\right)\ ,\ where\ h_1=w_2 \quad (39)$$

$$\rho_2=\alpha\,w_2+\beta\,h_2\,. \quad (40)$$

$$w_2^*=w_1^*+L_t^{-1}\left(2\left(\mu_1+\frac{\mu_2}{3}\right)\frac{\partial^2 u_1}{\partial x^2}\right)\ , \quad (41)$$

$$h_2^*=h_1^*+L_t^{-1}\left(-2\left(w_1^*\frac{\partial u_1}{\partial x}\right)-4k\frac{\partial\rho_1}{\partial x}-2\left(u_1\frac{\partial w_1^*}{\partial x}\right)\right)\ ,\ where\ h_1^*=w_2^* \quad (42)$$

$$u_2=\alpha\,w_2^*+\beta\,h_2^* \quad (43)$$

$$\vdots$$

so on.

The successive solutions that can be written as a sum;

$$\rho(x,t)=\rho_0+\rho_1+\rho_2+\cdots=\sum_{n=0}^{\infty}\rho_n(x,t)$$

and

$$u(x,t)=u_0+u_1+u_2+\cdots=\sum_{n=0}^{\infty}u_n(x,t).$$

The convergence of these series will be proved theoretically in the next section. However, for some problems these series cannot be determined, so we use an approximation of the solution from truncated series:

$$V_M=\sum_{n=0}^{\infty}\rho_n(x,t)$$

with

$$\lim_{M\to\infty}V_M=\rho$$

and

$$U_M=\sum_{n=0}^{\infty}u_n(x,t)$$

with

$$\lim_{M\to\infty}U_M=u.$$

The acceleration for this convergence means the need to few terms of the above equation, for obtaining the formula which approximates to the exact solution.

2.2. Algorithm analysis of SDHPM for two-dimensional CNSE

Consider the unsteady state two-dimensional compressible Navier-Stokes equation as the form:

$$\rho_t + \rho\, u_x + \rho\, v_y + u\, \rho_x + v\, \rho_y = 0, \tag{44}$$

$$\rho\left(u_t + u u_x + v v_y\right) - \mu_1(u_{xx} + u_{yy}) - \tfrac{\mu_2}{3}(u_{xx} + v_{xy}) + nk\rho^{n-1}\rho_x = 0, \tag{45}$$

$$\rho\left(v_t + u v_x + v v_y\right) - \mu_1(v_{xx} + v_{yy}) - \tfrac{\mu_2}{3}(v_{yy} + u_{xy}) + nk\rho^{n-1}\rho_y = 0, \tag{46}$$

with the initial conditions:

$$u_0 = u(x,0),\ v_0 = v(x,0),\ \rho_0 = \rho(x,0). \tag{47}$$

where u and v represent the velocity component of the fluid, ρ its density and the parameters μ_1 and μ_2 are the kinematic viscosities of the fluid.

As the same manner in section (2-1), we can apply algorithm of SDHPM which is represented by Eqs. 30-43 on Eqs. 44-46 after the division of Eqs. 45- 46 on ρ with $n = 2$, to obtain:

$$\rho_0 = \rho(x,0), u_0 = u(x,0),\ v_0 = v(x,0),$$

$$\left.\begin{aligned} w_1 &= w_0 + L_t^{-1}\left(-2\, u_0 \frac{\partial \rho_0}{\partial x} - 2 v_0 \frac{\partial \rho_0}{\partial y}\right),\ where\ w_0 = \rho_0 \\ h_1 &= h_0 + L_t^{-1}\left(-2\left(w_1 \frac{\partial u_0}{\partial x}\right) - 2\left(w_1 \frac{\partial v_0}{\partial y}\right) - \frac{\partial w_1}{\partial t}\right),\ where\ h_0 = w_1 \end{aligned}\right\} \to \rho_1 = \alpha\, w_1 + \beta\, h_1$$

$$\left.\begin{aligned} w_1^* &= w_0^* + L_t^{-1}\left(2\left(\mu_1 + \frac{\mu_2}{3}\right)\frac{\partial^2 u_0}{\partial x^2} + 2\mu_1 \frac{\partial^2 u_0}{\partial y^2} + 2\frac{\mu_2}{3}\frac{\partial}{\partial x}\left(\frac{\partial v_0}{\partial y}\right)\right),\ where\ w_0^* = u_0 \\ h_1^* &= h_0^* + L_t^{-1}\left(-2\left(w_1^* \frac{\partial w_1^*}{\partial x}\right) - 4k\frac{\partial \rho_0}{\partial x} - 2\left(v_0 \frac{\partial v_0}{\partial y}\right) - \frac{\partial w_1^*}{\partial t}\right),\ where\ h_0^* = w_1^* \end{aligned}\right\} \to u_1 = \alpha\, w_1^* + \beta\, h_1^*$$

$$\left.\begin{aligned} w_1^{**} &= w_0^{**} + L_t^{-1}\left(2\left(\mu_1 + \frac{\mu_2}{3}\right)\frac{\partial^2 v_0}{\partial y^2} + 2\mu_1 \frac{\partial^2 v_0}{\partial x^2}\right),\ where\ w_0^{**} = v_0 \\ h_1^{**} &= h_0^{**} + L_t^{-1}\begin{pmatrix} -2\left(w_1^{**} \frac{\partial w_1^{**}}{\partial y}\right) - 4k\frac{\partial \rho_0}{\partial y} - \\ 2\left(u_0 \frac{\partial w_1^{**}}{\partial x}\right) + 4\frac{\mu_2}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial y}\right) \\ -2\frac{\mu_2 \rho_0}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial y}\right) - \frac{\partial w_1^{**}}{\partial t} \end{pmatrix},\ where\ h_0^{**} = w_1^{**} \end{aligned}\right\} \to v_1 = \alpha\, w_1^{**} + \beta\, h_1^{**}.$$

Then, after repeating this procedure between two schemes (ADM &HPM) by exchange, we have:

$$w_2 = w_1 + L_t^{-1}\left(-2\, u_1 \frac{\partial \rho_1}{\partial x} - 2 v_1 \frac{\partial \rho_1}{\partial y}\right)$$

$$h_2 = h_1 + L_t^{-1}\left(-2\left(\rho_1 \frac{\partial u_0}{\partial x}\right) - 2\left(\rho_0 \frac{\partial u_1}{\partial x}\right) - 2\left(\rho_1 \frac{\partial v_0}{\partial y}\right) - 2\left(\rho_0 \frac{\partial v_1}{\partial y}\right)\right),\ where\ h_1 = w_2$$

$$\rho_2 = \alpha\, w_2 + \beta\, h_2$$

$$w_2^* = w_1^* + L_t^{-1}\left(2\left(\mu_1 + \frac{\mu_2}{3}\right)\frac{\partial^2 u_1}{\partial x^2} + 2\mu_1 \frac{\partial^2 u_1}{\partial y^2} + 2\frac{\mu_2}{3}\frac{\partial}{\partial x}\left(\frac{\partial v_1}{\partial y}\right)\right)$$

$$h_2^* = h_1^* + L_t^{-1}\left(-2\left(u_0 \frac{\partial u_1}{\partial y}\right) - 2\left(u_1 \frac{\partial u_0}{\partial y}\right) - 4k\frac{\partial \rho_1}{\partial x} - 2\left(v_0 \frac{\partial v_1}{\partial y}\right) - 2\left(v_1 \frac{\partial v_0}{\partial y}\right)\right),\ where\ h_1^* = w_2^*$$

$$u_2 = \alpha\, w_2^* + \beta\, h_2^*$$

$$w_2^{**} = w_1^{**} + L_t^{-1}\left(2\left(\mu_1 + \frac{\mu_2}{3}\right)\frac{\partial^2 v_1}{\partial y^2} + 2\mu_1 \frac{\partial^2 v_1}{\partial x^2}\right)$$

$$h_2^{**} = h_1^{**} + L_t^{-1}\begin{pmatrix} 2\left(v_0 \frac{\partial v_1}{\partial y}\right) - 2\left(v_1 \frac{\partial v_0}{\partial y}\right) - 4k\frac{\partial \rho_1}{\partial y} - 2\left(u_0 \frac{\partial v_1}{\partial x}\right) - 2\left(u_1 \frac{\partial v_0}{\partial x}\right) + \\ 4\frac{\mu_2}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial y}\right) - 2\frac{\mu_2 \rho_1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial y}\right) - 2\frac{\mu_2 \rho_0}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial y}\right) \end{pmatrix},\ where\ h_1^{**} = w_2^{**}$$

$$v_2 = \alpha\, w_2^{**} + \beta\, h_2^{**}$$

$\vdots$

So on.

3. Numerical test and discussion

The theoretical analysis of SDHPM is applied here to find the analytical approximate solution of two test problems: The first is one-dimensional CNSE and the second is two-dimensional CNSE.

Test problem 1D (P1) CNSE: The one-dimensional CNSE (18, 19) with exact solutions

$u = a_0 + a_1 \tanh(\xi)$, $\rho = c_0 + c_1 \tanh(\xi)$, $where\ \xi = x - at$

and initial conditions

$u_0 = a_0 + a_1 \tanh(x)$, $\rho_0 = c_0 + c_1 \tanh(x)$

where;

$$a_0 = a + \frac{a^2 k\mu_2}{-ak\mu_2 - \sqrt{a^2k^2\mu_2^2 - 4ac_0k^2\mu_2^2 + 8c_0k^3\mu_2^2}} - \frac{2ac_0k\mu_2}{-ak\mu_2 - \sqrt{a^2k^2\mu_2^2 - 4ac_0k^2\mu_2^2 + 8c_0k^3\mu_2^2}} + \frac{4c_0k\mu_2}{-ak\mu_2 - \sqrt{a^2k^2\mu_2^2 - 4ac_0k^2\mu_2^2 + 8c_0k^3\mu_2^2}} + \frac{a\sqrt{k^2(a^2 - 4ac_0 + 8c_0k)\mu_2^2}}{-ak\mu_2 - \sqrt{a^2k^2\mu_2^2 - 4ac_0k^2\mu_2^2 + 8c_0k^3\mu_2^2}}$$

$$a_1 = 2\left(2a\mu_2 - \frac{a^2\mu_2}{c_0} - 4k\mu_2 - \frac{a\sqrt{k^2(a^2 - 4ac_0 + 8c_0k)\mu_2^2}}{c_0k}\right)/3c_0k$$

$$c_1 = 2\left(-ak\mu_2 - \sqrt{a^2k^2\mu_2^2 - 4ac_0k^2\mu_2^2 + 8c_0k^3\mu_2^2}\right)/3c_0k^2$$

and a is the constant speed.

The iterative solutions for this problem with $n = 2$ by using SDHPM can be obtained after we split the linear operator of time of Eqs. 18-19 as in Eqs. 16-17, then by using algorithm of SDHPM for Eqs. 18-19 that is represented by Eqs. 30-43, we get the successive analytical approximate solution of Eqs. 18 and 19 as the following:

$$\rho_0 = c_0 + c_1 \tanh(x),\ u_0 = a_0 + a_1 \tanh(x),$$
$$\rho_1 = c_0 + c_1 \tanh(x) + a_1 c_1 t^2 (a_0 + a_1 \tanh(x))(\tanh(x)^2 - 1)^2,$$
$$u_1 = a_0 + a_1 \tanh(x) + \frac{16 a_1^2 q^2 t^3 \sinh(x)(\cosh(x)^2 - \sinh(x)^2)^2 (3\sinh(x)^2 - \cosh(x)^2)}{3\cosh(x)^7},$$
$$\rho_2 = c_0 + c_1 \tanh(x) + a_1^2 c_1 q t^4 (a_0 + a_1 \tanh(x))(\tanh(x)^2 - 1)^3 (3\tanh(x)^2 - 1) + \frac{2a_1 c_1 t^5 (\tanh(x)^2 - 1)^2 (5a_1 \tanh(x)^2 + 2a_0 \tanh(x) - a_1)[60\, t a_1^2 q^2 \sinh(x)^7]}{45\cosh(x)^7} + \frac{2a_1 c_1 t^5 (\tanh(x)^2 - 1)^2 (5a_1 \tanh(x)^2 + 2a_0 \tanh(x) - a_1)[9k a_1 c_1 \cosh(x)^7 - 36 k a_1 c_1 \sinh(x)^2 \cosh(x)^5]}{45\cosh(x)^7} + \frac{2a_1 c_1 t^5 (\tanh(x)^2 - 1)^2 (5a_1 \tanh(x)^2 + 2a_0 \tanh(x) - a_1)[27 k a_1 c_1 \sinh(x)^4 \cosh(x)^3 + 18 k a_0 c_1 \sinh(x)\cosh(x)^6]}{45\cosh(x)^7},$$
$$u_2 = a_0 + a_1 \tanh(x) + \frac{16 t^3 k a_1 c_1 q \cosh(x)^9}{3\cosh(x)^9} - \frac{4 a_1 q t^4 \sinh(x)(\cosh(x)^2 - \sinh(x)^2)^2 (60 k a_1 c_1 \cosh(x)^6 \sinh(x))}{15\cosh(x)^{11}}.$$

where; $q = \mu_1 + \frac{\mu_2}{3}$.

Table 1 shows the comparison of numerical results between the present study, ADM and HPM, and Fig. 1 illustrates the exact and analytic approximate solutions for the present study at $t = 1$ and the analytic approximate solutions in different t for u_2, ρ_2. Fig. 2 explains the surface plot of exact and analytic approximate solutions resulted from the present study. Moreover, Fig. 3 shows the comparison of the exact and analytic approximate solutions for three method (SDHPM, ADM, HPM) at $t = 1$.

Table 1: L_2 and L_∞ comparison of present study, ADM and HPM for p1

Error Measurmets / *Methods*	$\frac{L_2}{L_\infty}(u_1)$	$\frac{L_2}{L_\infty}(u_2)$	$\frac{L_2}{L_\infty}(\rho_1)$	$\frac{L_2}{L_\infty}(\rho_2)$
ADM	1.13×10^{-2}	8.20×10^{-3}	1.60×10^{-2}	6.50×10^{-3}
	5.38×10^{-3}	2.80×10^{-3}	6.88×10^{-3}	2.90×10^{-3}
HPM	6.94×10^{-3}	6.62×10^{-3}	1.60×10^{-2}	1.58×10^{-3}
	2.64×10^{-3}	2.51×10^{-3}	6.88×10^{-3}	6.33×10^{-4}
SDHPM	6.18×10^{-4}	5.97×10^{-4}	1.59×10^{-3}	4.27×10^{-4}
	2.72×10^{-5}	2.62×10^{-5}	8.80×10^{-4}	1.86×10^{-5}

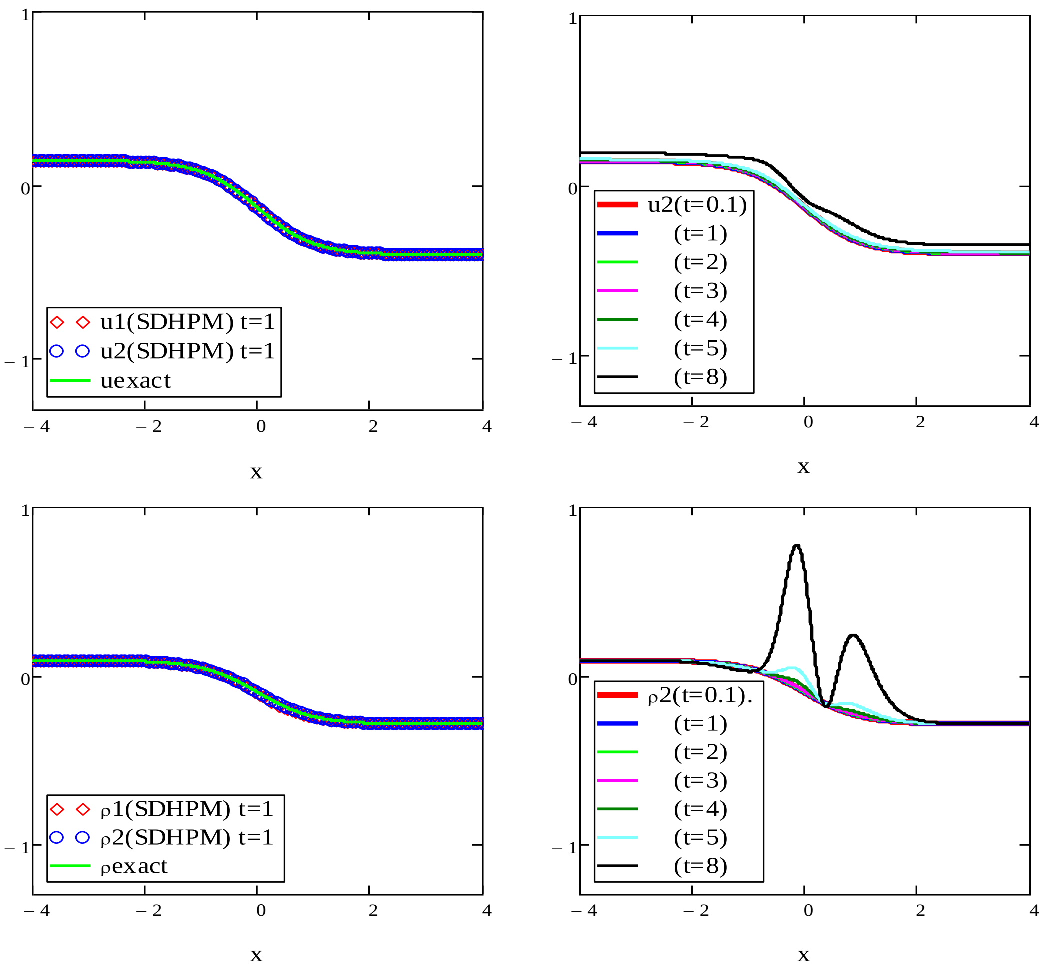

Fig. 1: Exact and analytic approximate solutions (u, ρ) and $u_2, \rho_2)$ with different times for P1

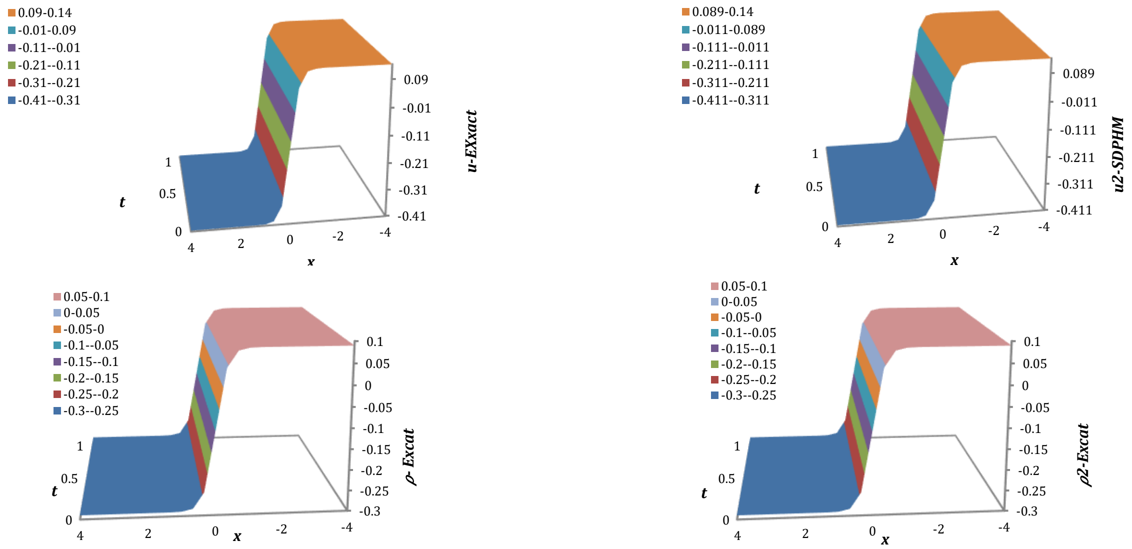

Fig. 2: Surface plot of exact and analytic approximate solutions resulting from SDHPM p1

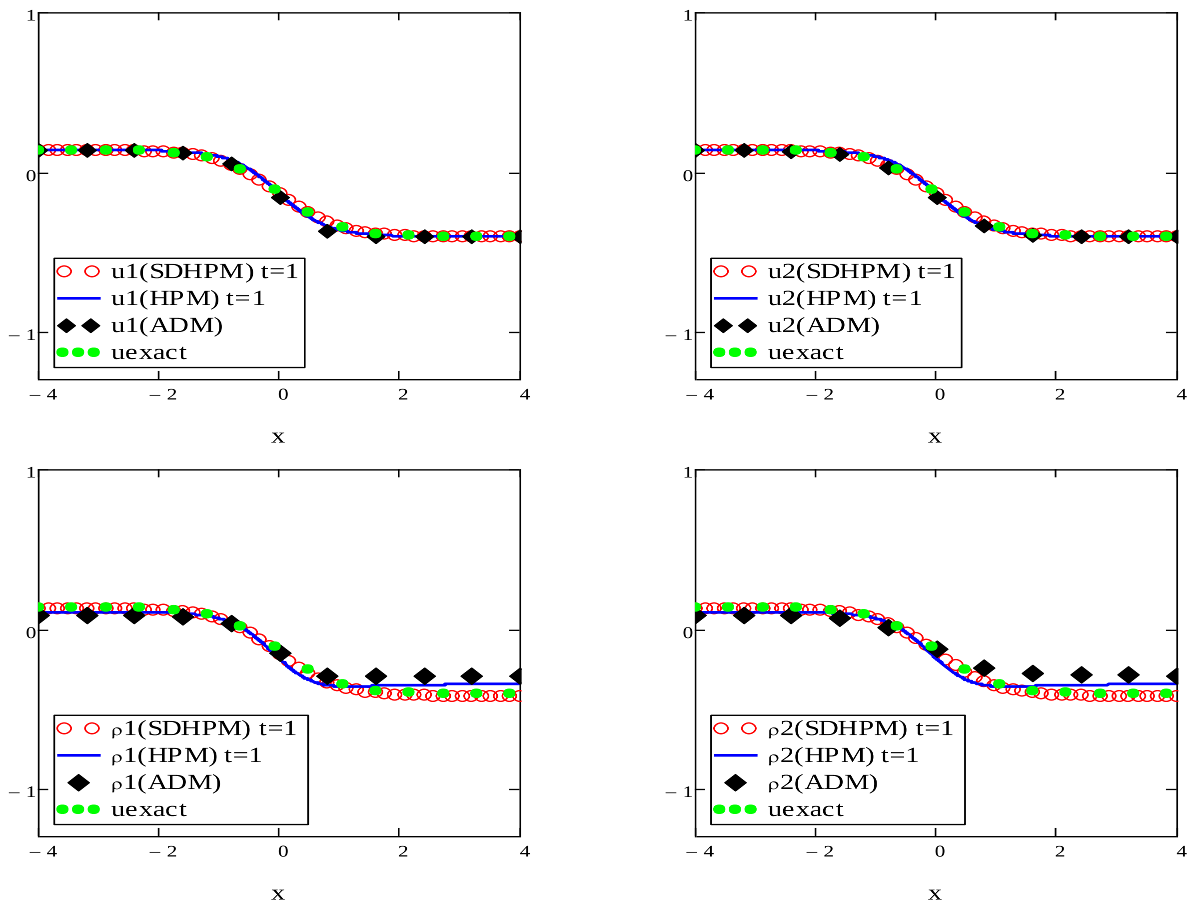

Fig. 3: Comparison between Exact and analytic approximate solutions (u, ρ) for P1

The measurement of L_2 and L_∞ errors for the velocity and density, which are shown in Table 1, ensure the ability of the suggested new method and its accuracy in finding the analytic approximate solutions of one-dimensional compressible Navier-Stokes equation. From our computations, which we found by using Mathcad.15 software; for SDHPM, we noted that the convergence of this method is increased at $t < 5$, while in the two other methods (ADM & HPM) the solutions are convergent to the exact solution at $t < 1$. Then; one can say that, the new method is effective and good method to find the solutions of one-dimensional CNSE compared to the two standard methods (ADM, HPM).

Test problem (P2) 2D CNSEs: The two-dimensional CNSEs (44-46) with exact solutions

$u = a - b_1 \tanh(\xi)\,,\ v = b_1 \tanh(\xi)\,, \rho = c_0 + \frac{2\mu_1 b_1}{k} tanh(\xi)^2,\ \ where\ \xi = x + y - at + \xi_0;$

and initial conditions

$u_0 = a - b_1 \tanh(x + y + \xi_0)$,
$v_0 = b_1 \tanh(x + y + \xi_0)$,
$\rho_0 = c_0 + \frac{2\mu_1 b_1}{k} tanh(x + y + \xi_0)^2$,

where, c_0, b_1, ξ_0, k are arbitrary constants and a is the speed constant. Now, by applying the algorithm of SDHPM for Eqs. 44-46 that is introduced in section 2.2, we get the successive analytical approximate solution of Eqs. 44-46 as the following:

$\rho_1 = c_0 + \frac{\mu_1 b_1}{k}\, \delta\, (2\delta + 4at(\delta^2 - 1))$,
$u_1 = a - b_1\delta - b_1 t(\delta^2 - 1)((a - b_1\delta) - 8\mu_1\delta)$,
$v_1 = b_1\delta + b_1 t(\delta^2 - 1)\left(\delta\left(b_1 + 8\mu_1 - \frac{4\mu_2}{3}\right) + (a - b_1\delta)\right)$,
$\rho_2 = \left(c_0 + \frac{\mu_1 b_1}{k}\delta^2\right)\left[1 + \left(\frac{b_1^2\mu_2 t^2(2\mu_1 b_1\delta^2 + c_0 k)(\delta^2-1)^2}{3k}\right) - \frac{b_1 p_2 t^3[8\delta(2-3\delta^2)(\delta^2-1)](a-b_1\delta)+2b_1(\delta^2-1)^2(3\delta^2-1)}{3}\right]$
$+ \frac{2ab_1\mu_1 t^2(3\delta^4-4\delta^2+1)}{k}\left((a - b_1\delta) + b_1 + \frac{4b_1 p_2 t\delta(a-b_1\delta)(\delta^2-1)}{3}\right) + \frac{4b_1\mu_1 t\delta(\delta^2-1)}{k}\left(a + \frac{4b_1^2 p_1^2 t^3\delta(3\delta^2-1)(\delta^2-1)^2}{27} + 4b_1\mu_1 t\delta(\delta^2 - 1) + \frac{b_1\mu_2 t\delta(2b_1\mu_1\delta^2+c_0 k)(\delta^2-1)}{3k}\right)$,
$u_2 = a - b_1\delta(1 + 10b_1\mu_1 t^2\delta^2 + 80\mu_1^2 t^2\delta^2) - b_1 t(a - b_1\delta)(\delta^2 - 1)$,
$v_2 = b_1\delta + b_1 t(\delta^2 - 1)(b_1\delta + (a - b_1\delta)) + \frac{1}{2}(b_1 t^2(a - b_1\delta)(\delta - 1)(\delta + 1)(2a\delta - 3b_1\delta + b_1))$.
⋮

where; $\delta = \tanh(x + y + \xi_0)$, $p_1 = 3q + 3\mu_1 - \mu_2$, $p_2 = q + \mu_1$, $q = \mu_1 + \frac{\mu_2}{3}$.

Tables 2 and 3 show the comparison of numerical results of the present study, ADM and HPM for p2 at $t = 0.1,1$. Fig. 4 explains the comparison of absolute errors between the present study, ADM and HPM for p2 at $t = 0.1,1$ for (u_2, v_2).

Fig. 5 illustrates the surface plot of exact and analytic approximate solutions for SDHPM to the two-dimensional compressible Navier-Stokes equations.

From the tables of errors which explain a comparison between the present study, ADM and HPM for different values of time by using the measurement of L_2 and L_∞ errors for the velocity and density, the effect and the accuracy of the present study are noted in comparison to the other methods (ADM, HPM). In addition, to a good convergence, and from our computations, which we found by using Mathcad.15 software for SDHPM, we note that the convergence of this method increases at $t < 5$. Moreover, from the plots of absolute errors for the three methods (SDHPM, ADM, HPM), we show the efficiency and the high accuracy of SDHPM. So, one can say that the new method is better and more accurate as compared to the standard methods (ADM & HPM).

Table 2: L_2 and L_∞ comparison of the present study, ADM and HPM for P2 at $t = 0.1$

Error Measurmets / *Methods*	$\frac{L_2}{L_\infty}(u_1)$	$\frac{L_2}{L_\infty}(u_2)$	$\frac{L_2}{L_\infty}(v_1)$	$\frac{L_2}{L_\infty}(v_2)$	$\frac{L_2}{L_\infty}(\rho_1)$	$\frac{L_2}{L_\infty}(\rho_2)$
ADM	7.48×10^-5^	1.14×10^-5^	7.45×10^-5^	7.25×10^-5^	5.46×10^-4^	1.07×10^-5^
	7.01×10^-5^	8.27×10^-6^	7.25×10^-5^	6.69×10^-5^	2.03×10^-4^	3.99×10^-6^
HPM	7.89×10^-5^	7.58×10^-5^	7.65×10^-5^	7.51×10^-5^	1.11×10^-5^	1.79×10^-6^
	8.14×10^-5^	7.53×10^-5^	7.68×10^-5^	7.38×10^-5^	3.95×10^-6^	1.53×10^-6^
SDHPM	7.61×10^-6^	4.03×10^-6^	3.19×10^-6^	3.95×10^-8^	6.03×10^-9^	7.03×10^-11^
	6.49×10^-7^	3.43×10^-7^	2.72×10^-6^	4.01×10^-8^	6.19×10^-9^	7.01×10^-11^

Table 3: L_2 and L_∞ comparison of the present study, ADM and HPM for P2 at $t = 1$

Error Measurmets / *Methods*	$\frac{L_2}{L_\infty}(u_1)$	$\frac{L_2}{L_\infty}(u_2)$	$\frac{L_2}{L_\infty}(v_1)$	$\frac{L_2}{L_\infty}(v_2)$	$\frac{L_2}{L_\infty}(\rho_1)$	$\frac{L_2}{L_\infty}(\rho_2)$
ADM	7.38×10^-4^	7.97×10^-5^	7.42×10^-5^	7.25×10^-5^	5.69×10^-4^	1.07×10^-4^
	6.67×10^-4^	4.73×10^-5^	7.22×10^-5^	4.55×10^-5^	2.39×10^-4^	3.99×10^-5^
HPM	7.71×10^-4^	7.43×10^-4^	7.50×10^-4^	7.40×10^-4^	1.12×10^-4^	1.83×10^-5^
	7.80×10^-4^	7.19×10^-4^	7.34×10^-4^	7.05×10^-4^	3.99×10^-5^	1.53×10^-5^
SDHPM	3.51×10^-5^	1.04×10^-6^	9.06×10^-6^	3.25×10^-6^	5.63×10^-7^	8.06×10^-8^
	3.03×10^-5^	9.15×10^-7^	7.31×10^-6^	3.10×10^-7^	5.94×10^-7^	8.63×10^-8^

4. Convergence analysis of SDHPM

In this section, we study the analysis of convergence in the same manner as (Alkalla et al., 2013; Inc, 2005; Jang, 2007) for the decomposition method to the nonlinear 1D compressible Navier-Stokes Eqs. 18-19. Let as consider the Hilbert space H which may be defined as $H = L^2(\Omega \times [0,1])$, the set of applications; $u: \Omega \times [0,1] \to \Re$ with

$$\int_{\Omega\times[0,1]} u^2 d\Omega < +\infty \quad (48)$$

and scalar product and induced norm:

$$(u, v) = \int_{\Omega\times[0,1]} u\, v\, d\Omega$$

and

$$\| u \|^2 = (u, u) \quad (49)$$

where, $\Re$ is real numbers.

The Adomian decomposition method is convergent if the following conditions are satisfied;

(I_u): $(L_t(\Delta u), \Delta u) \geq k_1 \| \Delta u \|^2, k_1 > 0, \forall\, u, \hat{u} \in H$
(II_u): Whatever may be $M > 0$, there exist a constant $C(M) > 0$ such that for $u, \hat{u} \in H$ with $\| u \| \leq M, \| \hat{u} \| \leq M$, we have:

$$(L_t(\Delta u), \mathrm{w}) \leq C\left(M, \left(\mu_1 + \frac{\mu_2}{3}\right)\right) \| \Delta u \| \| w \|$$

for every $w \in H$.
(I_ρ): $(L_t(\Delta\rho), \Delta\rho) \geq k_1 \| \Delta\rho \|^2, k_1 > 0, \forall\, \rho, \hat{\rho} \in H$
(II_ρ): Whatever may be $M > 0$, there exist a constant $C(M) > 0$ such that for $\rho, \hat{\rho} \in H$ with $\| \rho \| \leq M, \| \hat{\rho} \| \leq M$, we have:

$(L_t(\Delta\rho), w) \le C(M) \parallel \Delta\rho \parallel\parallel w \parallel$

for every $w \in H$.

Now, we will use the following theorem to satisfy the above conditions as (Alkalla et al., 2013; Inc, 2005).

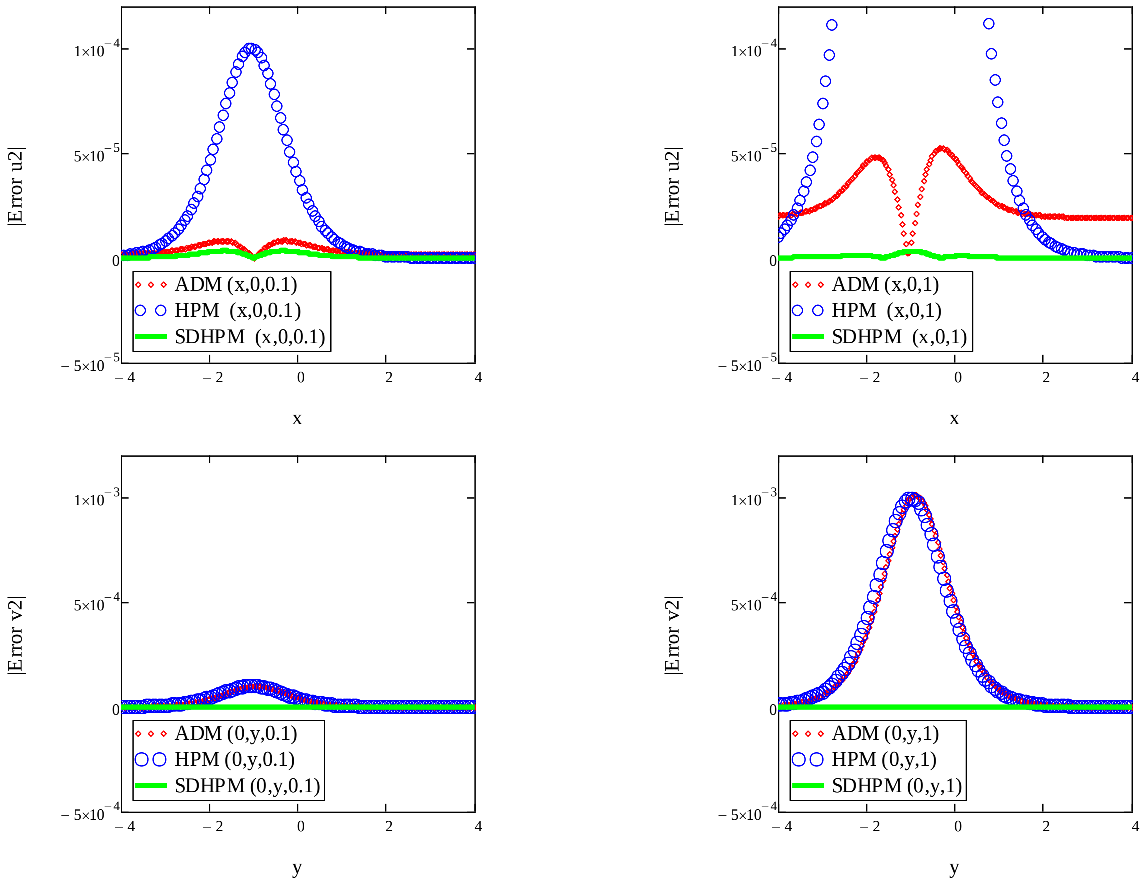

Fig. 4: Comparison of absolute errors of(u, v) between SDHPM, HPM and ADM

Theorem 1: If (I_ρ) and (II_ρ) are satisfied, then ADM of Eq. 21 is convergent.

Proof: It is easy to prove (I_ρ) and(II_ρ) as the same manner in (Al-Saif, 2015; Alkalla et al., 2013; Inc, 2005) to obtain on the results: Then condition (I_ρ) holds with$k_1 = -M(1-\delta_3)$, where δ_3 are constant and the condition (II_ρ) is satisfied with $C(M) = -M(1-\delta_4)$, where δ_4 is constant. Hence prove is complete.

Theorem 2: If (I_u) and (II_u) are satisfied, then ADM of Eq. 22 is convergent.

Proof: It is easy to prove (I_u) and (II_u) as the same manner in (Al-Saif, 2015; Alkalla et al., 2013; Inc, 2005) to obtained on the results: Then condition (I_u) holds with$k_1 = \delta_2 M - \left(\mu_1 + \frac{\mu_2}{3}\right)\delta_1$, where δ_1,δ_2 are constants and the condition (II_u) is satisfied with $C\left(M, \left(\mu_1 + \frac{\mu_2}{3}\right)\right) = M - \left(\mu_1 + \frac{\mu_2}{3}\right)$. Hence the prove is complete. Let us consider Eqs. 18 and 19 (after we apply the HPM) in the following form:

$$\left.\begin{array}{c} L_t(f) = L_t(\rho_0) + p\,[-fL_x(v) - vL_x(f) - L_t(\rho_0)] \\ L_t(v) = L_t(u_0) + p\left[-vL_x(v) - 2kL_x(f) + \frac{\mu_1}{f}L_{xx}(v) + \frac{\mu_2}{3f}L_{xx}(v) - L_t(u_0)\right] \end{array}\right\} \quad (50)$$

applying the inverse operator, L_t^{-1} to both sides of Eq. 50, we obtain

$$\left.\begin{array}{c} f = \rho_0 + p\,L_t^{-1}[-fL_x(v) - vL_x(f) - L_t(\rho_0)] \\ v = u_0 + p\,L_t^{-1}\left[-vL_x(v) - 2kL_x(f) + \frac{\mu_1}{f}L_{xx}(v) + \frac{\mu_2}{3f}L_{xx}(v) - L_t(u_0)\right] \end{array}\right\} \quad (51)$$

suppose that

$v = \sum_{i=0}^{\infty} p^i u_i$

and

$$f = \sum_{i=0}^{\infty} p^i \rho_i \quad (52)$$

Substituting (52) into the right-hand side of Eq. 51, yields

$$\left.\begin{array}{l} f = \rho_0 + p\, L_t^{-1}\left[-\sum_{i=0}^{\infty} p^i \rho_i\, L_x\left(\sum_{i=0}^{\infty} p^i u_i\right) - \sum_{i=0}^{\infty} p^i u_i\, L_x\left(\sum_{i=0}^{\infty} p^i \rho_i\right) - L_t(\rho_0)\right] \\ v = u_0 + p\, L_t^{-1}\left[\begin{array}{c} -\left(\sum_{i=0}^{\infty} p^i u_i\, L_x\left(\sum_{i=0}^{\infty} p^i u_i\right)\right) - 2kL_x\left(\sum_{i=0}^{\infty} p^i \rho_i\right) + \\ \frac{\mu_1}{\sum_{i=0}^{\infty} p^i \rho_i} L_{xx}\left(\sum_{i=0}^{\infty} p^i u_i\right) + \frac{\mu_2}{3\sum_{i=0}^{\infty} p^i \rho_i} L_{xx}\left(\sum_{i=0}^{\infty} p^i u_i\right) - L_t(u_0) \end{array}\right] \end{array}\right\} \quad (53)$$

if $p \to 1$, the exact solution may be obtained as;

$$\left.\begin{array}{l} \rho = L_t^{-1}\left[-\left(\sum_{i=0}^{\infty} \rho_i\, L_x\left(\sum_{i=0}^{\infty} u_i\right)\right) - \left(\sum_{i=0}^{\infty} u_i\, L_x\left(\sum_{i=0}^{\infty} \rho_i\right)\right)\right] \\ u = L_t^{-1}\left[-\left(\sum_{i=0}^{\infty} u_i\, L_x\left(\sum_{i=0}^{\infty} u_i\right)\right) - 2kL_x\left(\sum_{i=0}^{\infty} \rho_i\right) + \frac{\mu_1}{\sum_{i=0}^{\infty} \rho_i} L_{xx}\left(\sum_{i=0}^{\infty} u_i\right) + \frac{\mu_2}{3\sum_{i=0}^{\infty} \rho_i} L_{xx}\left(\sum_{i=0}^{\infty} u_i\right)\right] \end{array}\right\}$$

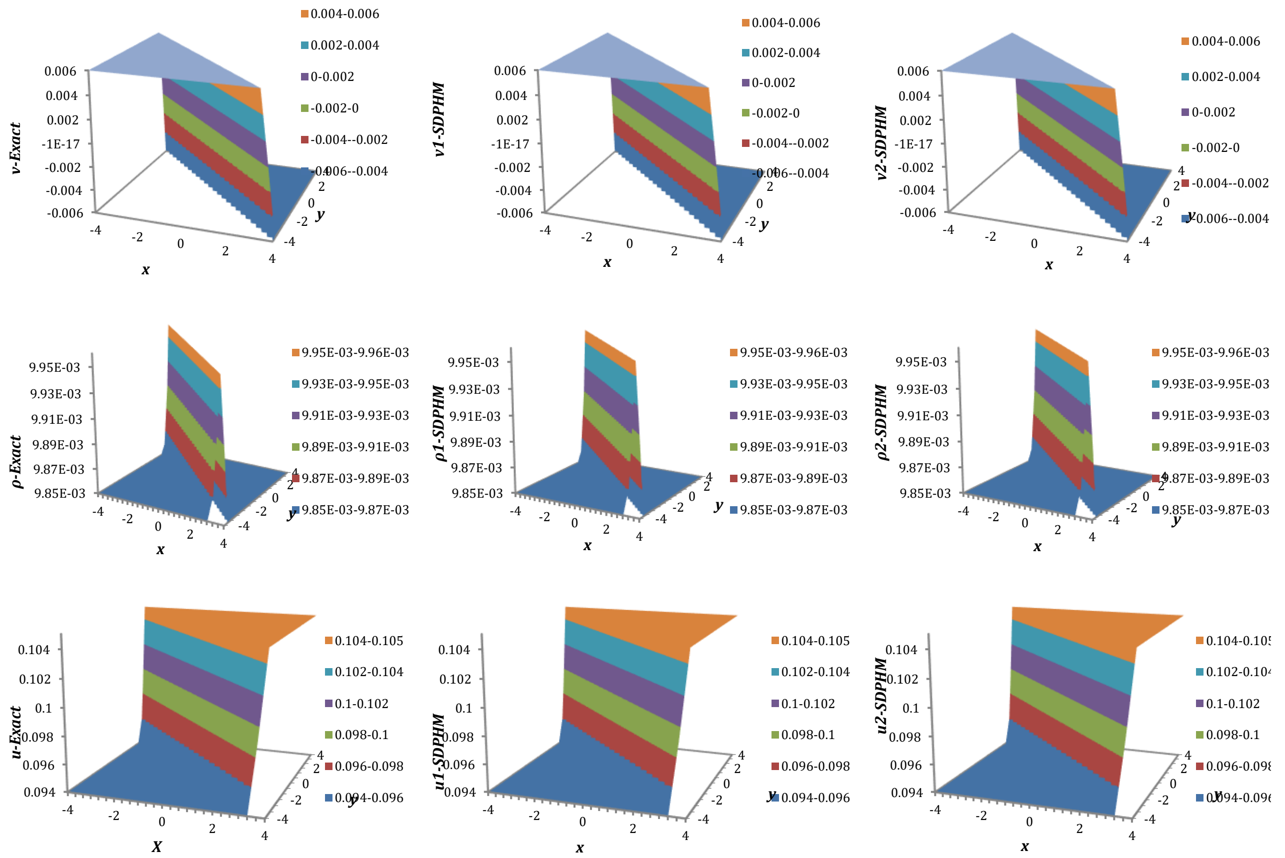

Fig. 5: Surface plot of exact and approximate solutions for SDHPM for P2

To study the convergence of this method, let us state the following theorem.

Theorem 3: (Sufficient Condition of Convergence (Biazar and Aminikhah, 2009)): Supposes that X and Y are Banach spaces and $N: X \to Y$ is a contractive nonlinear mapping, which is:

$\forall\, w, w^* \in X;\ \| N(w) - N(w^*) \| \le \gamma \| w - w^* \|,\ 0 < \gamma < 1.$

Then according to Banach's fixed point theorem N has a unique fixed point u, that is $N(u) = u$. Assume that the sequence generated by homotopy perturbation method can be written as;

$$W_n = N(W_{n-1}), \quad W_{n-1} = \sum_{i=0}^{n-1} W_i\ , \quad n = 1,2,3, \ldots$$

and suppose that:

$$W_0 = w_0 \in B_r(w) \ where\ B_r(w) = \{w^* \in X \mid \| w - w^* \| < r\}. \quad (54)$$

Then we have:

$(i) W_n \in B_r(w),. \quad (ii) \lim_{n\to\infty} W_n = w.$

As the same manner above we can study the convergence for two-dimensional compressible Navier-Stokes equations. Depending on the above theorems and their proofs, the convergence of SDHPM (sufficient condition of convergence) is to be hold. Also, the combination of the theorems gives us guarantee for convergence of the solutions that are obtained by SDHPM.

We illustrate the convergence of splitting Adomian decomposition homotopy perturbation method theoretically by applying the sufficient condition of convergence. According to the theorems of convergence, the convergence of splitting

Adomian decomposition homotopy perturbation method for the non-linear CNSEs (18-19) and (40-46) will be illustrated as follows respectively. By using definitions (48) and (49) and supposing that

$W_n = N(W_{n-1}),\ W_{n-1} = u_{n-1} = \rho_{n-1}$
$\rho_n = \sum_{j=0}^{n} \int_0^t \left(-u_j \frac{\partial \rho_j}{\partial x} - \rho_j \frac{\partial u_j}{\partial x}\right) dt\,, \quad n = 1,2,3,\dots$
$u_n = \sum_{j=0}^{n} \int_0^t \left(\frac{(3\mu_1+\mu_2)}{3\rho} \frac{\partial^2 u_j}{\partial x^2} - 2k \frac{\partial \rho_j}{\partial x} - \sum_{k=0}^{j} u_k \frac{\partial u_{j-k}}{\partial x}\right) dt\,, \quad n = 1,2,3,\dots$

with the theorem 3 (Sufficient Condition of Convergence) for the nonlinear mapping N, a sufficient condition for convergence of the SDHPM is the strict contraction of N.

For problem 1, we have:

$\| u_0 - u \| = \| a_0 + a_1 \tanh(x) - a_0 - a_1 \tanh(x - at) \|$, $\| \rho_0 - \rho \| = \| c_0 + c_1 \tanh(x) - c_0 - c_1 \tanh(x - at) \|$
$\| u_1 - u \| \le \| u_0 - u \| \gamma,\ \gamma = 0.994 < 1, \quad \| \rho_1 - \rho \| \le \| \rho_0 - \rho \| \gamma,\ \gamma = 0.985 < 1,$
$\| u_2 - u \| \le \| u_0 - u \| \gamma^2,\ \gamma^2 = 0.012 < 1, \quad \| \rho_2 - \rho \| \le \| \rho_0 - \rho \| \gamma^2,\ \gamma^2 = 0.791 < 1,$
$\vdots$
$\| u_n - u \| \le \| u_0 - u \| \gamma^n, \quad \| \rho_n - \rho \| \le \| \rho_0 - \rho \| \gamma^n$

Therefore,

$\lim_{n\to\infty} \| u_n - u \| \le \lim_{n\to\infty} \| u_0 - u \| \gamma^n = 0,\ \lim_{n\to\infty} \| \rho_n - \rho \| \le \lim_{n\to\infty} \| \rho_0 - \rho \| \gamma^n = 0,$

be hold.

For the Problem 2, we have

$\| u_0 - u \| = \| a_0 - b_1 \tanh(x + y + \xi_0) - a_0 + b_1 \tanh(x + y + \xi_0 - at) \|$, $\| v_0 - v \| = \| b_1 \tanh(x + y + \xi_0) - b_1 \tanh(x + y + \xi_0 - at) \|$, $\| \rho_0 - \rho \| = \| c_0 + \frac{2b_1\mu_1}{k} \tanh(x + y + \xi_0) - c_0 - \frac{2b_1\mu_1}{k} \tanh(x + y + \xi_0 - at)$
$\| u_1 - u \| \le \| u_0 - u \| \gamma, \gamma = 0.9813 < 1, \| v_1 - v \| \le \| v_0 - v \| \gamma,\ \gamma = 0.0636 < 1$
$\| \rho_1 - \rho \| \le \| \rho_0 - \rho \| \gamma, \gamma = 0.0613 < 1,$
$\| u_2 - u \| \le \| u_0 - u \| \gamma^2, \gamma^2 = 0.8911 < 1, \| v_2 - v \| \le \| v_0 - v \| \gamma^2, \gamma^2 = 0.0274 < 1,$
$\| \rho_2 - \rho \| \le \| \rho_0 - \rho \| \gamma^2,\ \gamma^2 = 0.0115 < 1,$
$\vdots$
$\| u_n - u \| \le \| u_0 - u \| \gamma^n, \quad \| v_n - v \| \le \| v_0 - v \| \gamma^n, \quad \| \rho_n - \rho \| \le \| \rho_0 - \rho \| \gamma^n$

therefore,

$\lim_{n\to\infty} \| u_n - u \| \le \lim_{n\to\infty} \| u_0 - u \| \gamma^n = 0\,, \lim_{n\to\infty} \| v_n - v \| \le \lim_{n\to\infty} \| v_0 - v \| \gamma^n = 0\,, \lim_{n\to\infty} \| \rho_n - \rho \| \le \lim_{n\to\infty} \| \rho_0 - \rho \| \gamma^n = 0,$
be hold.

5. Conclusion

In this paper, we proposed a new scheme called splitting decomposition homotopy perturbation method (SDHPM) to solve one and two-dimensional compressible Navier-Stokes equations. The results show that the SDHPM is an efficient method with good convergence and high accuracy to find analytical approximate solutions of two test unsteady state compressible problems. The convergence of this method increased at $t < 5$ while for the standard two methods ADM and HPM at $t < 1$. In addition to the measurement of L_2 and L_∞ errors for the velocity and density for the two problems explained, the high accuracy of the present study compared to the other two methods (ADM, HPM) is proved. Also, the application of SDHPM gave a simple powerful tool to obtain the solutions. Then, we conclude that the SDHPM is an efficient method with reasonable convergence and high accuracy to find analytic approximate solutions of one and two-dimensional compressible Navier-Stokes equations compare with ADM and HPM. Finally, from analysis of results, we can say that the tests are confirming the validity of a new method SDHPM to handle current complicated problems.

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