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Differential Quadrature Method for Steady Flow of an Incompressible Second-Order Viscoelastic Fluid and Heat Transfer Model

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Abstract The two-dimensional steady flow of an incompressible second-order viscoelastic fluid between two parallel plates was studied in terms of vorticity, the stream function and temperature equations. The governing equations were expanded with respect to a small parameter to get the zeroth- and first-order approximate equations. By using the differential quadrature method with only a few grid points, the high-accurate numerical results were obtained.

Key words differential quadrature method(DQM), second-order viscoelastic fluid, steady flow, heat transfer.
MSC 2000 65M99

1 Introduction

Although the computation of the fluid mechanics has great development during the last decades, due to the model in nonlinear fluid mechanics is very complicated, the more efficient techniques for solving this problem still attract the interest of many researchers. The differential quadrature method (DQM) introduced by Bellman, *et al.*[1,2] is an efficient numerical method for solving partial differential equations. In recent years, the DQM has been widely used for solving the problems of engineering and physical sciences[3-7]. The advantage of the DQM lies in that the information on all grid points is used to approximate the derivatives of unknown functions, so that accurate results can be obtained by using this method with a few grid points.

In recent years, there appeared many models in non-Newtonian fluids. Some models only consist of the momentum equation[8,9], and other models not only consist of momentum equation but also involve energy equation. The unsteady convection-diffusion equation in a viscoelastic fluid flowing through a tube was computed by using the implicit finite difference scheme[10]. The momentum equation and energy equation of a non-isothermal viscoelastic fluid were solved by using Galerkin's approach and B-splines[11]. The laminar convection heat transfer in a second-order viscoelastic fluid moving past a porous regime was solved by using a Keller-box implicit finite difference scheme[12]. A three-dimensional flow of non-Newtonian fluid was simulated by using a shooting method and fourth-order Runge-Kutta procedure, and the effect of the elasticity of fluid on velocity and temperature distributions was examined qualitatively in Ref. [13].

In this paper a fully developed problem of second-order viscoelastic fluid coupled with heat transfer is considered. Firstly by using the perturbation procedure we get the zeroth- and first-order approximate equations. Then, by using the DQM these equations are solved numerically. The numerical results obtained are in agreement with existing results qualitatively. The present paper is organized as follows. In Section 2, we describe the mathematical model of fluids. The standard perturbation procedure is applied to yield the zeroth- and first-order approximate equations in Section 3. The numerical formulations obtained by the DQM are given in Section 4. In Section 5, there will be a discussion of the numerical results. In the last section, we give our conclusions and remarks.

2 Mathematical Model

Let us consider the two-dimensional steady flow of

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an incompressible second-order viscoelastic fluid between parallel plates, as shown in Fig.1, in which the x-axis is horizontal and the y-axis is vertical upward. The governing equations of our problem are given as

Continuity equation $\nabla \cdot V = 0$, (1)

Momentum equation $\rho V \cdot \nabla V = \nabla \cdot \sigma + \rho g$, (2)

Energy equation $C_p \rho V \cdot \nabla T = K \nabla^2 T$, (3)

where V denotes the velocity field, T is the temperature, g is the gravitational acceleration, ρ is the density, K is the coefficient of thermal conduction, C_p is the specific heat at constant pressure, ∇ and ∇^2 are the gradient and Laplacian operators respectively, and σ is the Cauchy stress tensor defined as

$$\sigma = -PI + \tau, \tag{4a}$$

$$\tau = \mu A_1(V) + \alpha_1 A_1^2(V) + \alpha_2 A_2(V), \tag{4b}$$

where P is the hydrostatic pressure, I is the unit tensor, τ is the deviatoric stress tensor , μ is the coefficient of viscosity, $\alpha_i\ (i = 1,2)$ are material constants characterizing the elasticity of fluid, and $A_i\ (i = 1,2)$ are the first and second Rivlin-Eriken tensor written as

$$\left.\begin{aligned} A_1 &= (\text{grad } V) + (\text{grad } V)^T, \\ A_2 &= \frac{dA_1}{dt} + A_1(\text{grad } V) + (\text{grad } V)^T A_1, \end{aligned}\right\} \tag{5}$$

in which d/dt is the material time differentiation.

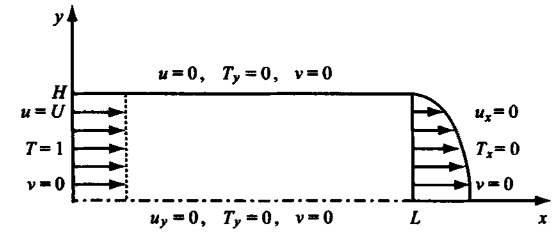

Fig.1 Geometry of flow model

The equation of state may be written as

$$\rho = \rho_0[1 - \beta(T - T_0)], \tag{6}$$

where β is the volume expansion coefficient, and T_0 is the temperature at which the fluid density is ρ_0.

We define dimensionless variables as

$$\hat{x} = \frac{x}{H}, \quad \hat{y} = \frac{y}{H}, \quad \hat{u} = \frac{u}{U}, \quad \hat{v} = \frac{v}{U},$$

$$\hat{P} = \frac{H}{\mu U}P, \quad \hat{\tau} = \frac{H}{\mu U}\tau, \quad \hat{\sigma} = \frac{H}{\mu U}\sigma, \quad \hat{T} = \frac{T - T_0}{\Delta T},$$

where H is the reference length of the entrance, U is the reference velocity, and $\Delta T = T_1 - T_0$ is the temperature difference. For simplicity, we omit the sign ^ in the following.

By using the Boussinesq approximation, the governing equations (1) - (3) can be reduced to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7a}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\text{Re}}\left(-\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right), \tag{7b}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\text{Re}}\left(-\frac{\partial P}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) - \frac{Ra}{\text{PrRe}^2}\mathscr{R}(T), \tag{7c}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\text{PrRe}}\nabla^2 T, \tag{7d}$$

where $\text{Re} = \rho UH/\mu$ is the Reynolds number, $Ra = C_p \rho g \Delta T H^3 / K\nu$ is the Rayleigh number, $\text{Pr} = \nu C_p \rho / K$ is the Prandtle number, and $\mathscr{R}(T) = (1 - \beta T \Delta T)/(\beta \Delta T)$, $\nu = \mu/\rho$ is the kinematics viscosity, and τ is the dimensionless deviatoric stress tensor that can be written as

$$\tau_{xx} = 2u_x + \beta_1[(2u_x)^2 + (u_y + v_x)^2] + \beta_2[2uu_{xx} + 2vu_{xy} + (2u_x)^2 + 2v_x(u_y + v_x)], \tag{8a}$$

$$\tau_{yy} = 2v_y + \beta_1[(2v_y)^2 + (u_y + v_x)^2] + \beta_2[2uv_{xy} + 2vv_{yy} + (2v_y)^2 + 2u_y(u_y + v_x)], \tag{8b}$$

$$\tau_{xy} = \tau_{yx} = u_y + v_x + \beta_2[u(u_y + v_x)_x + v(u_y + v_x)_y + 2u_x u_y + 2v_x v_y], \tag{8c}$$

where,

$$\begin{pmatrix} (\ )_x \\ (\ )_y \end{pmatrix} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix}, \quad \begin{pmatrix} (\ )_{xx} \\ (\ )_{yy} \end{pmatrix} = \begin{pmatrix} \partial^2/\partial x^2 \\ \partial^2/\partial y^2 \end{pmatrix},$$

$$\begin{pmatrix} (\ )_{xy} \\ (\ )_{yx} \end{pmatrix} = \begin{pmatrix} \partial^2/\partial x \partial y \\ \partial^2/\partial y \partial x \end{pmatrix}, \quad \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \frac{U}{\mu H}\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}.$$

The velocity components u and v may be written in the term of the stream function Ψ as

$$u = \Psi_y, v = -\Psi_x, \tag{9}$$

and vorticity (designated by ω) is defined by

$$\omega = v_x - u_y. \tag{10}$$

By substituting (8) into (7b) and (7c), eliminating pressure P, and using Eqs.(10) and (7a) the governing equations in terms of stream function, vorticity, and temperature are given by

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\text{Re}}\nabla^2 \omega + \beta_2\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]\nabla^2 \omega +$$

$$\frac{Ra}{\mathrm{PrRe}^2}\frac{\partial T}{\partial x}, \tag{11a}$$

$$\nabla^2 \Psi + \omega = 0, \tag{11b}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{PrRe}}\nabla^2 T. \tag{11c}$$

3 Perturbation Method

When the elasticity of the fluid is slight, β_1 and β_2 can be considered as small parameters. According to Ref.[14], we have $\beta_1 > 0$, $\beta_2 < 0$ and $\beta_1 = -c\beta_2$, where $c \approx 1.6$, so we can introduce only one parameter $\varepsilon = -\beta_2$. Obviously ε characterizes the elasticity of fluid. For clarity of the effect of the elasticity upon the flow we expand the governing equations with the respect to ε. Based on perturbation method, the variables ω, Ψ, T, u, v, and τ are written in the following forms:

$$\begin{pmatrix} u \\ v \\ \Psi \\ \omega \\ T \\ \tau \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \\ \Psi_0 \\ \omega_0 \\ T_0 \\ \tau^0 \end{pmatrix} + \varepsilon \begin{pmatrix} u_1 \\ v_1 \\ \Psi_1 \\ \omega_1 \\ T_1 \\ \tau^1 \end{pmatrix} + O(\varepsilon^2). \tag{12}$$

By substituting (12) into (8) - (11), and neglecting $O(\varepsilon^2)$, we get the following systems:

(1) The zeroth-order approximate equations

$$u_0\frac{\partial \omega_0}{\partial x} + v_0\frac{\partial \omega_0}{\partial y} = \frac{1}{\mathrm{Re}}\nabla^2\omega_0 + \frac{Ra}{\mathrm{PrRe}^2}\frac{\partial T_0}{\partial x}, \tag{13a}$$

$$\nabla^2\Psi_0 + \omega_0 = 0, \tag{13b}$$

$$u_0\frac{\partial T_0}{\partial x} + v_0\frac{\partial T_0}{\partial y} = \frac{1}{\mathrm{PrRe}}\nabla^2 T_0, \tag{13c}$$

where

$$u_0 = (\Psi_0)_y, \quad v_0 = -(\Psi_0)_x, \tag{13d}$$

$$\tau^0_{xx} = 2(u_0)_x, \quad \tau^0_{yy} = 2(v_0)_y,$$
$$\tau^0_{xy} = \tau^0_{yx} = (u_0)_y + (v_0)_x. \tag{14}$$

Eq.(14) is the constitutive relation of the viscous fluid. So, the zeroth-order approximation is an incompressible Newtonian fluid.

(2) The first-order approximate equations

$$u_0\frac{\partial \omega_1}{\partial x} + v_0\frac{\partial \omega_1}{\partial y} + u_1\frac{\partial \omega_0}{\partial x} + v_1\frac{\partial \omega_0}{\partial y} = \frac{1}{\mathrm{Re}}\Big[\nabla^2\omega_1 - \Big(u_0\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\Big)\nabla^2\omega_0\Big] + \frac{Ra}{\mathrm{PrRe}^2}\frac{\partial T_1}{\partial x}, \tag{15a}$$

$$\nabla^2\Psi_1 + \omega_1 = 0, \tag{15b}$$

$$u_0\frac{\partial T_1}{\partial x} + v_0\frac{\partial T_1}{\partial y} + u_1\frac{\partial T_0}{\partial x} + v_1\frac{\partial T_0}{\partial y} = \frac{1}{\mathrm{PrRe}}\nabla^2 T_1, \tag{15c}$$

where

$$u_1 = (\Psi_1)_y, \quad v_1 = -(\Psi_1)_x, \tag{15d}$$

$$\tau^1_{xx} = 2(u_1)_x + c[(2(u_0)_x)^2 + ((u_0)_y + (v_0)_x)^2] - [2u_0(u_0)_{xx} + 2v_0(u_0)_{xy} + (2(u_0)_x)^2 + 2(v_0)_x((u_0)_y + (v_0)_x)], \tag{16a}$$

$$\tau^1_{yy} = 2(v_1)_y + c[(2(v_0)_y)^2 + ((u_0)_y + (v_0)_x)^2] - [2u_0(v_0)_{xy} + 2v_0(v_0)_{yy} + (2(v_0)_y)^2 + 2u_y((u_0)_y + (v_0)_x)], \tag{16b}$$

$$\tau^1_{xy} = (u_1)_y + (v_1)_x - [u_0((u_0)_y + (v_0)_x)_x + v_0((u_0)_y + (v_0)_x)_y + 2(u_0)_x(u_0)_y + 2(v_0)_x(v_0)_y]. \tag{16c}$$

Eqs. (16a), (16b), (16c) are the constitutive relations characterizing the elasticity of fluid. So the solution of the first-order approximate equations represents the effect of elasticity upon the solution of Eq.(11).

In our problem, assume that the velocity components $u = 1/2$ and $v = 0$ at the entrance, $u = v = 0$ on the plate, the flow is in full development at the exit. Based on the symmetry, the boundary conditions are given as

$$\left.\begin{aligned} &\Psi_i = \delta_{i0}(y/2), \omega_i - (v_i)_x = 0, u_i = \frac{1}{2}\delta_{i0}, \\ &\quad v_i = 0, T_i = 1, \text{ for } x = 0,\ 0 \leqslant y \leqslant 2, \\ &(\Psi_i)_x = 0, (\omega_i)_x = 0, (u_i)_x = 0, \\ &\quad v_i = 0, (T_i)_x = 0, \text{ for } x = 4,\ 0 < y < 2, \\ &\Psi_i = 0, \omega_i = 0, (u_i)_y = 0, \\ &\quad v_i = 0, (T_i)_y = 0, \text{ for } y = 0,\ 0 < x < 4, \\ &\Psi_i = \delta_{i0}, \omega_i + (u_i)_y = 0, u_i = 0, \\ &\quad v_i = 0, (T_i)_y = 0, \text{ for } y = 2,\ 0 \leqslant x \leqslant 4, \end{aligned}\right\} \tag{17}$$

where $i = 0, 1$, $\delta_{00} = 1$, $\delta_{10} = 0$.

4 Numerical Formaluations

In the DQM the partial derivatives of a function are approximated by weighted linear sum of function values at discrete points in the computational domain. According to the DQM, for any function ( ), one has

$$\begin{pmatrix} \dfrac{\partial(\ )}{\partial x} \\ \dfrac{\partial(\ )}{\partial y} \end{pmatrix} = \begin{Bmatrix} \sum_{k=1}^{N} C^{(1)}_{ik}(\ )_{kj} \\ \sum_{l=1}^{M} \bar{C}^{(1)}_{jl}(\ )_{il} \end{Bmatrix}, \tag{18a}$$

$$\begin{pmatrix} \frac{\partial^2(\ )}{\partial x^2} \\ \frac{\partial^2(\ )}{\partial y^2} \\ \frac{\partial^2(\ )}{\partial x \partial y} \end{pmatrix}_{ij} = \begin{Bmatrix} \sum_{k=1}^{N} C_{ik}^{(2)}(\ )_{kj} \\ \sum_{l=1}^{M} \bar{C}_{jl}^{(2)}(\ )_{il} \\ \sum_{k=1}^{N}\sum_{l=1}^{M} C_{ik}^{(1)}\bar{C}_{jl}^{(1)}(\ )_{kl} \end{Bmatrix}, \tag{18b}$$

where $C_{ik}^{(1)}$, $C_{ik}^{(2)}$, $\bar{C}_{jl}^{(1)}$, and $\bar{C}_{jl}^{(2)}$ are the weighting coefficient matrices corresponding to first- and second-order derivatives respectively, N and M are the number of grid points along the x and y axes respectively. From above approximations, one can realize that the key issue in the DQM is how to determine its weighting coefficients and choose the grid points. The formula for calculating weighting coefficients has been given in Refs.[15] and [16], and grid distribution used in this paper is the same as in Ref.[16].

Substituting (18) into (13) and (14) in Part (1), yields

$$\sum_{k=1}^{N}\left[u_0(i,j)C_{ik}^{(1)} - \frac{C_{ik}^{(2)}}{\mathrm{Re}}\right]\omega_0(k,j) + \sum_{l=1}^{M}\left[v_0(i,j)\bar{C}_{jl}^{(1)} - \frac{\bar{C}_{jl}^{(2)}}{\mathrm{Re}}\right]\omega_0(i,l) = \frac{Ra}{\mathrm{PrRe}^2}\sum_{k=1}^{N} C_{ik}^{(1)} T_0(k,j), \tag{19a}$$

$$\sum_{k=1}^{N} C_{ik}^{(2)}\Psi_0(k,j) + \sum_{l=1}^{M}\bar{C}_{jl}^{(2)}\Psi_0(i,l) + \omega_0(i,j) = 0, \tag{19b}$$

$$\sum_{k=1}^{N}\left[u_0(i,j)C_{ik}^{(1)} - \frac{C_{ik}^{(2)}}{\mathrm{PrRe}}\right]T_0(k,j) + \sum_{l=1}^{M}\left[v_0(i,j)\bar{C}_{jl}^{(1)} - \frac{\bar{C}_{jl}^{(2)}}{\mathrm{PrRe}}\right]T_0(i,l) = 0, \tag{19c}$$

$$u_0(i,j) = \sum_{l=1}^{M}\bar{C}_{jl}^{(1)}\Psi_0(i,l), \quad v_0(i,j) = -\sum_{k=1}^{N} C_{ik}^{(1)}\Psi_0(k,j), \tag{19d}$$

where $i = 2, \cdots, N-1$, $j = 2, \cdots, M-1$.

Eq. (14) and the boundary conditions (17) are approximated by the DQM for all.

Similarly, substituting (18) into (15) and (16) in Part (2), gives

$$\sum_{k=1}^{N}\left[u_0(i,j)C_{ik}^{(1)} - \frac{C_{ik}^{(2)}}{\mathrm{Re}}\right]\omega_1(k,j) + \sum_{l=1}^{M}\left[v_0(i,j)\bar{C}_{jl}^{(1)} - \frac{\bar{C}_{jl}^{(2)}}{\mathrm{Re}}\right]\omega_1(i,l) = F_1(i,j) + \frac{Ra}{\mathrm{PrRe}^2}\sum_{k=1}^{N} C_{ik}^{(1)} T_1(k,j), \tag{20a}$$

$$\sum_{k=1}^{N} C_{ik}^{(2)}\Psi_1(k,j) + \sum_{l=1}^{M}\bar{C}_{jl}^{(2)}\Psi_1(i,l) + \omega_1(i,j) = 0, \tag{20b}$$

$$\sum_{k=1}^{N}\left[u_0(i,j)C_{ik}^{(1)} - \frac{C_{ik}^{(2)}}{\mathrm{PrRe}}\right]T_1(k,j) + \sum_{l=1}^{M}\left[v_0(i,j)\bar{C}_{jl}^{(1)} - \frac{\bar{C}_{jl}^{(2)}}{\mathrm{PrRe}}\right]T_1(i,l) = F_2(i,j), \tag{20c}$$

$$u_1(i,j) = \sum_{l=1}^{M}\bar{C}_{jl}^{(1)}\Psi_1(i,l), \quad v_1(i,j) = -\sum_{k=1}^{N} C_{ik}^{(1)}\Psi_1(k,j), \tag{20d}$$

where

$$\begin{pmatrix} F_1(i,j) \\ F_2(i,j) \end{pmatrix} = -\begin{Bmatrix} u_1(\omega_0)_x + v_1(\omega_0)_y - \frac{1}{\mathrm{Re}}\cdot [u_0(\nabla^2\omega_0)_x + v_0(\nabla^2\omega_0)_y] \\ u_1(T_0)_x + v_1(T_0)_y \end{Bmatrix}_{i,j}, \tag{21}$$

for $i = 2, \cdots, N-1$, $j = 2, \cdots, M-1$.

We also employ the DQM to approximate Eq.(16), F_1 and F_2, and the boundary conditions (17) for the first-order approximation. For the above two nonlinear systems (19) and (20) with discrete boundary conditions, we adopt an iterative method to solve them and the numerical results obtained are discussed in the next section.

5 Results and Discussion

An iterative method for solving nonlinear systems (19) and (20) with discrete boundary conditions is presented as follows.

If the n-th iterations $\omega_i^{(n)}$, $\Psi_i^{(n)}$, $T_i^{(n)}$, $u_i^{(n)}$, $v_i^{(n)}$ are known, the $(n+1)$-th iteration can be obtained in terms of the following steps:

(1) Solve the energy Eq.(19c) or Eq.(20c) and (21) with $u_i = u_i^{(n)}$, $v_i = v_i^{(n)}$ under the corresponding boundary conditions of T_i;

(2) Solve the vorticity Eq.(19a) or Eq.(20a) and (21) with $u_i = u_i^{(n)}$, $v_i = v_i^{(n)}$, $T_i = T_i^{(n+1)}$ under the corresponding boundary conditions of ω_i;

(3) Solve the stream function Eq.(19b) or Eq.(20b) with $\omega_i = \omega_i^{n+1}$ under the corresponding boundary conditions of Ψ_i;

(4) Compute the velocity components from Eq.(19d) or Eq.(20d) .

If the condition $\max|\omega^{n+1} - \omega^n| \leqslant e$ is true, the iteration stops and the $(n+1)$-th iteration is the solutions expected, where, $e = 10^{-5}$ is the given iteration accuracy. The linear systems of equations in Steps 1–3 are solved by the SOR iterative procedure.

In the following computation, we take $N = 21$ and $M = 11$, and Re lies in $(0, 12]$. By using the DQM and iterative procedure mentioned above the numerical experiment for our model was successfully conducted. The results of velocity are given in Table 1, and the maximum absolute errors for vorticity are given in Table 2.

Table 1(a) Zeroth-order velocity at the centerline with Pr = 0.71, Er = 0.01, $Ra = 10^4$

Re \ X	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
									u_0								
4	0.35	0.21	0.60	0.81	0.85	0.87	0.90	0.91	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
8	0.49	0.47	0.71	0.83	0.86	0.88	0.90	0.91	0.91	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
12	0.54	0.56	0.75	0.84	0.86	0.88	0.90	0.90	0.91	0.91	0.92	0.92	0.92	0.92	0.92	0.92	0.92

Table 1(b) First-order velocity at the centerline with Pr = 0.71, Er = 0.01, $Ra = 10^4$

Re \ X	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
									$-u_1$								
4	0.25	0.56	0.38	0.22	0.16	0.12	0.08	0.05	0.03	0.02	0.01	0.01	0	0	0	0	0
8	0.10	0.25	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.02	0.01	0.01	0.01	0	0	0
12	0.05	0.13	0.12	0.10	0.09	0.08	0.08	0.07	0.07	0.05	0.04	0.02	0.02	0.01	0.01	0.01	0

Table 2 Maximum absolute error for ω_0 and ω_1 (Pr = 0.71, Re = 8)

Ra	10^1	10^2	10^3	10^4	10^5
$\max\lvert\omega_0\rvert$	5.80×10^{-6}	5.26×10^{-7}	7.49×10^{-9}	9.44×10^{-11}	3.38×10^{-13}
$\max\lvert\omega_1\rvert$	3.53×10^{-5}	4.33×10^{-6}	1.15×10^{-6}	1.07×10^{-6}	1.02×10^{-6}

The streamlines for the zeroth- and first- order systems are drawn in Fig. 2. From Fig. 2, the effect of the elastic of the second-order viscoelastic fluids is weak far from the entrance. This qualitative property agrees with that in Ref.[8].

The zeroth- and first-order velocity profiles at the centerline are drawn in Figs.3(a) and 3(b) respectively. From Table 1 the entrance lengths of the zeroth-order approximation system for Re = 4, 8, 12 are about 1.8, 2.0, 2.2 respectively, and the entrance lengths of first order approximation system for Re = 4, 8, 12 are about 2.6, 3.0, 3.4 respectively. This shows that the entry length increases as Re increases. The isotherm lines for different parameters are shown in Figs.4 and 5. It is shown that the isotherm lines for T_0 and T_1 are pushed towards the exit. The temperatures are however augmented with the increase in Re or Pr.

In Figs. 6 and 7, the graphs are plotted to show the effect of the elasticity upon the deviatoric stress τ^0_{xx}, τ^1_{xx}, τ_{xx}, τ^0_{yy}, τ^1_{yy}, τ_{yy}, at the centerline and near the wall respectively, where $c \approx 1.6$ and $\varepsilon = -\beta_2 = 0.1$. We observe that the shear stress is small near the centerline and it is large near the wall. Thus the stress of the second-order fluid is close to Newtonian fluid at the centerline, and evidently different from it near the wall. These qualitative properties obtained agree with those in Ref.[9] and [12] as Re is in the range$(0,10]$.

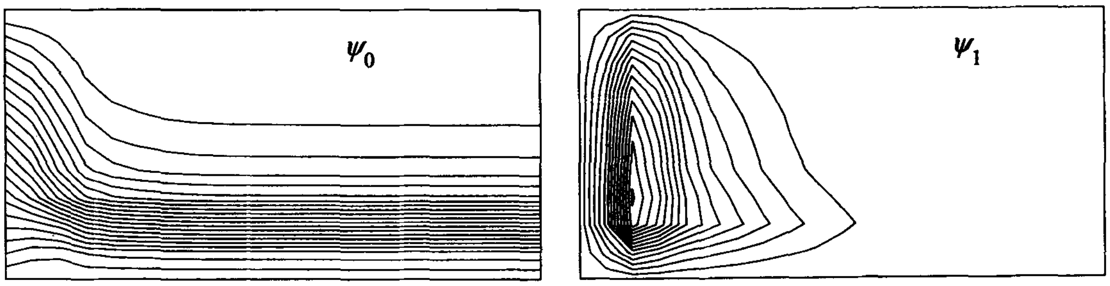

Fig.2 Streamlines for the zeroth- and first-order systems for Re = 8, Pr = 0.71, $Ra = 10^4$

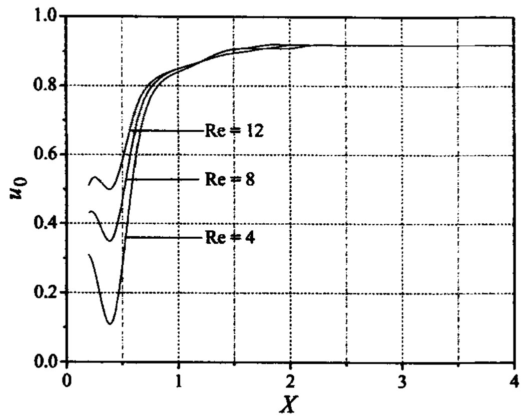

Fig.3(a) Zeroth-order velocity profiles at the centerline

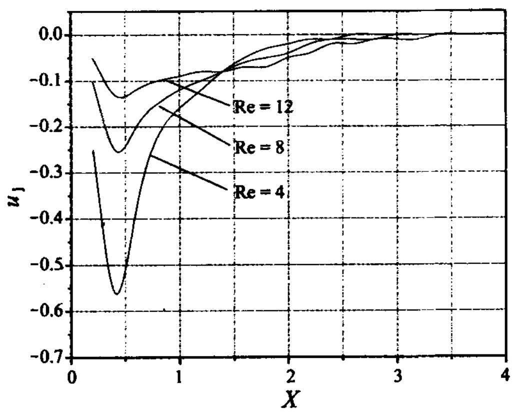

Fig.3(b) First-order velocity profiles at the centerline

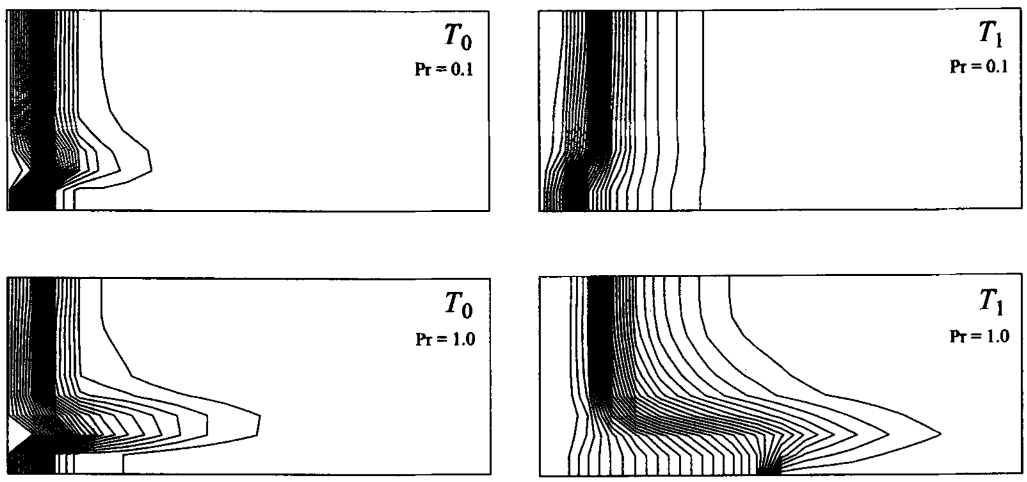

Fig.4 Isotherm lines with Re = 8 and different values of Pr = 0.1, 1.0

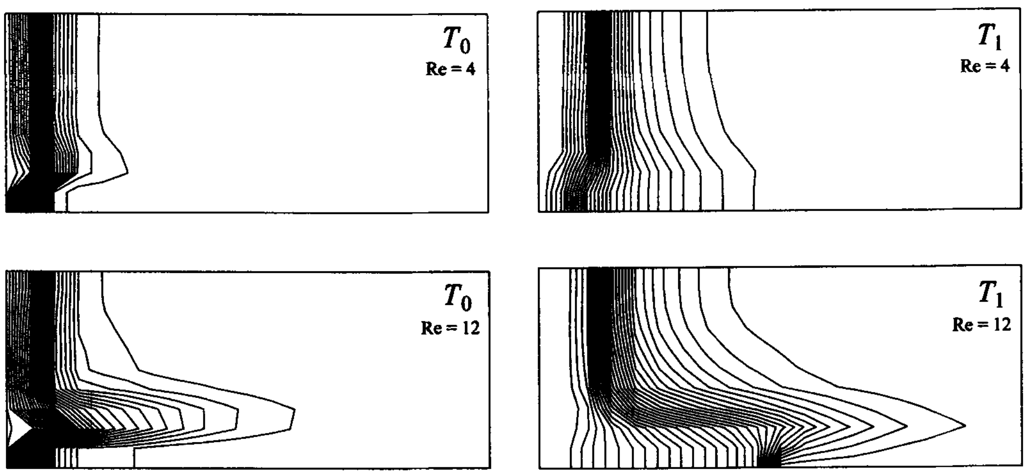

Fig.5 Isotherm lines with Pr = 0.71 and different values of Re = 4, 12

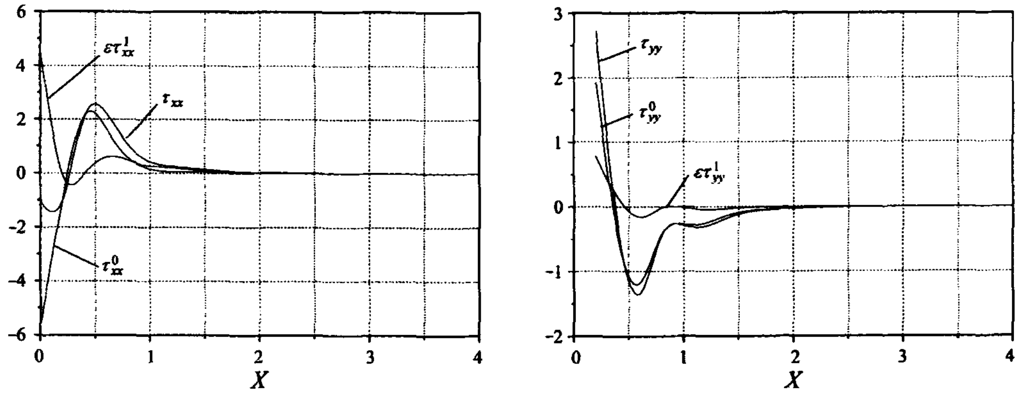

Fig.6 Stress components at the centerline with Re = 8, Pr = 0.71, $Ra = 10^4$

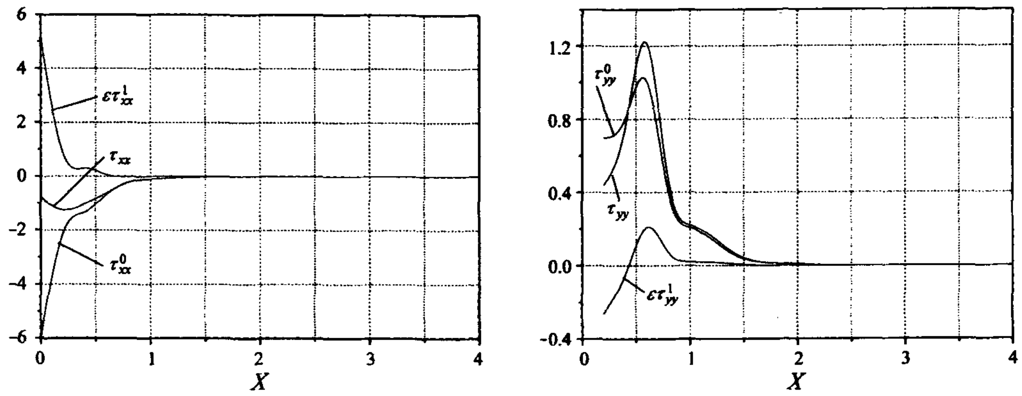

Fig.7 Stress components near the wall with Re = 8, Pr = 0.71, $Ra = 10^4$

6 Conclusions

The two-dimensional steady flow of an incompressible second-order viscoelastic fluid between two parallel plates has been investigated in terms of vorticity, the stream function and temperature equations in this paper. The governing equations have been expanded with respect to a small parameter to get the zeroth-and first-order approximate equations. By using the differential quadrature method and an iterative technique the numerical solution has been successfully obtained.

The numerical results show that the effect of elasticity of fluid upon flow is evident at the entrance near the wall, but weak far from the entrance. The results also show that the DQM can be used to obtain the high-accuracy solutions with a few grid points.

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