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Application of Mixed Differential Quadrature Method for Solving the Coupled Two-Dimensional Incompressible Navier-Stokes Equation and Heat Equation

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Abstract The traditional differential quadrature method was improved by using the upwind difference scheme for the convective terms to solve the coupled two-dimensional incompressible Navier-stokes equations and heat equation. The new method was compared with the conventional differential quadrature method in the aspects of convergence and accuracy. The results show that the new method is more accurate, and has better convergence than the conventional differential quadrature method for numerically computing the steady-state solution.

Key words coupled N-S equation and heat equation, differential quadrature method, upwind difference scheme.
MSC 2000 65M 99

1 Introduction

Various numerical techniques, such as the finite difference method and finite elements method, have been used in the past to solve the incompressible flow problem numerically. Usually, these methods require a large number of discretized points in the computational domain for accurate results. Because the information on all grid points is used to fit derivatives at grid points in the differential quadrature (DQ) method, it is enough to use only few grid points to obtain high-accuracy numerical solutions. Therefore, the number of grid points can be greatly reduced while still obtaining accurate results by using the DQ method. The DQ method was introduced by Bellman and his associates[1,2], and it has been successfully employed to obtain numerical solutions in engineering and physical sciences[3]. There are many papers discussing the two-dimensional (2D) incompressible Navier-stokes equations without heat equation by the DQ method (for examples see[4 - 6]). But, maybe there are only a few (if there are) papers dealing with the coupled incompressible Navier-Stokes equations and heat equation by means of the DQ method. Some investigators have been studied this kind of problems in different cases[7-10], especially with our problems.

In our previous paper[11] the incompressible plane flow field subjected to a force field that is dependent upon the temperature T and has not potential was discussed. Also, in that paper the numerical computation by using the DQ method with a few grid points was satisfactory with good accuracy. But, for some (higher and lower) values of Re and α, the parameter including in the force field term, and some number and positions of the sampling points, the numerical results had some disadvantages (oscillating or divergent). The purpose of this paper is to introduce a method to overcone the disadvantages in the previous paper. With this aim, a mixed DQ method is presented by using the traditional upwind difference method (UDM) for the convective term and the traditional DQ method for the other terms. Using the mixed DQ method for solving the coupled incompressible (2D) Navier-stokes equations and heat equation the excellent numerical results are obtained. It is shown that the mixture DQ method is wider in use and more accurate than the traditional DQ method.

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2 Mathematical Formulations

Let us consider the incompressible flow problem in a rectangular region Ω in Fig. 1. The non-dimensional governing equations for vorticity, stream function and temperature are

$$\frac{\partial \omega}{\partial t}+u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}-\frac{1}{Re}\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)=F(\Psi,\omega,T,x,y) \tag{1}$$

$$\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}+\omega=0 \tag{2}$$

$$u=\frac{\partial \Psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial \Psi}{\partial x} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}-\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0 \tag{4}$$

where, Ψ the stream function, u and v the velocity components, $\omega=v_x-u_y$ the vorticity, T the temperature, F is a term caused by the force field that dose not have potential and is dependent upon temperature, Re the Reynolds number.

The boundary conditions (B.Cs) considered in this paper are

$$u=0, v=0, T=0, \omega=\frac{\partial v}{\partial x} \quad \text{at } x=0, x=1$$

$$u=0, v=0, T=1, \omega=-\frac{\partial u}{\partial y} \quad \text{at } y=0$$

$$u=1, v=0, T=0, \omega=-\frac{\partial u}{\partial y} \quad \text{at } y=1$$

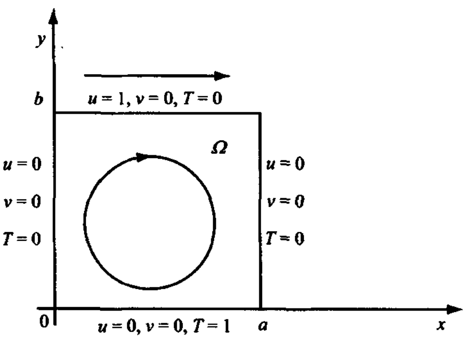

Fig. 1 Geometry of the problem

In this paper the mixed DQ method with the Chebyshev-Gauss-Lobatto spacing points is used to obtain high-accuracy steady-state solutions of the incompressible flow Navier-Stokes equations and heat equation.

3 Differential Quadrature Formulations

The essence of the DQ method is that the partial derivatives of a function with respect to a variable in governing equations are approximated by a weighted linear sum of function values at all discrete points in that direction. We consider a function $f=f(x,y)$ defined in a rectangular domain $0\leqslant x\leqslant a$, $0\leqslant y\leqslant b$, with a and b fixed. Suppose that, the grid is obtained by taking N and M points in the x and y directions, respectively. Then, an rth-order x-partial derivative of the function $f(x,y)$ at a point $x=x_i$ along any line $y=y_j$ parallel to the x-axis, and an sth-order y-partial derivative of the function $f(x,y)$ at a point $y=y_j$ along any line $x=x_i$ parallel to the y-axis, respectively may be approximately written as

$$\frac{\partial^r f(x_i,y)}{\partial x^r}=\sum_{k=1}^{N}A_{ik}^{(r)}f(x_k,y),$$
$$\frac{\partial^s f(x,y_j)}{\partial y^s}=\sum_{l=1}^{M}A_{jl}^{(s)}f(x,y_l) \tag{5}$$

where $r=1, 2, \cdots, N-1$, $s=1, 2, \cdots, M-1$, $i=1, 2, \cdots, N$, $j=1, 2, \cdots, M$, $A_{ik}^{(r)}$ and $A_{jl}^{(s)}$ are the respective weighting coefficients. The determinant of the weighting coefficients and the choice of sampling points are very important factors for the accuracy of the DQ solution. In fact, the weighting coefficients obtained by directly solving the Vandermonde equations becomes increasingly inaccurate as an increasing number of sampling points. The weighting coefficients for the derivatives may be obtained directly, and most accurately, irrespective of the number and positions of the sampling points[5,12]. From [5] and [12], the weighting coefficient of DQ are determined as follows.

The weighting coefficients for the first-order derivative are given as

$$A_{ik}^{(1)}=\frac{C^{(1)}(x_i)}{(x_i-x_k).C^{(1)}(x_k)} \quad \text{for } i,k=1,2,\cdots N \text{ but } k\neq i, \tag{6}$$

where

$$C^{(1)}(x_i)=\prod_{\mu=1,\mu\neq i}^{N}(x_i-x_\mu), C^{(1)}(x_k)=\prod_{\mu=1,\mu\neq k}^{N}(x_k-x_\mu),$$

The weighting coefficients for he second-order and high-order derivatives are given as

$$A_{ik}^{(r)}=r\left[A_{ii}^{(r-1)}A_{ik}^{(1)}-\frac{A_{ik}^{(r-1)}}{x_i-x_k}\right] \quad \text{for } i,k=1,2,\cdots N \text{ but } k\neq i,\ 2\leqslant r\leqslant(N-1), \tag{7}$$

where

$$A_{ii}^{(r)} = -\sum_{\mu=1,\mu\neq i}^{N} A_{i\mu}^{(r)} \quad \text{for } i=1,2,\cdots N,\ 1\leqslant r\leqslant (N-1) \tag{8}$$

The weighting coefficients of derivatives with respect to y can be also obtained in similar forms of Eqs. (6), (7) and (8). Two kind of sampling points of the type (I) equally spaced nodes and the type(II) the Chebyshev-Gauss-Lobatto points are used in this paper.

3.1 Traditional differential quadrature method (TDQM)

The TDQM can be obtained by applying (5) to Eqs. (1) - (4), in the form

$$\frac{\partial \omega}{\partial t}\Big|_{ij} + \sum_{k=1}^{N}(u_{ij}A_{ik}^{(1)} - A_{ik}^{(2)}/Re)\omega_{kj} + \sum_{l=1}^{M}(v_{ij}A_{jl}^{(1)} - A_{jl}^{(2)}/Re)\omega_{il} = F_{ij} \tag{9}$$

$$\sum_{k=1}^{N}A_{ik}^{(2)}\Psi_{kj} + \sum_{l=1}^{M}A_{jl}^{(2)}\Psi_{il} = -\omega_{ij} \tag{10}$$

$$u_{ij} = \sum_{l=1}^{M}A_{jl}^{(1)}\Psi_{il} \text{ and } v_{ij} = -\sum_{k=1}^{N}A_{ik}^{(1)}\Psi_{kj} \tag{11}$$

$$\frac{\partial T}{\partial t}\Big|_{ij} + \sum_{k=1}^{N}(u_{ij}A_{ik}^{(1)} - A_{ik}^{(2)})T_{kj} + \sum_{l=1}^{M}(v_{ij}A_{jl}^{(1)} - A_{jl}^{(2)})T_{il} = 0 \tag{12}$$

where (i,j)'s should be all internal grid points $(2\leqslant i\leqslant(N-1), 2\leqslant j\leqslant(M-1))$. The linear system of algebraic equations in (9), (10), (12) may be solved by the iterative method, and at every iterative step the boundary conditions of ω can be computed by the DQM as follows:

$$\left.\begin{aligned} \omega_{1j} &= \sum_{k=1}^{N}A_{1k}^{(1)}v_{kj}\cdots,\cdots \omega_{Nj} = \sum_{k=1}^{N}A_{Nk}^{(1)}v_{kj} \\ \omega_{i1} &= -\sum_{l=1}^{M}A_{1l}^{(1)}u_{il}\cdots,\cdots \omega_{iM} = -\sum_{l=1}^{M}A_{Ml}^{(1)}u_{il} \end{aligned}\right\} \tag{13}$$

where u and v are computed from Eq. (11).

The TDQM is applied to solve this problem previously[1] with the sampling points of type (I). Here we add other sampling points of type (II) with this method to solve the above equations, and compare these two kind of sampling points in iteration required for the steady-state solution(see Table 1). The type (II) leads to good results for different values of Re and more accurate than the type (I); but, also there are some disadvantage for some higher and lower values of Re and for large values of α which included in the term of force field. Therefore, in the next section we will suggest a new procedure to treat these difficulties, which is able to give reasonable treatment for this problem.

3.2 Mixed differential quadrature method (MDQM)

Here a MDQM will be suggested to solve the coupled two-dimensional incompressible Navier-Stokes equations and heat equation. This method includes two points. First, the upwind difference scheme is used to approximate the convective terms subject to mechanism of flow directions, and the DQ for other terms in space. The second is to adopt two dependent stages for calculation of results at every step, similar to the time splitting technique[13]. In order to demonstrate these ideas: we replace a convective terms by the upwind difference scheme, after the average axial has been evaluated at half a grid forward and backward from (x, y) in the x- and y-direction, respectively. It can easily be verified that the upwind difference form is automatically preserved when the following numerical formulas are used.

Table 1 Iteration required for the steady state-solution with $\Delta t = 0.025$ and $\alpha = 400$

Type of sampling points		Re 0.1	1	10	50	100	400	800	Regions of no convergent or convergent slowly
7×7	I	-	95	44	39	38	37	35	$Re<1$
	II	-	-	47	42	39	38	36	$Re<4$
9×9	I	-	-	-	-	-	-	-	-
	II	-	-	-	144	136	131	130	$Re<13$
11×11	I	-	-	-	-	-	-	-	-
	II	-	-	-	155	137	124	123	$Re<33$

- Divergent

$$\left.\begin{aligned} u\frac{\partial \Omega}{\partial x}\Big|_{ij} &= U_b(\Omega_{ij}^n - \Omega_{i-1,j}^n) + U_f(\Omega_{i+1,j}^n - \Omega_{ij}^n) \\ v\frac{\partial \Omega}{\partial y}\Big|_{ij} &= V_b(\Omega_{ij}^n - \Omega_{i,j-1}^n) + V_f(\Omega_{i,j+1}^n - \Omega_{ij}^n) \end{aligned}\right\} \tag{14}$$

where $\Omega = (\omega \quad T)^{\mathrm{T}}$,

$$U_b = \frac{u_{ij}^n + |u_{ij}^n|}{2(x_i - x_{i-1})}, U_f = \frac{u_{ij}^n - |u_{ij}^n|}{2(x_{i+1} - x_i)},$$

$$V_b = \frac{v_{ij}^n + |v_{ij}^n|}{2(y_j - y_{j-1})}, V_f = \frac{v_{ij}^n - |v_{ij}^n|}{2(y_{j+1} - y_j)}$$

The remained terms are approximated by the DQ method in space and the forward difference method in time. The formula for the first computed stage at every step is resulted from applying (5) and (14) to the system (1) - (4) as

$$\frac{\Omega_{ij}^* - \Omega_{ij}^n}{\Delta t} + u\frac{\partial \Omega}{\partial x}\Big|_{ij}^* + v\frac{\partial \Omega}{\partial y}\Big|_{ij}^* - \Gamma\left(\sum_{k=1}^{N} A_{ik}^{(2)}\Omega_{kj}^* + \sum_{l=1}^{M} A_{jl}^{(2)}\Omega_{il}^*\right) = \Phi_{ij} \tag{15}$$

$$\sum_{k=1}^{N} A_{ik}^{(2)}\Psi_{kj}^* + \sum_{l=1}^{M} A_{jl}^{(2)}\Psi_{il}^* = -\omega_{ij}^* \tag{16}$$

$$u_{ij}^* = \sum_{l=1}^{M} A_{jl}^{(1)}\Psi_{il}^* \quad \text{and} \quad v_{ij}^* = -\sum_{k=1}^{N} A_{ik}^{(1)}\Psi_{kj}^* \tag{17}$$

where $\Gamma = (1/Re, 1)^{\mathrm{T}}$ and $\Phi = (F^m, 0)^{\mathrm{T}}$

Then, the formula for the second computed stage at every step is resulted from applying (5) to the system (1) - (4) as

$$\frac{\Omega_{ij}^{n+1} - \Omega_{ij}^*}{\Delta t} + \left(\sum_{k=1}^{N}(u_{ij}^* A_{ik}^{(1)} - \Gamma A_{ik}^{(2)})\Omega_{kj}^{n+1} + \sum_{l=1}^{M}(v_{ij}^* A_{jl}^{(1)} - \Gamma A_{jl}^{(2)})\Omega_{il}^{n+1}\right) = \Phi_{ij}^* \tag{18}$$

$$\sum_{k=1}^{N} A_{ik}^{(2)}\Psi_{kj}^{n+1} + \sum_{l=1}^{M} A_{jl}^{(2)}\Psi_{il}^{n+1} = -\omega_{ij}^{n+1} \tag{19}$$

$$u_{ij}^{n+1} = \sum_{l=1}^{M} A_{jl}^{(1)}\Psi_{il}^{n+1} \text{ and } v_{ij}^{n+1} = -\sum_{k=1}^{N} A_{ik}^{(1)}\Psi_{kj}^{n+1} \tag{20}$$

where (i, j)'s should be all internal grid points ($2 \leqslant i \leqslant (N-1), 2 \leqslant j \leqslant (M-1)$). Generally, if we assume that

$$Z_1 = [\Omega(2,2), \cdots, \Omega(M-1,2), \cdots, \Omega(2,N-1), \cdots, \Omega(M-1,N-1)]^{\mathrm{T}}$$

$$Z_2 = [\Psi(2,2), \cdots, \Psi(M-1,2), \cdots, \Psi(2,N-1), \cdots, \Psi(M-1,N-1)]^{\mathrm{T}}$$

We can write Eqs. (15), (16), (18) and (19) in the form of a linear system of algebraic equations, respectively as

$$[\boldsymbol{I} + \Delta t \boldsymbol{P}_{RC}]Z_1^* = Z_1^n - \Delta t\, h_R + \Delta t \boldsymbol{\Phi} \tag{21}$$

$$Q_{RC}Z_2^* = -Z_1^* - g_R^* \tag{22}$$

$$[\boldsymbol{I} + \Delta t \boldsymbol{P}_{RC}^*]Z_1^{n+1} = Z_1^* - \Delta t\, h_R^* + \Delta t \boldsymbol{\Phi}^* \tag{23}$$

$$Q_{RC}Z_2^{n+1} = -Z^{n+1} - g_R^{n+1} \tag{24}$$

where, $\boldsymbol{I}$ is the $(N-2) \times (M-2)$ identity matrix, $\boldsymbol{P}$, $\boldsymbol{P}^*$ and $\boldsymbol{Q}$ are the $(N-2) \times (M-2)$ unsymmetrical matrices,

$$P_{RC} = \delta_{jl}[-(U_b + U_f) - \Gamma A_{ik}^{(2)}] + \delta_{ik}[-(V_b + V_f) - \Gamma A_{jl}^{(2)}]$$

$$P_{RC}^* = \delta_{jl}[u_{ij}^* A_{ik}^{(1)} - \Gamma A_{ik}^{(2)}] + \delta_{ik}[v_{ij}^* A_{jl}^{(1)} - \Gamma A_{jl}^{(2)}]$$

$$Q_{RC} = \delta_{jl}A_{ik}^{(2)} + \delta_{ik}A_{jl}^{(2)}$$

$$h_R = [-(U_b + U_f) - \Gamma A_{i1}^{(2)}]\Omega_{1j}^n + [-(U_b + U_f) - \Gamma A_{iN}^{(2)}]\Omega_{Nj}^n + [-(V_b + V_f) - \Gamma A_{j1}^{(2)}]\Omega_{i1}^n + [-(V_b + V_f) - \Gamma A_{jM}^{(2)}]\Omega_{iM}^n$$

$$h_R^* = [u_{ij}^* A_{i1}^{(1)} - \Gamma A_{i1}^{(2)}]\Omega_{1j}^* + [u_{ij}^* A_{iN}^{(1)} - \Gamma A_{iN}^{(2)}]\Omega_{Nj}^* + [v_{ij}^* A_{j1}^{(1)} - \Gamma A_{j1}^{(2)}]\Omega_{i1}^* + [v_{ij}^* A_{jM}^{(1)} - \Gamma A_{jM}^{(2)}]\Omega_{iM}^*$$

$$g_R^* = A_{i1}^{(2)}\Psi_{1j}^* + A_{iN}^{(2)}\Psi_{Nj}^* + A_{j1}^{(2)}\Psi_{i1}^* + A_{jM}^{(2)}\Psi_{iM}^*$$

$$g_R^{n+1} = A_{i1}^{(2)}\Psi_{1j}^{n+1} + A_{1N}^{(2)}\Psi_{Nj}^{n+1} + A_{j1}^{(2)}\Psi_{i1}^{n+1} + A_{jM}^{(2)}\Psi_{iM}^{n+1}$$

The subscripts R and C are defined by i, j, l, k in the following form:
$C = (l-2)(N-2) + k - 1$ and
$R = (j-2)(N-2) + i - 1$,
$i, k = 1, 2, \cdots, N$ and $j, l = 1, 2, \cdots, M$. δ_{jl} and δ_{ik} are the Kronecker delta, defined by

$$\delta_{il} = \begin{cases} 1 & \text{if } j = l \\ 0 & \text{if } j \neq l \end{cases} \quad \text{and} \quad \delta_{ik} = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{if } i \neq k \end{cases}$$

At the first step we assume that ω^n, T^n, Ψ^n, u^n and v^n are known, and then use Eqs. (21) and (22) to compute ω^*, T^*, Ψ^*; and use Eq. (17) to compute u^*, v^* and the boundary conditions of ω^* by Eq. (13). From Eqs. (23) and (24), we compute ω^{n+1}, T^{n+1}, Ψ^{n+1}; and Eq. (20) to compute u^{n+1}, v^{n+1} and the boundary conditions of ω^{n+1} by Eq. (13). The sufficient condition for the convergence of numerical solution given by the DQ method which should satisfy $\max |\omega^{n+1} - \omega^n| \leqslant \varepsilon$ (where ε is an iterative accuracy given previously, n and $n+1$ denote the local values at the beginning and the end of nth iteration). Then the steady-state solutions of the incompressible flow can be obtained.

4 Numerical Results and Discussion

The computationally efficiency of the MDQM with the Chebyshev-Gauss-Lobatto points for the numerically accurate results has been well demonstrated here. The results presented in this section are attempted to illustrate our study has extend the application of the TDQM, and the effect of sampling grid spacing with the force field on the numerical solution. The numerical computations were carried out by employing the TDQM & MDQM for the incompressible flow coupled 2D Navier-Stokes equations and heat equation with some values of $0 < Re < 1000, 0 < \alpha < 410$, and iterative accuracy $\varepsilon = 10^{-6}$, $a = b = 1$, and $F = \alpha T$, $\alpha > 0$. The numerical results for 9×9 grid DQ model with $Re = 100, \Delta t = 0.025$ and different values for α are shown in Figs 2 and 3, and for $\alpha = 400$ with different values for Re in Fig 4. The comparisons between two methods (*i.e.*, the TDQM & MDQM) in terms of iteration number for convergence for some values of Re and maximum absolute error of vortex calculation for some values of α, are listed in Tables 2 and 3. respectively with the Chebyshev-Gauss-Lobatto spacing points.

Table 2 Iteration number in the numerical solution obtained by the TDQM & MDQM, with $\Delta t = 0.025$ and $\alpha = 400$

(a) (9×9) DQ Model

Re	0.1	1	10	50	100	400	800
TDQM	–	–	–	155	137	124	123
MDQM	324	188	74	51	46	42	41

(b) (11×11) DQ Model

Re	0.1	1	10	50	100	400	800
TDQM	–	–	–	144	136	131	130
MDQM	218	132	59	45	43	41	40

The result with the MDQM for each value of $Re < 1000$ and $\alpha < 410$ is convergent, with limited values of number of iteration at very high values of Re. From Table 2 and 3, we note that the MDQM has better convergency and accuracy than the TDQM. Also, we see Re and α are the controlling parameters in the results. Moreover, we notice that the MDQM is faster than the TDQM to tend the steady state solution. From the present calculation for $Re = 100$ with 9×9 grid DQ model, at the beginning, we notice convective motion starts weakly and the numerical solution of this model problem is unstable. Then in the final stage(see Figs. 2,3,4) this motion becomes stronger and the results more stable.

All the test cases for $Re = 100$ with the different values of α are convergent by using the SOR iterative method to solve asymmetrical linear systems (21) – (24) for a damping factor $0 < \Theta < 1$ and a residual $R_\varepsilon = 10^{-5}$. Furthermore, for Re larger than 1000, the solution with the DQM will either oscillate or converge only slowly. Also the DQ method does not converge when α becomes large. From Table 1 we note that the TDQM with the Chebyshev-Gauss-Lobatto points (II) gives good results for three DQ grid models, while equally sampling spacing (I) is not convergent for the last two grid models, irrespective of the regions which does not converge or converge slowly. Generally, the type (II) gives results more accurate than the type (I), and this fact has been shown in many papers for solving different problems. The velocities (u & v) are increased (see Fig. 2.), when the force field increases (dependent on the values of temperature and parameter α). The convective motion becomes stronger and the results more stable. The temperature profiles are not plotted in the output because the results vary slowly with respect to time and they are still accurate in the present computation. The numerical solution of the governing equations leads to a final state, at which the system may be said to be most stable under imposed temperature conditions.

5 Conclusions

The improved method (MDQM) with the sampling points of type (Ⅱ) is used for solving the coupled 2D Navier-Stokes equations and heat equation. The accurate numerical results can be obtained by the MDQM using only a few grid points and requires much less

Table 3 Maximum absolute error of vortex calculation, $Re = 100$

α	$\Delta t = 0.025$			$\Delta t = 0.03125$		
	1	10	20	1	10	20
TDQM	6.68×10^{-6}	7.22×10^{-6}	6.19×10^{-6}	7.63×10^{-6}	8.34×10^{-6}	9.54×10^{-6}
MDQM	1.87×10^{-8}	6.41×10^{-7}	5.27×10^{-6}	4.66×10^{-9}	3.27×10^{-7}	5.36×10^{-7}

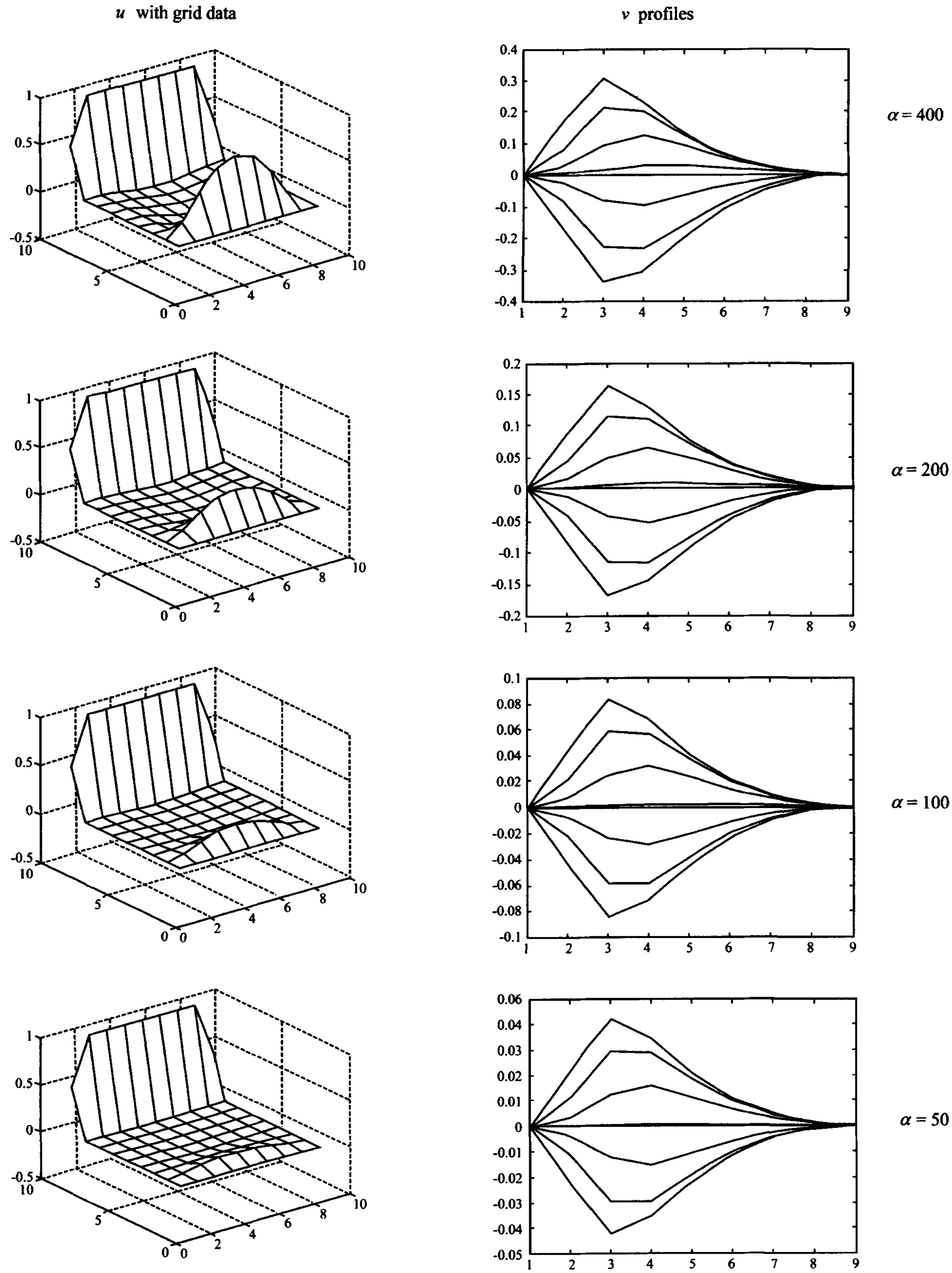

Fig. 2 The effect of the force field on the velocity components for $Re = 100$, $\Delta t = 0.025$

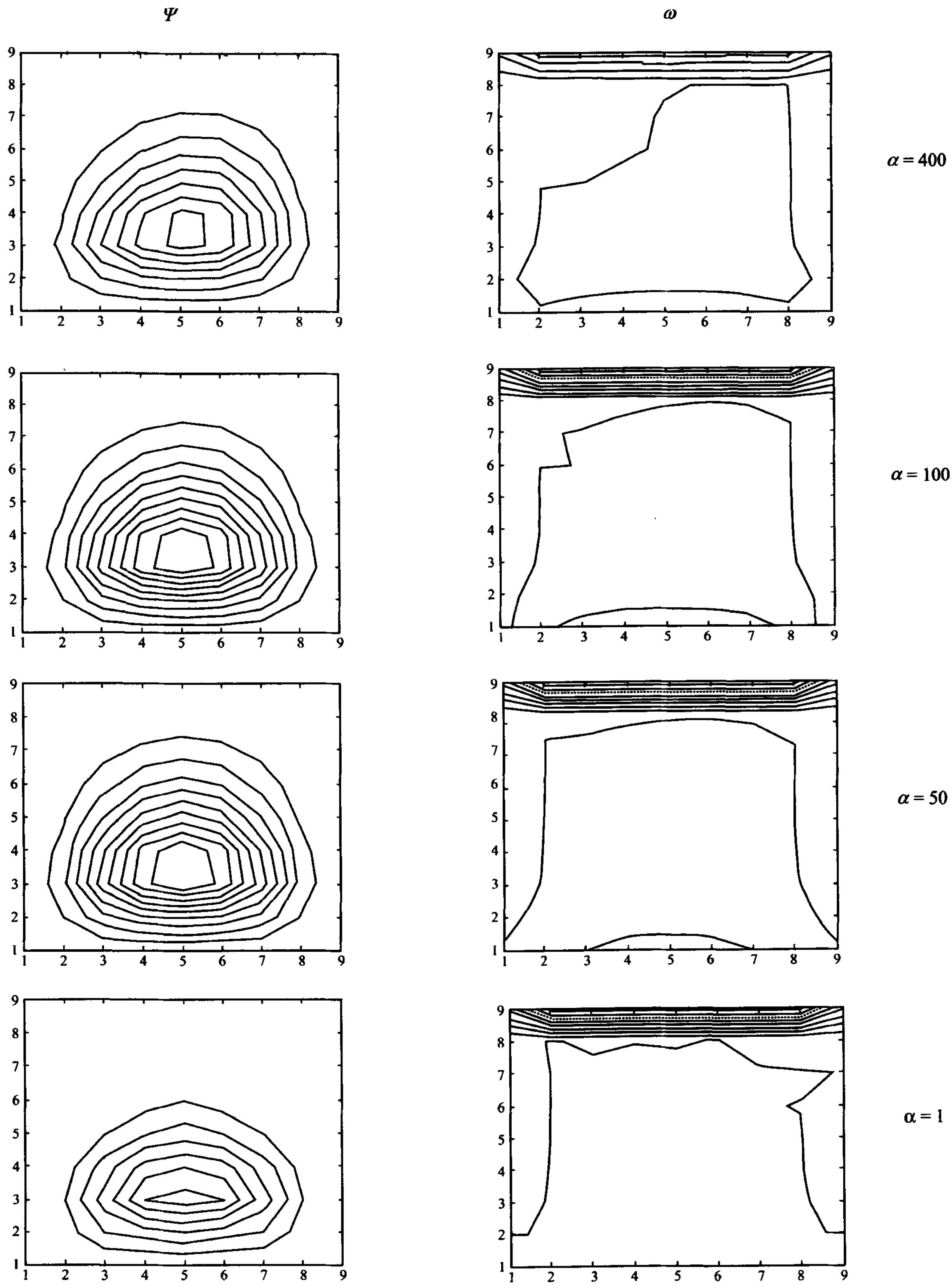

Fig. 3 The streamlines and vortex contour for $Re = 100, \Delta t = 0.025$

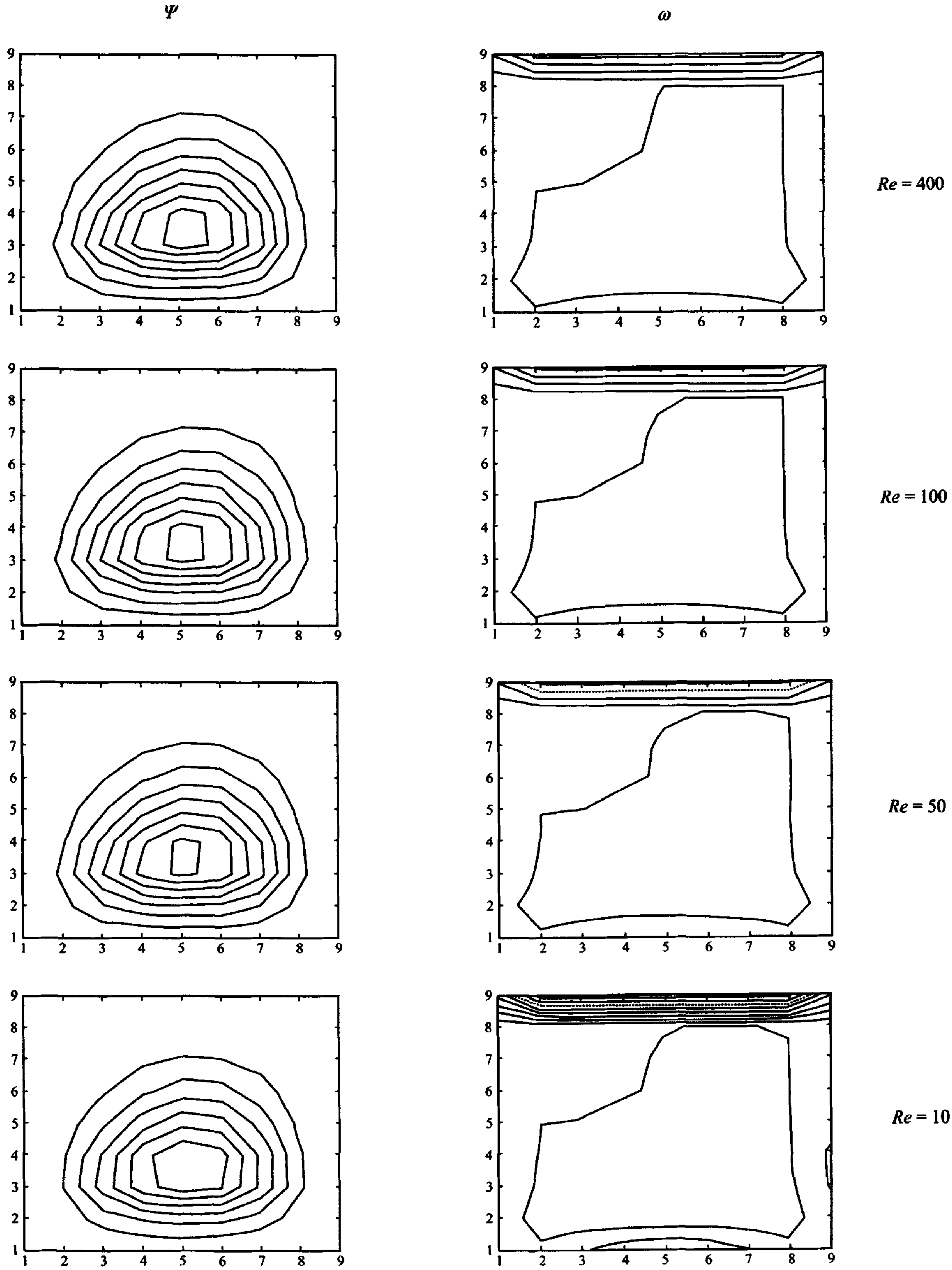

Fig.4 The streamlines and vortex contour for $\alpha = 400, \Delta t = 0.025$

storage and computational effort, compared with the conventional low-order finite difference method, in which a large number of grid points, must usually be used. The effect of the force field is clear when α increases (see Figs. 2, 3 and Table 3). Consequently. The excellent numerical solutions are obtained and are in good agreement with existing results. The MDQM with the Chebyshev-Gauss-Lobatto points has been successfully applied to overcome the divergent cases in Ref. [11]. The application of the MDQM to solve this problem in the case of α varying as a function with respect to a variable and more general problems will be reported in future work.

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