# An IBM description of <sup>76</sup>Se and neighbouring Se-isotopes

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Abstract. The level structure of <sup>76</sup>Se is discussed within the framework of Interacting Boson Models. The calculated energy levels, transition probabilities, mixing ratios,  $Q_{2_1}$  and  $g_{2_1}$  are compared with experimental data. Mixed-symmetry states are investigated in the isotopes <sup>72</sup>Se, <sup>74</sup>Se, <sup>76</sup>Se, <sup>78</sup>Se and <sup>80</sup>Se. Two mixed symmetry states with spin 1<sup>+</sup> and 3<sup>+</sup> are proposed for <sup>76</sup>Se, and also six other 1<sup>+</sup> and 2<sup>+</sup> levels in the neighbouring selenium isotopes.

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## 1 Introduction

The even-even selenium isotopes (Z = 34, N = 38-48) are situated away from both the proton closed shell number at 28 and neutron closed shell at 50, and used to be considered as being nearly spherical so that their structure may be described by the vibrational model. In particular the appearance of the  $0_2^+ 2_2^+$  and  $4_1^+$  triplet in all of these isotopes, at nearly twice the energy of the first excited state  $2_1^+$ , resembles closely the one phonon singlet state and the two phonon triplet state predicted by the vibrational model. However, a number of experiments [1-3] have explored the structure of these nuclei and it has been shown that their structures cannot be explained by the simple vibrational model. Previous studies of <sup>76</sup>Se within IBM-2 have been reported [3-5], but their calculations were limited to energy fitting and some B(E2)s, so it is worthwhile to reinvestigate the level structure and to make further calculations of electromagnetic properties and mixing ratios in order to obtain more detailed information from the interacting boson model. One of the aims of this paper is to investigate mixed-symmetry states in <sup>76</sup>Se and, by also including the other neighbouring selenium isotopes in our IBM calculations, we can further examine these states in the isotopic chain. Additionally, electromagnetic properties are calculated and compared with experiment.

#### 2 Interacting boson model (IBM)

The IBM [6–8] provides a unified description of collective nuclear states in terms of a system of interacting bosons. The shell closures for the selenium isotopes are taken at the neutron number N = 50 and the proton number at Z = 28, resulting in boson numbers, formed by proton (particle) pairs and neutron (hole) pairs. When no distinction between the proton and neutron pairs is made then the IBM is referred to as IBM-1. If protons and neutrons are explicitly introduced then the model is known as IBM-2.

IBM-1 calculations for the selenium isotopes were performed using the computer code PHINT [9] for energies. The IBM-1 Hamiltonian we used has the standard form as given in [6]. Our measurements (see Sect. 3.1) of the  $\gamma$ -rays following the  $\beta^-$  decay of <sup>76</sup>As agree very well with the decay scheme published in [10–12] and our calculations were used to fit these low-lying experimental energy levels. Values of the interaction parameters in the IBM-1 Hamiltonian of [6], which gave the best fit to the experimental data, are shown in Table 1. The calculated energy levels are shown in Fig. 1 and compared with the experimental levels.

The advantage of IBM-2 [13], in which the neutron (v) and proton ( $\pi$ ) bosons are treated separately, over IBM-1 is that, as well as energies and B(E2)s, the model can describe mixed-symmetry states [14, 15] and M1 transitions between low-lying collective states. Mixed-symmetry

 Table 1. The parameters of the IBM-1 Hamiltonian used for the descriptions of the Se-isotopes

Nucleus	3	C <sub>0</sub>	C <sub>2</sub>	$C_4$	<i>v</i> <sub>2</sub>	v <sub>o</sub>
<sup>72</sup> Se <sup>74</sup> Se <sup>76</sup> Se <sup>78</sup> Se <sup>80</sup> Se	$\begin{array}{c} 1.00 \\ 0.725 \\ 0.622 \\ 0.655 \\ 0.700 \end{array}$	0.16 0.16 0.31 0.31 0.31	$\begin{array}{r} - \ 0.31 \\ - \ 0.19 \\ - \ 0.615 \\ - \ 0.16 \\ - \ 0.15 \end{array}$	0.11 0.155 0.18 0.20 0.22	0.15 0.12 0.0915 0.062 0.075	$\begin{array}{r} - \ 0.028 \\ - \ 0.028 \\ - \ 0.028 \\ - \ 0.028 \\ - \ 0.028 \\ - \ 0.028 \end{array}$

All parameters in MeV

 Table 2. The parameters of the IBM-2 Hamiltonian used for the description of the Se-isotopes

	<sup>72</sup> Se	<sup>74</sup> Se	<sup>76</sup> Se	<sup>78</sup> Se	<sup>80</sup> Se
$N_{\pi}$	3	3	3	3	3
N <sub>v</sub>	6	5	4	3	2
ε (MeV)	1.09	0.84	0.805	0.925	0.931
κ (MeV)	-0.055	-0.055	-0.086	-0.135	- 0.139
χπ	-1.20	-1.20	-1.20	-1.20	-1.20
χv	0.02	0.14	0.38	0.65	0.80
$C_{0\pi} = C_{0\nu} \text{ (Mev)}$	-0.95	-0.65	-0.40	-0.34	-0.30
$C_{4\pi} = C_{0\nu} \text{ (Mev)}$	0.35	0.22	0.18	0.14	0.12
$\xi_1$ (Mev)	-0.43	-0.43	-0.43	-0.43	-0.43
$\xi_2$ (Mev)	0.32	0.32	0.30	0.22	0.20
$\xi_3$ (Mev)	-0.28	-0.28	-0.28	-0.28	-0.28

states arise from the out-of-phase collective motion of protons and neutrons, and are allowed in IBM-2 [16] because of extra  $\nu\pi$  degrees of freedom present in this model. They have been observed in vibrational [16], rotational [17] and  $\gamma$ -unstable [18] nuclei. In vibrational nuclei, the lowest mixed-symmetry state is a 2<sup>+</sup> level and



appears at about 2 MeV [16]. The mixed-symmetry states are expected to decay predominantly to the regular symmetric states by essentially pure M1 transitions characterised by small  $\delta$ -values.

The mixed-symmetry identity of a state is indicated by the dependence of its fitted energy on the parameters of the Majorana term [19]. The three independent Majorana parameters  $\xi_1, \xi_2$  and  $\xi_3$  influence the energy of mixed-symmetry levels without much affecting the energy of the fully symmetric states which correspond to the lowest regular IBM-1 states. A further indication of the mixed-symmetry character of a state is given by its *F-spin* [20]. States which are symmetric have  $F_{max}(F_{max} = \frac{N}{2})$  and those with  $F < F_{max}$  are not symmetric with respect to neutron and proton collective motion.

Earlier IBM-2 investigators [3, 4] chose not to fit the  $0_2^+$  state found experimentally and [4] considered it as a non-collective state. Subber [5], as a starting point, used the IBM-2 parameters of Matsuzaki and Taketani [3] to describe <sup>76</sup>Se. The parameters of their model fitted the energy of the  $0_2^+$  state correctly, but predicted poorly the  $Q_{2_1^+}$ . They investigated mixed-symmetry states, and suggested a mixed-symmetry for the  $2_3^+$  state.

Fig. 1. Positive energy states of  $^{76}$ Se from IBM calculations in comparison with the established experimental levels. The energy levels shown dashed were observed from reaction studies [3,24]. The energy levels shown dotted were not reported in [5] but were calculated from their parameters

The IBM-2 Hamiltonian is written as [13]

$$H = H_{\pi} + H_{\nu} + V_{\pi\nu} \tag{1}$$

where  $H_{\pi}$  and  $H_{\nu}$  are the proton (neutron) boson Hamiltonian, while  $V_{\pi\nu}$  is the proton-neutron interaction. A simplified Hamiltonian may be written as [21]

$$H = \varepsilon (n_{d\pi} + n_{d\nu}) + \kappa Q_{\pi} \cdot Q_{\nu} + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}$$
(2)

where Q are the quadrupole operators, the terms  $V_{\pi\pi} + V_{\nu\nu}$  represent d-boson conserving residual neutronneutron and proton-proton interactions and the last term,  $M_{\pi\nu}$ , is the Majorana interaction.

The program NPBOS [22] was used to diagonalize the Hamiltonian. The parameters in the IBM-2 Hamiltonian are obtained from a fit to the experimental data, starting from those parameters given by Kaup [4], but also including  $V_{\pi\pi} + V_{\nu\nu}$  terms. This was done because the  $V_{\pi\pi} + V_{\nu\nu}$  interaction terms were found [23] to be especially important for nuclei near closed shells. These terms, in the vibrational limit, lead to a splitting of the SU(5)two-phonon triplet, L = 0, 2, 4. The IBM-2 parameters [21] which gave the best fit are shown in Table 2. The energy level fit with these parameters is shown in Fig. 1 along with IBM-1 prediction, and the IBM-2 levels of [5]. As can be seen, the agreement between experiment and theory is quite good and the general features are reproduced well. However, the  $3^+$  and  $2^+$  states at 1689 and 2515 keV respectively were not fitted as well by IBM-2 as by the IBM-1 calculations. These levels are largely affected by the inclusion of the  $V_{\pi\pi} + V_{\nu\nu}$  terms, but the present choice of model parameters is such as to produce the best overall agreement with experimental data.

#### 3 The electromagnetic properties

Calculations of electromagnetic transitions give a good test of nuclear model wave functions. In this section we discuss the calculation of these properties, and compare them with the available experimental data. The *E*2 transition operator is given by

$$T^{(E2)} = \begin{cases} e_B Q & \text{in IBM-1} \\ e_\pi Q_\pi + e_y Q_y & \text{in IBM-2} \end{cases}$$
(3)

In IBM-1 the overall strength of *E*2 transitions was adjusted with one boson effective charge  $e_B$  but in the case of IBM-2 the boson effective charges of protons (neutrons)  $e_{\pi}(e_v)$  were considered separately. The experimental  $B(E: 2_1^+ \rightarrow 0_1^+)$  value was fixed in order to find the best fit for the remaining B(E2) values and also to determine the value of boson effective charges.

The M1 transition operator  $T^{(M1)}$  [25] in IBM-2 can be written in terms of the proton (neutron) boson *g*-factors  $g_{\pi}(g_{\nu})$ . Their values were determined by fitting to the experimental value of the *g*-factor of the  $2_1^+$  state  $(g_{2_1^+})$  and found to be  $g_{\nu} = 0.15 \mu_N$  and  $g_{\pi} = 0.70 \mu_N$ .

### 3.1 Experimental results

<sup>76</sup>As  $(T_{1/2} = 26.2 \text{ h})$  sources were prepared by thermal neutron irradiation of 99.99% pure Arsenic powder at the

**Table 3.** Relative intensities  $(I_{\gamma})$  of gamma-ray transitions following the  $\beta^-$ -decay of <sup>76</sup>As

Energy	Intensity related to $I_{\gamma}$ (559.11) = 100								
(keV)	Present		Peikun et al	Peikun et al. [26]					
114.80	0.018	(0.005)	0.097	$(0.011)^{a}$					
302.26	0.019	(0.002)	0.020	(0.004)					
357.77	0.026	(0.004)	$\simeq 0.03^{b}$						
403.10	0.052	(0.002)	0.050	(0.004)					
438.30	0.012	(0.004)	0.003	(0.001)					
456.79	0.084	(0.003)	0.076	(0.003)					
463.65	0.008	(0.003)	0.002	(0.001)					
465.60	0.016	(0.008)	0.018	(0.005)°					
472.75	0.12	(0.03)	0.113	(0.006)					
484.67	0.017	(0.001)	0.011	(0.004)					
559.11	100	· /	100	· /					
563.53	2.66	(0.03)	2.72	(0.04)					
571.48	0.29	(0.01)	0.28	(0.01)					
575.52	0.15	(0.02)	0.146	(0.007)					
641.20	0.008	(0.004)	0.008	$(0.003)^{d}$					
657.03	14.05	(0.07)	13.70	(0.29)					
665.00	*	· /		· /					
665.35	0.89	(0.05)	0.91	(0.02)					
695.20	0.020	(0.002)	0.019	(0.003)					
727.03	0.045	(0.003)	0.036	(0.006)					
740.20	0.256	(0.005)	0.265	(0.011)					
755.82	0.008	(0.004)	0.001	$(0.001)^{d}$					
771.74	0.258	(0.006)	0.251	(0.007)					
797.80	0.009	(0.003)	0.004	$(0.001)^{d}$					
809.85	0.039	(0.001)	0.036	(0.002)					
863.52	0.027	(0.003)	0.025	$(0.002)^{d}$					
867.59	0.28	(0.01)	0.28	(0.02)					
882.25	0.14	(0.01)	0.121	(0.02)					
954.76	0.008	(0.001)	0.007	(0.002)					
980.73	0.095	(0.002)	0.088	(0.006)					
1060.73	0.010	(0.002)	0.004	(0.001)°					
1098.36	0.010	(0.002)	0.007	(0.003)					
1129.88	0.310	(0.005)	0.289	(0.007)					
1130.14	*	(0.000)		(*****)					
1212.62	3.15	(0.06)	3.15	(0.06)					
1216.14	8.45	(0.32)	7.53	(0.10)					
1228.38	2.78	(0.06)	2.60	(0.04)					
1393.10	0.0043	$(0.001)^{\circ}$		(****)					
1439.14	0.61	(0.02)	0.60	(0.02)					
1453.78	0.236	(0.009)	0.23	(0.02)					
1533.06	0.057	(0.003)	0.051	(0.003)					
1568.28	0.020	(0.002)	0.015	(0.002)					
1611.54	0.017	(0.002)	0.028	(0.002)					
1787.69	0.62	(0.01)	0.62	(0.02)					
1804 70	0.005	(0.001)	0.003	$(0.02)^{\circ}$					
1870.08	0.111	(0.001)	0.11	(0.002)					
1881.35	0.003	(0.001)	0.002	$(0.001)^{d}$					
1955.83	0.021	(0.001)	0.019	(0.002)					
2096.35	0.17	(0.03)	1 19	(0.02)					
2110.79	0.71	(0.03)	0.68	(0.02)					
2127.17	0.004	(0.001)	0.003	(0.001)					
2429.10	0.074	(0.006)	0.068	(0.001)					
2655.36	0.086	(0.006)	0.090	(0.003)					
	0.000	(	0.020	()					

<sup>a</sup> Intensity reported by [27]

<sup>b</sup> Intensity reported in [11] by Funel [28]

<sup>c</sup> Intensity reported in [11] by Nagahara [29]

<sup>d</sup> Intensity reported in [11] by Kaur [10]

<sup>e</sup> Gamma ray reported in [11] by Ardisson [30] in coincidence only

\* Resolved in coincidence spectra, also by [27] and in [11] by [10]

$B(E2) (e^2b^2)$					$B(M1)$ ( $\mu_N^2$ )				
Case	ade	Experiment	Experiment		IBM			IBM-2	
$\overline{J_i^+}$	$J_f^+$	Present	Ref. [11]	IBM-1	IBM-2	Present	Ref. [11]	$\mathbf{N}^{\mathbf{a}}$	Sp
21	01	0.084(2)	0.083(2)	0.083	0.083				
$2_{2}^{-}$	$0_1$	0.0022(5)	0.0024(6)	0.0068	0.0005				
$2_{3}^{-}$	$0_{1}^{-}$	$0.00007(\frac{7}{3})$	$0.00006(^{6}_{4})$	0.0	0.0001				
$0_2$	$2_{1}^{2}$	0.0091(42)	0.09(4)	0.09	0.07				
$2^{\tilde{2}}_{2}$	$2_{1}^{1}$	0.083(17)	0.082(21)	0.010	0.12	0.0015(2)	0.0009(2)	0.0028	0.0094
$2\bar{3}$	$2_{1}^{1}$	$0.0004(\frac{4}{2})$	0.0004(2)	0.0	0.0001	$0.0017(\frac{17}{6})$	0.0021(11)	0.0013	0.0041
24	$2_{1}^{1}$	(2)		0.0001	0.0002	(0 )	( )	0.077	0.26
$2_{5}^{-}$	2,				0.0002	0.0002		0.031	0.10
41	$2_{1}^{1}$	0.137(13)	0.136(4)	0.14	0.13				
3	21	0.0062	< 0.024	0.010	0.0010	0.0062	< 0.025	0.0013	0.0042
1	$0_{1}^{-1}$							0.020	0.066
1	2				0.0001			0.01	0.024
3,	$\frac{-1}{2}$				0.0001			0.0034	0.011
3.	4.		< 0.49	0.031	0.037		< 0.047	0.026	0.087
$\frac{2}{2}$	4.	$0.008(^{8}_{2})$	0.006(2)	0.025	0.025		2 0.0 17	0.020	0.007
3	4.	0.000(2)	0.000(4)	0.025	0.005			0.032	0.11
4.	4.	0.056(32)	0.056(28)	0.053	0.064	0.007(3)	0.0072(35)	0.002	0.020
6.	$\underline{4}$	0.000(02)	0.16(3)	0.055	0.15	0.007(5)	0.0072(00)	0.000	0.020
1.	0.		0.10(5)	0.10	0.15			0.065	0.21
1.	$\frac{0}{2}$				0.0001			0.005	0.41
3	$\frac{2}{2}$	0.35(20)	< 0.78	0.10	0.0001	0.08(5)	< 0.125	0.033	0.11
$\frac{J_1}{2}$	$\frac{2}{2}$	0.55(20)	< 0.011	0.10	0.10	0.00(3)	< 0.123	0.000	0.0040
23	$\frac{2}{2}$		< 0.011	0.0000	0.015		< 0.0025	0.0012	0.0040
52 1	$\frac{2}{2}$	0.067(1)	0.07(5)	0.082	0.075			0.032	0.11
+2 a	$\begin{pmatrix} 2 \\ (\mu) \end{pmatrix}$	0.007(1)	$0.07(_2)$	0.082	0.075		+ 0.40(12)	+ 0.40	0.46
$g_{2_{1}^{+}}$	$(\mu_N)$						+ 0.40(12)	+ 0.40	+ 0.40

**Table 4.** Experimental B(E2) ( $e^2b^2$ ) values and B(M1) ( $\mu_N^2$ ) values in <sup>76</sup>Se compared with IBM predictions. The  $g_{2^+}$  is also given

<sup>a</sup> N: nonstandard values used for the boson *g*-factors,  $g_v = 0.15 \mu_N$  and  $g_{\pi} = 0.70 \mu_N$ 

<sup>b</sup> S: standard values used for the boson *g*-factors,  $g_v = 0.0 \ \mu_N$  and  $g_\pi = 1 \ \mu_N$ 

Imperial College Reactor Centre. Any one source was left for 20 h before measurements were initiated to allow short-lived activities to die away and to provide a source strength of about  $3.7 \times 10^5$  Bq ( $10_{\mu}$ Ci), sufficient to give about 2000 counts s<sup>-1</sup> at a source-to-detector distance of 25 cm. Gamma-rays following the  $\beta$ -decay of <sup>76</sup>As were detected by HPGe detector (24% efficient with 1.8 keV resolution, FWHM for the 1332 keV peak of <sup>60</sup>Co) and Ge(Li) detector (10%, 2.3 keV resolution).

The present singles and  $\gamma$ - $\gamma$  coincidence measurements confirm the main results of previous work as published in Nuclear Data Sheets [11]. The energies and relative intensities of all transitions observed in the present work, together with the relative intensities of the gamma-rays reported by Peikun [26] are listed in Table 3. Our gamma-ray intensities were used to calculate the present experimental B(E2) and B(M1) values which are shown in Table 4 for a comparison with [11] and our IBM calculations.

#### 3.2 Reduced transition probabilities

The wave functions obtained by diagonalization of the IBM-1 Hamiltonian have been used by the program FBEM [9] to calculate the electric quadrupole transition

probabilities. Those from the IBM-2 Hamiltonian have been used by the program NPBEM [22] to calculate the reduced transitions probabilities for E2 and M1 transitions. The complete range of B(E2) values, given in Table 4, were calculated with boson effective charges  $e_B =$ 0.103 eb (the equivalent parameter in FBEM is E2SD  $= e_B$ ) and  $e_{\pi} = 0.089$  eb and  $e_v = 0.087$  eb in NPBEM. The experimental and calculated B(E2) values are compared in Table 4. The predictions of the two models only show a very significant difference in the case of the  $2_2^+ \rightarrow 0_1^+$  transition, and are otherwise in good agreement with experiment. The quadrupole moment of the first excited state  $Q_{2_1^+}$  is also calculated. The predicted IBM-1 and IBM-2 values are -0.38 (eb) and -0.23 (eb) respectively. Although the IBM-2 value is low and just outside the error in the experimental value -0.34(7) (eb) [11], it is closer than the value of -0.014 eb predicted by the IBM-2 model parameter of Subber [5]. They used large and opposite values of  $\chi_{\pi}(=1.0)$  and  $\chi_{\nu}(=-1.0)$  to obtain a best fit to the observed energies but at the expense of the quadrupole moment [16, 31].

The  $g_{2_1^+}$  and B(M1) values obtained from NPBEM [22], with boson *g*-factors as described at the beginning of this section, are compared with experiment in Table 4. In general, good agreement between the IBM-2 prediction and the experimental results is achieved. Having obtained

the value of the reduced E2 and M1 matrix elements, we can calculate the multiple mixing ratios ( $\delta$ ). They are defined as [32]

$$\delta\left(\frac{E2}{M1}\right) = 0.0835E_{\gamma}(\text{MeV}).\,\Delta \tag{4}$$

where  $\Delta$  is the ratio of the reduced E2 matrix elements to the reduced M1 matrix element, and determines the sign of  $\delta$ . The predicted  $\delta$ -values are shown in Table 5 to allow a comparison with the available experimental data. It is seen that there is a good agreement between the magnitude of the mixing ratios as calculated from the IBM-2 and those obtained from experiment. Notice again that the predicted sign of the multipole mixing ratio of all transitions is negative, consistent with experiment, the exception being for the transition at 1129.88 keV for which, experimentally, the sign of the mixing ratio is positive. It may be noted that so far such properties  $(B(M1) \text{ and } \delta \text{ values})$  have not been reported in the literature, but to date few experimental data are available to test such IBM-2 predictions.

The values of  $g_{\nu}$  and  $g_{\pi}$  used here are close to, but with some deviation from, the standard values of the boson g-factor  $(g_v = 0 \mu_N \text{ and } g_\pi = 1 \mu_N)$  which follow from the microscopic calculations of Sambataro [25]. Such deviation, however, is justified by Lipas [33] who concludes that the values of  $g_{\pi}$  and  $g_{\nu}$  depend on the nuclear configuration and they should be different for different nuclei. It has been found that with the standard boson g-factor the experimental  $\delta$ -values are not described properly (Table 5), and also  $g_{2_1^+}$  would be +0.46  $\mu_N$  (Table 4). Thus the measured value of the *g*-factor of the  $2_{2_1}^+$  state  $(g_{2_1}^+)$  is reproduced very well with  $g_{\pi} = 0.70 \mu_N$  and  $g_{\nu} = 0.15 \mu_N$ and not with the standard values. Wolf [35] points out that in calculating  $g_{2_1^+}$  at least part of the empirical devi-

**Table 5.** IBM-2 mixing ratios ( $\delta$ ) in comparison with available experimental data in 76Se

		$J_f^+$	Mixing Ratio ( $\delta$ ) (e.b/ $\mu_N$ )					
Transition	$J_i^+$		IBM-2		Experiment			
(kev)			S <sup>a</sup>	N <sup>b</sup>				
657.04	22	21	- 1.92	- 3.50	$-4.7(^{11}_{20})^{\rm c}$ or $4.15(20)^{\rm d}$			
1228.60	23	21	- 0.19	- 0.34	$-0.49(5)^{e}$ or			
1129.88	31	21	- 0.44	-0.80	$-0.53(8)^{a}$ 1.08(10) <sup>f</sup> or $+0.45 \le \delta \le 1.5^{g}$			
695.13 571.40	$4_{2} \\ 2_{3}$	$4_{1} \\ 2_{2}$	-1.0 -0.94	-1.83 - 1.70	$+ 0.43 < 0 < 1.5^{\circ}$ $- 1.7(^{6}_{1})^{\circ}$ $> 1.37^{d} \text{ or}$ $0.12(24)^{d}$			
472.92	31	22	- 0.36	- 0.66	$-0.13(34)^{-1}$ $-0.24(^{95}_{85})^{\circ}$ or			
575.34	32	23	- 0.22	-0.40	$(-0.75(_{44}^{+3})^{\circ})^{\circ}$ $(-1.18(35)^{f})^{\circ}$			

<sup>a</sup> S: standard values used for the boson *g*-factors,  $g_v = 0.0 \mu_N$ , and  $g_{\pi} = 1.0 \ \mu_N$ <sup>b</sup> N: nonstandard values used for the boson *g*-factors,  $g_{\nu} = 0.15 \ \mu_N$ 

and  $g_{\pi} = 0.70 \ \mu_N$ <sup>c,d,e,f,g</sup> Experimental values from [3], [5], [34], [12], [10]

ation of the boson g-factors from their "bare" values of 1 and 0 is due to the presence of some F-spin admixture in the state which is therefore not a fully symmetric

#### 4 Mixed-symmetry states in <sup>76</sup>Se

state.

The calculation by IBM-2 of all energy levels above  $4_1^+$  (except the  $6_1^+$  level) in <sup>76</sup>Se, as shown in Fig. 1, was found to be very sensitive to the variation in the parameters  $\xi_i$  of the Majorana term, which also affect the magnitude and the sign of the multipole mixing ratios of some transitions. This indicates these levels are either of a mixed-symmetry character or contain mixed-symmetry contributions. Those with mixed-symmetry character have no counterpart in IBM-1. An inspection of Fig. 1 shows that each level predicted by IBM-2 has an equivalent energy level from IBM-1 calculations except the  $1_1^+$  and  $3_2^+$  levels. It is noticed that these two levels have already been seen as mixed-symmetry candidates in vibrational-like nuclei [36, 37] and therefore a mixed-symmetry description can be given for these states. Possible experimental energy level candidates for the predicted  $1_1^+$  and  $3_2^+$  states are the levels at 1881 and 2363 keV which have suggested spins of  $(1, 2)^+$  and  $3^+$  respectively. More experimental information is needed, especially on B(E2) and B(M1) decay modes as they would assist a mixed-symmetry description. The predicted B(M1)values listed in Table 4 for transitions from the  $1_1^+$  and  $3_2^+$  states to the regular symmetric states, however, show their predominance over the B(E2) components, and this is consistent with a mixed-symmetry character. Stronger B(M1) transitions can be obtained (see Table 4) when the standard values of the boson g-factors  $g_{\pi}(=1 \mu_N)$  and  $g_{\nu}(=0.0 \ \mu_N)$  are considered, but the agreement with experiment of the mixing ratios of most transitions becomes worse (see Table 5). Although the fitted energy of the  $2_3^+$  level has been found sensitive to the Majorana parameters, as discussed above, the present IBM analysis indicates mixed-symmetry contributions, rather than the mixed-symmetry character suggested by Subber [5], since it has been fitted successfully by IBM-1 as well as by IBM-2 (see Fig. 1).

## 5 The neighbouring isotopes <sup>72</sup>Se, <sup>74</sup>Se, <sup>78</sup>Se, <sup>80</sup>Se

By including the other four even mass members of the isotopic chain from A = 72-80 in a similar IBM-1 and IBM-2 analysis we were able to extend our discussion of mixed-symmetry states.

The parameters for both models were constrained to vary smoothly with neutron number in order to fit the experimental energies. The final parameters are given in Tables 1 and 2 and it is seen that there is a sharp change in  $\chi_{\nu}$ ,  $C_{0(\pi)\nu}$ ,  $C_0$  and  $C_2$  between <sup>72,74</sup>Se and <sup>76</sup>Se. This is to counteract the trend that the calculated energies of some  $2^+$  and  $0^+$  levels become much higher than experiment. The correspondence between the fitted IBM energy levels and those measured experimentally is shown in Fig. 2. The agreement is quite good, except in the case of <sup>72</sup>Se where



E(keV) Exp.[38] IBM-1 IBM-2 Exp.[39] IBM-1 IBM-2 Exp.[40] IBM-1 IBM-2 Exp.[41] IBM-1 IBM-2 3000

**Fig. 2.** Comparison between experimental energy levels and IBM models for <sup>72</sup>Se, <sup>74</sup>Se, <sup>78</sup>Se and <sup>80</sup>Se isotopes

the two phonon triplet is expected to be accounted for by the residual interaction between like nucleons in the IBM-2 Hamiltonian, as is expected for the  $0_2^+$  853 keV level in <sup>74</sup>Se. The 6<sup>+</sup> level 2466 keV is predicted rather poorly by IBM-2, whereas the predicted *B*(*E*2) value, Table 6, is in very good agreement with experiment.

Both models work well below 2 MeV excitation energy for  $^{74,78,80}$ Se, and the IBM-2 is good between 2–3 MeV. One reason is that levels above 2 MeV are mostly 2<sup>+</sup> states which are readily influenced by the Majorana parameters. The slightly better fits that can be obtained for the energy levels of a single nucleus were rejected in the spirit of obtaining a smoothly varying global fit.

Associated with the energy spectra the electromagnetic properties of the four neighbouring isotopes were also calculated, as these properties depend on the Boson Model wave functions and help identify mixed-symmetry states. The  $B(E: 2_1^+ \rightarrow 0_1^+)$  experimental value was used to determine  $e_B$  for IBM-1 and  $e_v$  and  $e_\pi$  for IBM-2, while  $g_v$  and  $g_\pi$  were normalized to the experimental *g*-factor of the  $2_1^+$  state. These are shown at the end of Table 6.

The experimental and calculated B(E2) values (several calculated for the first time) are compared in Table 6: very good agreement has been obtained for most values, notable exceptions being for the few cases where  $0^+$  states are involved.

A	Energy (keV)	Cascade	$B(E2) (e^2b^2)^{(e)}$			$B(M1) \; (\mu_N^2)^{({ m f})}$		$(\delta) (eb/\mu_N)$	
		$J_i^+ \ J_f^+$	Expt.	IBM-1	IBM-2	Expt.	IBM-2	Expt.	IBM-2
72 <sup>(a)</sup>	$\begin{array}{c} 862.03 \\ 075.00 \\ 379.55 \\ 1316.70 \\ 774.73 \\ 830.10 \\ Q_{2_1^+}(eb) \\ g_{2_1^+}(\mu_N) \end{array}$	$\begin{array}{cccc} 2_1 & 0_1 \\ 0_2 & 2_1 \\ 2_2 & 0_2 \\ 2_2 & 0_1 \\ 4_1 & 2_1 \\ 6_1 & 4_1 \end{array}$	0.040(4) 0.28(5) 0.12(2) 0.00068(10) 0.09(1) 0.089(13)	$\begin{array}{c} 0.040\\ 0.080\\ 0.021\\ 0.0055\\ 0.075\\ 0.094\\ -\ 0.34\end{array}$	$\begin{array}{c} 0.040\\ 0.052\\ 0.0047\\ 0.00\\ 0.070\\ 0.087\\ -\ 0.15\end{array}$		0.35		
74 <sup>(b)</sup>	634.78 219.05 634.32 1269.06 728.37 1203.90 615.17 744.75 838.97 1437.21 868.21 777.61 $Q_{27}^+$ (eb) $q_{2+}^+$ ( $\mu_x$ )	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 0.078(2)\\ 0.14(2)\\ 0.089(26)\\ 0.0015(4)\\ 0.15(1)\\ \end{array}$ $\begin{array}{c} 0.20(9)\\ < 0.032\\ 0.05(2)\\ 0.007(3)\\ 0.13(3)\\ 0.089(33)\\ - 0.36(7)\\ \end{array}$	$\begin{array}{c} 0.077\\ 0.12\\ 0.082\\ 0.0094\\ 0.14\\ \end{array}$ $\begin{array}{c} 0.10\\ 0.043\\ 0.080\\ 0.0082\\ 0.165\\ 0.068\\ -\ 0.44\\ \end{array}$	$\begin{array}{c} 0.078\\ 0.10\\ 0.11\\ 0.0002\\ 0.13\\ 0.0009\\ 0.10\\ 0.065\\ 0.080\\ 0.00\\ 0.156\\ 0.07\\ -\ 0.20\\ \end{array}$	0.00072(54) < 0.013	0.0009 0.00 0.0151 0.002	-5.6(16), -2.6(2) 0.18(9), 1.5(3) 0.31(1) -4.3(3), 2.4(2)	- 6.21 - 6.54 - 1.32 - 3.95
78 <sup>(c)</sup>	$g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$ $g_{21}^{-}(\mu_{N})$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 0.079(13)\\ 0.11(3)\\ 0.047(^{2}_{14})\\ 0.0021(1)^{*}\\ 0.058(10)^{*}\\ 0.066(5)^{*}\\ 0.0014(^{2}_{8})\\ 0.020(10)\\ 0.0016(^{2}_{14})\\ 0.00018\\ 0.063(26)\\ 0.0089(35)\\ 0.00022(12)\\ - 0.26(9) \text{ or}\\ - 0.30 (11)\\ \end{array}$	$\begin{array}{c} 0.081\\ 0.13\\ 0.10\\ 0.0025\\ 0.06\\ 0.12\\ 0.0034\\ 0.054\\ 0.0032\\ 0.00\\ 0.962\\ 0.026\\ 0.00\\ 0.00\\ - 0.22 \end{array}$	$\begin{array}{c} 0.079\\ 0.12\\ 0.076\\ 0.0002\\ 0.05\\ 0.11\\ 0.002\\ 0.00\\ 0.004\\ 0.00\\ 0.004\\ 0.00\\ 0.057\\ 0.026\\ 0.00\\ 0.0001\\ - 0.20\\ \end{array}$	$0.06(\frac{2}{3})$ $0.0089(\frac{53}{35})$ 0.0062(26) 0.14(5) 0.0061(34) 0.39(11)	0.073 0.019 0.0022 0.0068 0.0018 0.06 0.0334 0.016 0.40	$0.42(4), 0.45(10)$ $3.5(5), 4.0(7), 2.7(\frac{9}{6})$ $- 0.41(\frac{13}{31})$ $> - 2.2 < - 0.01$ $- 0.30(18)$ $> - 0.49 < - 0.12$ $- 1.8(5), 3.3(\frac{17}{1}), - 0.1(1)$	-0.45 -1.40 -0.281 0.028 0.014 -0.58 0.023
80 <sup>(d)</sup>	$g_{21}$ (µN) 666.14 783.14 1449.30 812.61 1035.26 1294.07 1343.00 793.6 1828.9 1644.80 $Q_{21}$ (eb)	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 0.051(\ )\\ 0.039(4)\\ 0.0072(2)\\ 0.0014(2)\\ 0.072(2)\\ 0.00012(\frac{1}{8}^2)\\ 0.00016(8)\\ 0.062(\frac{6}{2}^1)\\ 0.0012(\frac{1}{6}^2)\\ -\ 0.31(7) \ \text{or}\\ -\ 0.35(12) \end{array}$	$\begin{array}{c} 0.051\\ 0.072\\ 0.0015\\ 0.041\\ 0.079\\ 0.00\\ 0.00\\ 0.00\\ 0.036\\ 0.00\\ 0.00\\ -\ 0.17\\ \end{array}$	$\begin{array}{c} 0.051\\ 0.073\\ 0.00\\ 0.033\\ 0.072\\ 0.0028\\ 0.0001\\ 0.001\\ 0.034\\ 0.00\\ 0.00\\ -\ 0.11\\ \end{array}$	0.00071(53) $0.025(\frac{25}{13})$	0.016 0.040 0.035 0.00	$-5(\frac{2}{6}), 0.71(\frac{12}{17}), 5(\frac{3}{2}),$ $-0.15(\frac{9}{5}), -0.4(1), > +8$ $-0.31(5), +10(\frac{10}{2})$ +1.1(1) $+1.95(\frac{5}{5}), -0.10(\frac{2}{6})$	-1.42 + 0.29 + 0.65 - 0.31
	$g_{2_{1}^{+}}(\mu_{N})$					0.42(24)	0.43		

**Table 6.** B(E2) and B(M1) values and mixing ratios ( $\delta$ ) obtained by IBM-1 and IBM-2 calculations in comparison with experimental results for transitions in the Se-isotopes. The  $Q_{2_1^+}$  and  $g_{2_1^+}$  are also given

<sup>(a, b, c, d)</sup> B(E2), B(M1) and ( $\delta$ ) experimental values from [38–41] <sup>(e)</sup> Boson effective charges and <sup>(f)</sup> g factors used in <sup>72</sup>Se:  $e_B = 0.065$  (eb),  $e_v = 0.040$  (eb),  $e_\pi = 0.090$  (eb) and  $g_v = 0.15 \ \mu_N, \ g_\pi = 0.70 \ \mu_N$ <sup>74</sup>Se:  $e_B = 0.093$  (eb),  $e_v = 0.080$  (eb),  $e_\pi = 0.089$  (eb) and  $g_v = 0.20 \ \mu_N, \ g_\pi = 0.70 \ \mu_N$ <sup>78</sup>Se:  $e_B = 0.11$  (eb),  $e_v = 0.086$  (eb),  $e_\pi = 0.098$  (eb) and  $g_v = 0.05 \ \mu_N, \ g_\pi = 0.71 \ \mu_N$ <sup>80</sup>Se:  $e_B = 0.097$  (eb),  $e_v = 0.080$  (eb),  $e_\pi = 0.094$  (eb) and  $g_v = 0.25 \ \mu_N, \ g_\pi = 0.65 \ \mu_N$ \* Experimental value from [1]

The inclusion of  $(V_{\pi\pi} + V_{\nu\nu})$  in the IBM-2 Hamiltonian lowers the value of  $Q_{2_1^+}$  (Table 6) below both the IBM-1 and experimental value, except in the case of <sup>80</sup>Se where the IBM-1  $Q_{2_1^+}$  value is also lower than experiment.

The B(M1) transition probabilities,  $g_{2_1^+}$  and mixing ratios were calculated by the IBM-2 for the first time and are shown in Table 6. As for <sup>76</sup>Se, a good agreement with experiment was only achieved when nonstandard values were used for  $g_v$  and  $g_{\pi}$ .  $\delta$ -values are also good; discrepancies in sign being caused by the use of different sign convensions in the experimental work.

The identification of mixed symmetry states is hampered by a lack of experimental B(E2) and B(M1) data, but a comparison between IBM-1 and IBM-2 results can still help their identification. The IBM-1 fitting above 2 MeV is not good, but the 2<sup>+</sup> levels in <sup>74</sup>Se and <sup>78</sup>Se at 2482 keV and 2536 keV respectively, which are well fitted by IBM-2, and have no counterpart in IBM-1 can therefore be regarded as being of mixed symmetry. The same argument applies to the  $(1, 2)^+$  level at 2378 keV in <sup>74</sup>Se. The small  $\delta$ -value, resulting from the *M*1 component of the 1923 keV transition from the 2536 keV level in <sup>78</sup>Se being stronger than its B(E2) component, supports this conclusion.

In <sup>80</sup>Se the energy levels at 2515  $(1^-, 2^+)$  and 2627  $(0^+-2^+)$  keV have no equivalent levels in the IBM-1, but they are predicted by the IBM-2 both with spin 2<sup>+</sup> at 2479 and 2677 keV respectively. The IBM-2 analysis also gives a 1<sup>+</sup> level at 2545 keV in <sup>78</sup>Se which could correspond to the experimental energy level not predictable by IBM-1 at 2647 keV.

#### 6 Summary and conclusion

The energy levels in <sup>76</sup>Se have been investigated by both IBM-1 and IBM-2, and in general a satisfactory agreement with the experimental data is obtained. Although the energy spectrum of <sup>76</sup>Se displays a vibrational-like structure, the use of the IBM-1 complete Hamiltonian rather than the SU(5) limit is an effort to account for the phase transition observed in this nucleus [3]. The use of the complete Hamiltonian shows that vibrational features are dominant in <sup>76</sup>Se, but with the presence of some O(6)characteristics [5]. A best fit for states above 1.5 MeV and  $Q_{2+}$  was obtained from the IBM-1 complete Hamiltonian, while the 2-phonon triplet states were produced very well by IBM-2 calculations. This was due to inclusion of the interaction between like nucleons, which on other hand lowers the predicted value of  $Q_{2_1^+}$ . With both models the  $0_3^+$  state occurs lower than the experimental value, and crosses-over with  $2_4^+$  state. This may result from some non-collective degree of freedom present in this state. For the first time the IBM-2 was used to calculate  $g_{2^+}$ , B(M1) transitions and multipole mixing ratios in <sup>76</sup>S<sup>4</sup>. A good agreement has been achieved with the experimental results using nonstandard values for the boson g-factors.

The IBM-2 analyses reveal that some levels in this selenium isotopic chain appear to have mixed-symmetry character. This is indicated by their sensitivity to the Majorana parameters and also they are fitted successfully in IBM-2 but have no counterpart in IBM-1. Further evidence for the mixed-symmetry of a state is the *F-spin* which is given by the IBM-2 calculations as a % of  $F_{max}$ . The states considered in this paper to be of mixed-symmetry do not have this F% greater than 77.

In <sup>76</sup>Se the  $1_1^+$  (F% = 55) and  $3_2^+$  (F% = 75) levels are both absent from the IBM-1 calculations and the possible experimental levels with spin  $(1, 2)^+$  and  $3^+$  are 1881 keV and 2363 keV. For the same reasons in <sup>74</sup>Se and <sup>78</sup>Se two levels with spin  $2^+$  at 2482 (F% = 77) and 2536 keV (F% = 50) respectively are suggested to have mixed-symmetry character, as is the  $(1, 2)^+$  level in <sup>74</sup>Se at 2378 keV which has the low F% = 25. The same holds for the levels at 2515 ( $1^-, 2^+$ ) (F% = 41) and 2627 ( $0^+-2^+$ ) keV (F% = 54) in <sup>78</sup>Se and the energy level at 2647 (1, 2)<sup>+</sup> keV (F% = 49) in <sup>78</sup>Se. However, the suggestion of mixedsymmetry character can not be conclusive until further experimental data is available.

Concerning the main argument about the nature of the  $2_3^+$  states in selenium, the IBM results indicate they have mixed-symmetry contributions rather than mixed-symmetry character, as they are fitted successfully by both IBM-1 and IBM-2.

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