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Low-lying positive parity yrast bands in Gd, Dy and Er nuclei for N = 96

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Abstract. The interacting boson model has been applied to calculate the low-lying positive parity yrast bands in Gd, Dy and Er nuclei for N = 96 neutrons. The results closed to the SU(3) limit in this model. Reasonable agreement with available energies and B(E2) transition rates. The potential energy surfaces (PESs) to the IBM Hamiltonian have been obtained using the intrinsic coherent state.
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1 Introduction

Arima and Iachello (1979) [1] have developed the interacting boson model (IBM), which is one of those attempts that has been successful in describing the low-lying nuclear collective motion in medium and heavy mass nuclei. The interacting boson model (IBM) is a powerful tool to describe even-even nuclei in term of an inner core plus the valence particles outside the nearest closed shells considered as boson. The IBM-1 formalism (no distinction between neutron and proton bosons), applicable to even-even nuclei, the basic building blocks are s and d bosons, with angular momentum $J^{\pi} = 0^{+}$ and 2^{+} [1, 2] and it is successful in the description of energy spectra and also electromagnetic transition strengths in a wide range of the nuclear chart. The underlying U(6) group structure of model give three dynamical symmetry limits, known as vibrational U(5), rotational SU(3) and gamma unstable O(6) [3-5]. The nuclei ^{160}Gd, ^{162}Dy, and ^{164}Er, have atomic number Z = 64, 66, and 68, respectively, and same neutron number N = 96, these nuclei are part of an interesting region beyond the closed proton shell at p= 50, while the number of neutrons in the open shell is much larger, as such these nuclei have been commonly considered to exhibit rotational-like properties.
Many experimental and theoretical studies on the structure of energy level and electromagnetic transition properties of the even-even Gd-Er isotopes had been investigated [6-15].
The aim of the present work by application of IBM-1 to predict the yrast level, reduced transition probabilities and PES to understand the type of dynamical symmetry which exist in Gd, Dy and Er nuclei for N = 96.

2 Method of Calculations

The IBM-1 Hamiltonian can be expressed as [1, 16-18]:

$$H = \varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L \left[\left[d^\dagger \times d^\dagger\right]^{(L)} \times \left[\tilde{d} \times \tilde{d}\right]^{(L)}\right]^{(0)} + \frac{1}{\sqrt{2}}\upsilon_2 \left[\left[d^\dagger \times d^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{s}\right]^{(2)} + \left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{d}\right]^{(2)}\right]^{(0)} + \frac{1}{2}\upsilon_0 \left[\left[d^\dagger \times d^\dagger\right]^{(0)} \times \left[\tilde{s} \times \tilde{s}\right]^{(0)} + \left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{d} \times \tilde{d}\right]^{(0)}\right]^{(0)} + \frac{1}{2}u_0 \left[\left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{s} \times \tilde{s}\right]^{(0)}\right]^{(0)} + u_2 \left[\left[d^\dagger \times s^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{s}\right]^{(2)}\right]^{(0)} \quad (1)$$

In this Hamiltonian, specified by nine parameters, two appearing in the one body terms (ε_s and ε_d, the dots indicate the scalar products), and seven in the two-body terms [c_L (L = 0, 2,4), v_L (L = 0, 2), u_L (L = 0, 2) the crosses indicate tensor products]. However, the total number of boson N_b (pairs) is conserved, $N_b = n_s + n_d$ [17].
Then the IBM-1 Hamiltonian in equation (1) can be written in general form as [17, 19]:

$$\hat{H}=\varepsilon\hat{n}_d+a_0\hat{P}.\hat{P}+a_1\hat{L}.\hat{L}+a_2\hat{Q}.\hat{Q}+a_3\hat{T}_3.\hat{T}_3+a_4\hat{T}_4.\hat{T}_4 \quad (2)$$

where $\hat{n}_d$, $\hat{p}$, $\hat{L}$, $\hat{Q}$ and $\hat{T}_r$ are the total number of d_{boson}, pairing, angular momentum quadrupole the octupole (r=3) and hexadecapole (r=4) operators, defined as:

$$\left.\begin{aligned} \hat{n}_d &= \left(d^\dagger.\tilde{d}\right) \\ \hat{p} &= 1/2\left[\left(\tilde{d}.\tilde{d}\right) - (\tilde{s}.\tilde{s})\right] \\ \hat{L} &= \sqrt{10}\left[d^\dagger \times \tilde{d}\right]^1 \\ \hat{Q} &= \left[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}\right]^{(2)} + \chi\left[d^\dagger \times \tilde{d}\right]^{(2)} \\ \hat{T}_r &= \left[d^\dagger \times \tilde{d}\right]^{(r)} \end{aligned}\right] \quad (3)$$

Equation (2) defines an IBM-1 Hamiltonian in terms of the six parameters ε (the boson energy) a_0, a_1, a_2, a_3 and a_4 (the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between the bosons, respectively), and χ is the quadrupole structure parameter and take the values 0 and $\pm\frac{\sqrt{7}}{2}$ [4, 20, 21]. In that case, one says that the Hamiltonian (H) has a dynamical symmetry. These symmetries are called U(5) vibrational, SU(3) rotational and O(6) γ-unstable[17, 22].
The eigenvalues for these three limits are given by [14, 23]:

$$\left.\begin{aligned} E &= \varepsilon\, nd + \beta\, nd\,(nd + 4) + 2\gamma\, \upsilon\,(\upsilon + 3) + 2\delta L\,(L + 1) \ldots\ldots\ldots\ldots\ldots\ldots U(5) \\ E &= \frac{a_2}{2}\left(\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)\right) + \left(a_1 - \frac{3a_2}{8}\right) L(L+1) \ldots\ldots\ldots\ldots\ldots\ldots SU(3) \\ E &= a_0/4\,(N - \sigma)(N + \sigma + 4) + a_3/2\ \tau(\tau + 3) + (a_{1-}\, a_3/10)\, L(L+1) \ldots O(6) \end{aligned}\right] \quad (4)$$

A geometric shape visualization of the even-even nuclei is made by plotting the potential energy surface $E(N_b, \beta, \gamma)$ in the (β,γ) plane. The technique described by Dieperink et al. [24] allows one also to give an algebraic description of the nature of the transition between one phase and another. The $E(N_b, \beta, \gamma)$ can be obtained by calculating the expectation value of the Hamiltonian (1) in the coherent state and is given in the following forms [4,17]:

$$E(N_b, \beta, \gamma) = \varepsilon\, N_b[\beta^2/(1 + \beta^2)], \ldots U(5) \quad (5)$$

$$E(N_b, \beta, \gamma) = a_2N_b(N_b - 1)[(1 + 3/4\beta^4 - \sqrt{2}\beta^3 \cos 3\gamma)/(1 + \beta^2)^2], \ldots SU(3) \quad (6)$$

$$E(N_b, \beta, \gamma) = a_0N_b(N_b - 1)[(1 - \beta^2)/(1 + \beta^2)]^2, \ldots O(6), \quad (7)$$

where N_b is number of bosons, β, γ are deformation parameters (usually, $\beta \geq 0$, $0^\circ \leq \gamma \leq 60^\circ$) which determine the geometrical shape of the nucleus and other terms are the same as in the Hamiltonian (1). These expression give (for large N_b) $\beta_{min} = 0$, $\sqrt{2}$, and 1 for U(5), SU(3), and O(6), respectively.

3 Results and discussion

Gd-Er isotopes have neutron number N = 96 which has fourteen neutrons than the magic number N = 82 with atomic number Z from 64 to 68 (N and Z values near mid shell would suggest rotational structure SU(3)). At neutron number N=96 for ^{160}Gd, ^{162}Dy and ^{164}Er isotopes the total boson numbers N_b = 14, 15 and 14, respectively. The degree (and type) of collectivity can be expressed in terms of the energy ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$, which used as a starting point and is a good indicator of the shape deformation of the nucleus and its value is 10/3 for the well-deformed nuclei SU(3), 2.5 for O(6) or γ-unstable nuclei and 2 for vibrational U(5)[1, 25-27]. The experimental values of $R = E4_1^+/E2_1^+$ of low–lying energy levels of ^{160}Gd, ^{162}Dy and ^{164}Er nuclei are shown in Table 1. From this Table, $R_{4/2}$ attains the SU(3) value of ~ 3.33 in ^{160}Gd, ^{162}Dy and ^{164}Er nuclei.

Table 1: The ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$ for ^{160}Gd, ^{162}Dy and ^{164}Er nuclei [28-31].

Nucleus	^{160}Gd	^{162}Dy	^{164}Er
$R4/2$	3.306	3.310	3.280

The calculations have been performed using IBM with PHINT code [32] and, hence, no distinction made between neutron and proton bosons which calculated from the sum of the proton bosons of the close shells (50 and 82) and the neutron bosons of the close shells (82 and 126). The number of bosons and the parameters of the IBM-1 Hamiltonian (2) which give the best fitting between theoretical and experimental energy levels [28-31] of the above isotopes are shown in Table 2.

Table 2: Adopted values for the parameters used for IBM-1 calculations. All parameters are given in MeV, excepted N_b and CHQ for Gd-Er isotopes.

Isotopes	N_b	ELL	QQ	CHQ
^{160}Gd	14	0.0161	-0.0226	-2.958
^{162}Dy	15	0.0197	-0.0185	-2.958
^{164}Er	14	0.0232	-0.0189	-2.958

(ELL = $2a_1$ and QQ = $2a_2$, CHQ= $\sqrt{5}\chi$)[17].

Figure 1 show that the calculated yrast band and the experimental data [28-31] for even-even Gd-Er isotopes. In the Figure 1, the energy levels of low lying states increased continuously with increase Z at N is constant (e.g: $E2^+$= 0.075, 0.080 and 0.091 MeV for ^{160}Gd, ^{162}Dy and ^{164}Er isotopes, respectively) and in general, the calculated energy levels are in good agreement with the experimental ones for all isotopes.

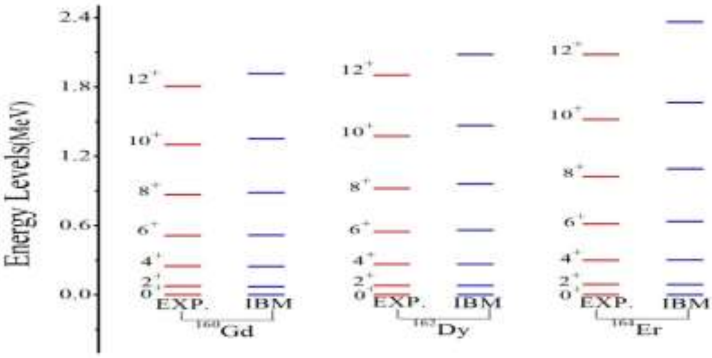

Figure 1: (Color online) comparisons between the calculated IBM-1 and the experimental data [28-31] for ^{160}Gd, ^{162}Dy and ^{164}Er with N=96.

Now, we discuss other information on the structure of nuclei is the transition strength between the excited states and can be expressed in terms of the reduced E2 matrix element which must be a Hermitian tensor of rank two when N_b must be conserved. The $B(E2)$ strength for the $E2$ transitions is given by [1, 17, 33]

$T^{E2}= \alpha_2 [d^\dagger s + s^\dagger d]^{(2)} + \beta_2[d^\dagger d]^{(2)} = e_B\hat{Q}$ (8),

where ($s^\dagger$, $d^\dagger$) and (s, d) are creation and annihilation operators for s and d bosons, respectively, while α_2 and β_2 are two parameters, and $\left(\beta_2 = \chi\alpha_2, \alpha_2 = e_B(\text{effective charge of boson})\right)$.

The reduced transition probability for the SU(3) limit is given by:[1, 16, 17, 34]

SU(3) $B(E2; L\rightarrow L-2) = {e_B}^2 \frac{3(L+2)(L+1)}{4(2L+3)(2L+5)} (2N_b - L)(2N_b + L + 3)$ (9)

where L is the angular momentum. From the given experimental value B(E2) of transition $(2_1^+ \rightarrow 0_1^+)$, one can calculate the value of the parameter $e_B = \alpha_2$for each isotope. This value is used to calculate the reduced transition probabilities$B(E2; L\rightarrow L-2)$. Table 3 shows the values of the α_2 and β_2 parameters, which were obtained in the present calculations. Table 4 shows the B(E2) in the low-lying positive parity yrast bands in Gd, Dy and Er nuclei and the ratio (R=$(E2; 4_1^+ \rightarrow 2_1^+)/(E2; 2_1^+ \rightarrow 0_1^+)$, which is equal 1.4 in SU(3) symmetry [1, 17, 35]) values for Gd-Er isotopes with neutron number N= 96. The reduced transition probabilities are increase as proton number increases and there are good agreement and strong between the B(E2) calculated with experimentally reported in Gd-Er isotopes, except ^{160}Gd isotope because there's no sufficient experimental data [28-31]. Moreover, the ratio R is almost constant with proton number increases towards the subshell between (Z = 50 and 82) *i.e.* the nucleus gets more deformed.

Table 3: Parameters (in eb) used to reproduce B(E2) values for ^{160}Gd, ^{162}Dy and ^{164}Er isotopes.

A	N_b	α_2	β_2
^{160}Gd	14	0.109	-0.323
^{162}Dy	15	0.103	-0.307
^{164}Er	14	0.115	-0.342

Table 4: The IBM-1 and Experimental [28-31] values of B(E2) (in e^2 b^2) and the ratio R $((E2;4_1^+ \rightarrow 2_1^+)/(E2;2_1^+ \rightarrow 0_1^+))$ for Gd-Er isotopes.

A	N_b		$2_1^+ \rightarrow 0_1^+$	$4_1^+ \rightarrow 2_1^+$	$6_1^+ \rightarrow 4_1^+$	$8_1^+ \rightarrow 6_1^+$	$10_1^+ \rightarrow 8_1^+$	ratio R
^{160}Gd	14	Exp.	1.037	--	--	--	--	--
		IBM-1	1.036	1.463	1.578	1.600	1.575	1.412
^{162}Dy	15	Exp.	1.071	1.517	1.668	1.727	1.716	1.416
		IBM-1	1.066	1.508	1.630	1.660	1.642	1.415
^{164}Er	14	Exp.	1.163	1.515	--	1.829	1.803	1.303
		IBM-1	1.161	1.640	1.768	1.793	1.765	1.414

In Figure 2, the contour plot of the potential energy surfaces, PES, show that Gd-Er isotopes under study are deformed and have rotational like characters SU(3).

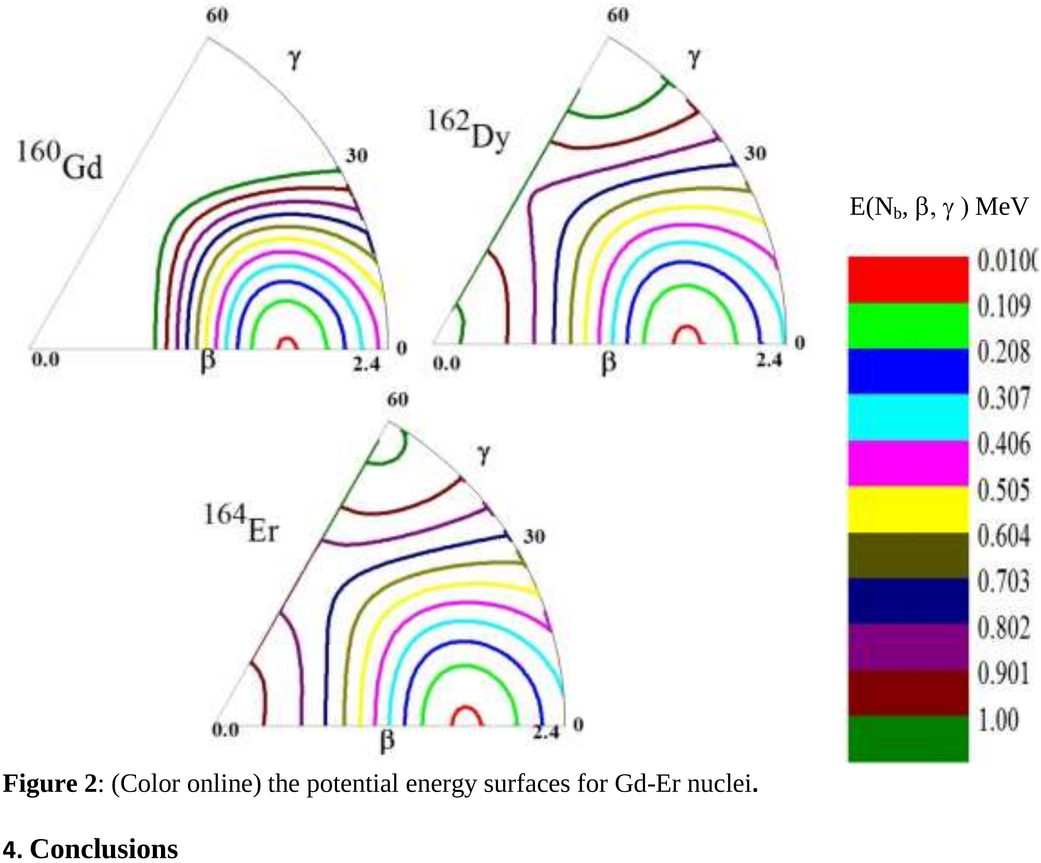

Figure 2: (Color online) the potential energy surfaces for Gd-Er nuclei**.**

4. Conclusions

The low-lying positive parity yrast bands are calculated using IBM-1 for ^{160}Gd, ^{162}Dy and ^{164}Er nuclei with neutron number N=96. The result shows good agreement with published experimental data. The reduced transition probabilities B(E2) values have been calculated using Interacting Boson Model (IBM). A good agreement is obtained for all the observed able studied. the ratio $R((E2;4_1^+ \rightarrow 2_1^+)/(E2;2_1^+ \rightarrow 0_1^+)$ for all nuclei under study and show this ratio constant with increasing Z from 64 to 66 which close to SU(3) limiting value of 1.4. The contour plot of PES show, that the ^{160}Gd, ^{162}Dy and ^{164}Er nuclei are deformed and have rotational-like characters.

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