THE EFFECT OF DEPENDENT DATA ON TYPE I ERROR RATES FOR MULTIPLE COMPARISON PROCEDURES FOR 3- WAY CROSSED BALANCED MODEL

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ABSTRACT

In this article determines the effect of dependence of observations on several multiple comparison procedures for 3- way crossed balanced model.

تأثير البيانات غير المستقلة في احتمالات الخطأ من النوع الأول لطرائق المقارنات المتعددة

لنموذج ذو ثلاثة عوامل متقاطع متوازن

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الخلاصة

في هذا البحث تم تحديد تأثير الترابط بين البيانات على عدد من طرائق المقارنات المتعددة لنموذج ذو ثلاثة عوامل متقاطع متوازن

1. INTRODUCTION

Independence of observations is considered as one of the standard assumption in the analysis of variance (ANOVA) which is seldom verified. Some researcher has been devoted to showing that slight dependence in the form of serial correlation and interclass correlation can quickly inflate the Type I error rate. Gastwirth and Rubin (1971), Smith and Lewis (1980), Al- Rabeaa (1995), and Gabbara and Al- Rabeaa (1996 a) and (1996b) have shown that in an experimental design, certain forms of dependence can quickly invalidate the result of ANOVA. How small correlation can be induced on the random component of a factorial experiment is shown in Pavur and Davenport (1985) for adjusting an ANOVA by correcting F statistic for 2- way crossed classification, in Pavur (1988) for studying the effect on several multiple comparison procedures (MCPs) for 1- way classification, in Gabbara and Al- Rabeaa (1996) for studying the effect on several MCPs for 2- way nested classification and in Al- Mouel, Al- Abdulla and Al- Rabeaa (1999) for studying the effect on several MCPs for 3- way nested classification.

The aim of this paper is to show that the model for 3- way crossed classification ANOVA design with a certain correlation pattern can be rewritten as 3- way crossed classification ANOVA design in which the error terms are independent and the design matrix is multiplied by a constant. With the model rewritten in this form, the effects of small correlations can be easily determined for several MCPs. Fisher's Least significant difference, Tukey's honestly significant difference, the student – Newman – Keuls significant difference, and Scheffe's significant difference are the multiple comparison procedures for which the effect of correlation is determined.

2. DEFINING THE MODEL

Consider the 3- way crossed model

$$Y= XB+ e \qquad \text{…(1)}$$

Where Y is an (abcd x1) vector of observations, X is the design Matrix which is equal to

$$1_d \otimes \left[1_{abc} : 1_{bc} \otimes I_a : 1_c \otimes I_b \otimes 1_a : I_c \otimes 1_{ab} : 1_c \otimes I_{ab} : I_c \otimes 1_b \otimes I_a : I_{bc} \otimes 1_a : I_{abc}\right]$$

e is a random vector of size abcd:e~N(0, σ^2 V),where V is a correlation matrix and σ^2 is the variance of each component of observations .Let,

$$B' = \begin{bmatrix} \theta, \alpha_1, ..., \alpha_a, \beta_1, ..., \beta_b, \gamma_1, ..., \gamma_c, (\alpha\beta)_{11}, ..., (\alpha\beta)_{ab}, (\alpha\gamma)_{11}, ..., (\alpha\gamma)_{ac} \\ , (\beta\gamma)_{11}, ..., (\beta\gamma)_{bc}, (\alpha\beta\gamma)_{111}, ..., (\alpha\beta\gamma)_{abc} \end{bmatrix}$$

$$= \left[\theta, \alpha', \beta', \gamma', (\alpha\beta)', (\alpha\gamma)', (\beta\gamma)', (\alpha\beta\gamma)'\right]$$

such that

$$\sum_i \alpha_i = \sum_j \beta_j = \sum_k \gamma_k = \sum_j (\alpha\beta)_{ij} = \sum_k (\alpha\gamma)_{ik} = \sum_k (\beta\gamma)_{jk} = \sum_k (\alpha\beta\gamma)_{ijk} = 0$$

then

$$\begin{aligned} \mu = X\beta = \theta 1_{abcd} + 1_{bcd} \otimes \alpha + 1_{cd} \otimes \beta \otimes 1_a + 1_d \otimes \gamma \otimes 1_{ab} + 1_{cd} \otimes (\alpha\beta) \\ + (1_d \otimes I_c \otimes 1_b \otimes I_a)(\alpha\gamma) + 1_d \otimes (\beta\gamma) \otimes 1_a + 1_d \otimes (\alpha\beta\gamma) \end{aligned} \qquad \text{…(2)}$$

where 1_s is a vector of one and $\otimes$ denotes the kronecker product of two Matrices . the kronecker product was defined by Graybill(1969).

3. TRANSFORMING THE MODEL

We have $$Y \sim N(\mu, \sigma^2 V) \qquad \text{... (3)}$$

where

$$\begin{aligned} V = (1-\rho_1) I_{abcd} + (\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) J_d \otimes \\ I_{abc} + (\rho_2 - \rho_5 - \rho_6 + \rho_8) J_{cd} \otimes I_{ab} + (\rho_3 - \rho_5 - \rho_7 + \rho_8) \\ J_d \otimes I_c \otimes J_b \otimes I_a + (\rho_5 - \rho_8) J_{bcd} \otimes I_a + (\rho_4 - \rho_6 - \rho_7 + \rho_8) \\ J_d \otimes I_{bc} \otimes J_a + (\rho_6 - \rho_8) J_{cd} \otimes I_b \otimes J_a + (\rho_7 - \rho_8) J_d \otimes I_c \otimes J_b \otimes \\ J_a + \rho_8 J_{abcd}) \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned} \qquad \text{....(4)}$$

Define $$M_s = \frac{1}{s} J_s = \frac{1}{s} 1_s 1'_S \; and \qquad N_s = I_s - M_s \qquad \text{…(5)}$$

Now it can be shown that V can be written as

$$\begin{aligned} V = \lambda_1 M_{abcd} + \lambda_2 M_{bcd} \otimes N_a + \lambda_3 M_{cd} \otimes N_b \otimes M_a + \lambda_4 M_d \otimes N_c \otimes M_{ab} + \lambda_5 M_{cd} \otimes N_{ab} \\ + \lambda_6 M_d \otimes N_c \otimes M_b \otimes N_a + \lambda_7 M_d \otimes N_{bc} \otimes M_a + \lambda_8 M_d \otimes N_{abc} + \lambda_9 N_d \otimes I_{abc} \end{aligned} \qquad \text{...(6)}$$

where

$$\lambda_1 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + cd(\rho_2 - \rho_5 - \rho_6 + \rho_8) + bd(\rho_3 - \rho_5 - \rho_7 + \rho_8) + ad(\rho_4 - \rho_6 - \rho_7 + \rho_8) + bcd(\rho_5 - \rho_8) + acd(\rho_6 - \rho_8) + abd(\rho_7 - \rho_8) + \rho_8 abcd$$

$$\lambda_2 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + cd(\rho_2 - \rho_5 - \rho_6 + \rho_8) + bd(\rho_3 - \rho_5 - \rho_7 + \rho_8) + bcd(\rho_5 - \rho_8)$$

$$\lambda_3 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + cd(\rho_2 - \rho_5 - \rho_6 + \rho_8) + ad(\rho_4 - \rho_6 - \rho_7 + \rho_8) + acd(\rho_6 - \rho_8)$$

$$\lambda_4 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + bd(\rho_3 - \rho_5 - \rho_7 + \rho_8) + ad(\rho_4 - \rho_6 - \rho_7 + \rho_8) + abd(\rho_7 - \rho_8)$$

$$\lambda_5 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + cd(\rho_2 - \rho_5 - \rho_6 + \rho_8)$$

$$\lambda_6 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + bd(\rho_3 - \rho_5 - \rho_7 + \rho_8)$$

$$\lambda_7 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8) + ad(\rho_4 - \rho_6 - \rho_7 + \rho_8)$$

$$\lambda_8 = (1-\rho_1) + d(\rho_1 - \rho_2 - \rho_3 - \rho_4 + \rho_5 + \rho_6 + \rho_7 - \rho_8)$$

$$\lambda_9 = (1-\rho_1) \qquad ...(7)$$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9$ are positive constants and represent the eigen values of the correlation matrix V repeated 1,a-1,b-1,c-1, (a-1) (b-1), (a-1) (c-1), (b-1) (c-1), (a-1) (b-1) (c-1), abc (d-1)times respectively.

Since V is a positive definite Matrix and each of the Matrices

$M_{abcd}, M_{bcd} \otimes N_a, M_{cd} \otimes N_b \otimes M_a, M_d \otimes N_c \otimes M_{ab}, M_{cd} \otimes N_{ab}, M_d \otimes N_c \otimes M_b \otimes N_a, M_d \otimes N_{bc} \otimes M_a, M_d \otimes N_{abc} and N_d \otimes I_{abc}$

are idempotent and the product of any two is zero Matrix , then there exist a unique square Matrix $V^{\frac{1}{2}}$ *such* that V=($V^{\frac{1}{2}}$)^2 , where

$$V^{-\frac{1}{2}} = \frac{1}{\sqrt{\lambda_1}} M_{abcd} + \frac{1}{\sqrt{\lambda_2}} M_{bcd} \otimes N_a + \frac{1}{\sqrt{\lambda_3}} M_{cd} \otimes N_b \otimes M_a + \frac{1}{\sqrt{\lambda_4}} M_d \otimes N_c \otimes M_{ab} + \frac{1}{\sqrt{\lambda_5}} M_{cd} \otimes N_{ab} + \frac{1}{\sqrt{\lambda_6}} M_d \otimes N_c \otimes M_b \otimes N_a + \frac{1}{\sqrt{\lambda_7}} M_d \otimes N_{bc} \otimes M_a + \frac{1}{\sqrt{\lambda_8}} M_d \otimes N_{abc} + \frac{1}{\sqrt{\lambda_9}} N_d \otimes I_{abc} \qquad \ldots(8)$$

Now

$$V^{-\frac{1}{2}}\mathrm{Y} = V^{-\frac{1}{2}}\mathrm{X}B + V^{-\frac{1}{2}}e \qquad \ldots(9)$$

$$\mathrm{Y}^{*} = V^{-\frac{1}{2}}\mathrm{X}B + e^{*}$$

such that

$$\mu^{*} = V^{-\frac{1}{2}}\mathrm{X}B = \mathrm{X}B^{*}$$

where $B^{*} = \left(\frac{1}{\sqrt{\lambda_1}}\theta, \frac{1}{\sqrt{\lambda_2}}\alpha', \frac{1}{\sqrt{\lambda_3}}\beta', \frac{1}{\sqrt{\lambda_4}}\gamma', \frac{1}{\sqrt{\lambda_5}}(\alpha\beta)', \frac{1}{\sqrt{\lambda_6}}(\alpha\gamma)', \frac{1}{\sqrt{\lambda_7}}(\beta\gamma)', \frac{1}{\sqrt{\lambda_8}}(\alpha\beta\gamma)'\right)'$ …(10)

so we can write model (1) as

$$\mathrm{Y}^{*} = \mathrm{X}B^{*} + e^{*} \qquad \ldots(11)$$

where X is the design Matrix for 3-way crossed balanced model.

$$\mathrm{Y}^{*} = V^{-\frac{1}{2}}\mathrm{Y} \quad \text{and} \quad e^{*} \sim N\left(0, \sigma^{2} I\right)$$

4. CORRECTING FOR CORRELATION

Multiple comparison procedures require the form

$$\overline{\mathrm{Y}}_{i} - \overline{\mathrm{Y}}_{i'} \pm (\mathit{factor})\sqrt{\frac{MSE}{n}}, \qquad i \neq i' \qquad \ldots(12)$$

Where the term factor depends on the procedure used and

$$MSE = \frac{SSE}{dfe} = \frac{\mathrm{Y}'(Nd \otimes I_{abc})\mathrm{Y}}{abc\,(d-1)} \qquad \ldots(13)$$

where dfe is the degrees freedom of error

It can be show that

$$MSE^{*} = \frac{SSE^{*}}{dfe} = \frac{\mathrm{Y}^{*\prime}(N_{d} \otimes I_{abc})\mathrm{Y}^{*}}{abc\,(d-1)}$$

$$= \left(\frac{1}{\lambda_{9}}\right)\frac{\mathrm{Y}'(N_{d} \otimes I_{abc})\mathrm{Y}}{abc\,(d-1)}$$

That is

$$MSE^{*} = \left(\frac{1}{\lambda_{9}}\right) MSE \qquad \textbf{…(14)}$$

Now multiple comparison procedures testing the following hypotheses

$$H_0 : \alpha_i = \alpha_{i'} \quad vs \quad H_1 : \alpha_i \neq \alpha_{i'}, \forall \quad i, i' = 1,..., a, i < i' \qquad ...(15)$$

$$H_0 : \beta_j = \beta_{j'} \quad vs \quad H_1 : \beta_j \neq \beta_{j'}, \forall \quad j, j' = 1,..., b, j < j' \qquad ...(16)$$

$$H_0 : \gamma_k = \gamma_{k'} \quad vs \quad H_1 : \gamma_k \neq \gamma_{k'}, \forall \quad k, k' = 1,..., c, k < k' \qquad ...(17)$$

$$H_0 : (\alpha\beta)_{ij} = (\alpha\beta)_{i'j'} \quad vs \quad H_1 : (\alpha\beta)_{ij} \neq (\alpha\beta)_{ij'}, \forall \quad i, i' = 1,..., a,$$

$$i < i' and \quad j, j' = 1,..., b, j < j' \qquad ...(18)$$

$$H_0 : (\alpha\gamma)_{ik} = (\alpha\gamma)_{i'k'} \quad vs \quad H_1 : (\alpha\gamma)_{ik} \neq (\alpha\gamma)_{i'k'}, \forall \quad i, i' = 1,..., a, i < i'$$

$$and \quad k, k' = 1,..., c, k < k' \qquad ...(19)$$

$$H_0 : (\beta\gamma)_{jk} = (\beta\gamma)_{j'k'} \quad vs \quad H_1 : (\beta\gamma)_{jk} \neq (\beta\gamma)_{j'k'}, \forall \quad j, j' = 1,..., b, j < j' \text{ and}$$

$$and \quad k, k' = 1,..., c, k < k' \qquad ...(20)$$

$$H_0 : (\alpha\beta\gamma)_{ijk} = (\alpha\beta\gamma)_{i'j'k'} \quad vs \quad H_1 : (\alpha\beta\gamma)_{ijk} \neq (\alpha\beta\gamma)_{i'j'k'}, \forall \; i, i' = 1,..., a, i < i', j, j'$$

$$= 1,..., b, j < j' and \; k, k' = 1,..., c, k < k' \qquad ...(21)$$

The confidence interval for the first testing hypothesis in model (1)written in the form

$$\overline{Y}_{i...} - \overline{Y}_{i'...} \pm (factor)\sqrt{\frac{MSE^*}{bcd}} \qquad ...(22)$$

Now, for the transformed model, (22) becomes

$$\overline{Y}^*_{i...} - \overline{Y}^*_{i'...} \pm (factor)\sqrt{\frac{MSE^*}{bcd}} \qquad ...(23)$$

Define $w'_{ii'} = \left(\frac{1}{bcd}\right)(0,..., 0, 1, 0,..., 0, -1, 0,..., 0) : w_{ii'} \perp 1_a$

It can be shown that

$$\overline{Y}^*_{i...} - \overline{Y}^*_{i'...} = (1'_{bcd} \otimes w'_{ii'})Y^* = \left(\frac{1}{\sqrt{\lambda_2}}\right)\left(\overline{Y}_{i...} - \overline{Y}_{i'...}\right) \qquad ...(24)$$

Then substituting (14)and (24)in (23),we get

$$\overline{Y}_{i...} - \overline{Y}_{i'...} \pm \sqrt{c_1}\ factor \sqrt{\frac{MSE}{bcd}} \qquad ...(25)$$

where the constant $c_1 = \frac{\lambda_2}{\lambda_9}$ represent the correction factor for the hypothesis(15).

Similarly we can be found constant $c_2 = \frac{\lambda_3}{\lambda_9}$ for the second testing hypothesis can be found by using the vector.

$$w'_{jj'} = \frac{1}{acd}\left(0,...,\ 0,1,0,...,\ 0,-1,0,...,\ 0\right):\ w_{jj'} \perp 1_b$$

So the confidence interval is

$$\overline{Y}_{.j..} - \overline{Y}_{.j'..} \pm \sqrt{c_2}\ factor \sqrt{\frac{MSE}{acd}} \qquad ...(26)$$

for the third test, define

$$w'_{kk'} = \left(\frac{1}{abd}\right)\left(0,...,0,1,0,...,0,-1,0,...,0\right):\ w_{kk'} \perp 1_c$$

The confidence interval is

$$\overline{Y}_{..k.} - \overline{Y}_{..k'.} \pm \sqrt{c_3}\ factor \sqrt{\frac{MSE}{abd}} \qquad ...(27)$$

where $c_3 = \frac{\lambda_4}{\lambda_9}$. The confidence interval for the fourth testing hypothesis in model (1) written in the form

$$\overline{Y}_{ij..} - \overline{Y}_{ij'..} - \overline{Y}_{i'j..} + \overline{Y}_{i'j'..} \pm \left(factor\right)\sqrt{\frac{MSE}{cd}} \qquad ...(28)$$

No for the transformed model, (28) becomes

$$\overline{Y}^*_{ij..} - \overline{Y}^*_{ij'..} - \overline{Y}^*_{i'j..} + \overline{Y}^*_{i'j'..} \pm \left(factor\right)\sqrt{\frac{MSE^*}{cd}} \qquad ...(29)$$

Define

$$w'_{jj'} = \left(\frac{1}{cd}\right)\left(0,...,0,1,0,...,0,-1,0,...,0\right):\ w_{jj'} \perp 1_b\ and$$
$$w'_{ii'} = \left(0,...,0,1,0,...,0,-1,0,...,0\right):\ w_{ii'} \perp 1_a$$

It can be shown that

$$\overline{Y}^*_{ij..} - \overline{Y}^*_{ij'..} - \overline{Y}^*_{i'j..} + \overline{Y}^*_{i'j'..} = \left(1'_{cd} \otimes w'_{jj'} \otimes w'_{ii'}\right)Y^*$$
$$= \left(\frac{1}{\sqrt{\lambda_5}}\right)\left(\overline{Y}_{ij..} - \overline{Y}_{ij'..} - \overline{Y}_{i'j..} + \overline{Y}_{i'j'..}\right) \qquad ...(30)$$

Then substituting (14) and (30) in (29), we get

$$\overline{Y}_{ij..} - \overline{Y}_{ij'..} - \overline{Y}_{i'j..} + \overline{Y}_{i'j'..} \pm \sqrt{c_4}\, factor \quad \sqrt{\frac{MSE}{cd}} \qquad ...(31)$$

Where $c_4 = \dfrac{\lambda_5}{\lambda_9}$

Similarly we can be found the constant c_5 for fifth testing hypothesis by using the vector

$$w'_{kk'} = \left(\frac{1}{bd}\right)(0,...,0,1,0,...,0,-1,0,...,0) : w_{kk'} \perp 1_c \text{ and}$$

$$w'_{ii'} = (0,...,0,1,0,...,0,-1,0,...,0) : w'_{ii'} \perp 1_a$$

Then the confidence interval is

$$\overline{Y}_{i.k.} - \overline{Y}_{i.k'.} - \overline{Y}_{i'.k.} + \overline{Y}_{i'.k'.} \pm \sqrt{c_5}\, factor \quad \sqrt{\frac{MSE}{bd}} \qquad ...(32)$$

Where $c_5 = \dfrac{\lambda_6}{\lambda_9}$

also for the sixth test, define

$$w'_{kk'} = \left(\frac{1}{ad}\right)(0,...,0,1,0,...,0,-1,0,...,0) : w_{kk'} \perp 1_c \text{ and}$$

$$w'_{jj'} = (0,...,0,1,0,...,0,-1,0,...,0) : w_{jj'} \perp 1_b$$

the confidence interval is

$$\overline{Y}_{.jk.} - \overline{Y}_{.jk'.} - \overline{Y}_{.j'k.} + \overline{Y}_{.j'k'.} \pm \sqrt{c_6}\, factor \quad \sqrt{\frac{MSE}{ad}} \qquad ...(33)$$

where $c_6 = \dfrac{\lambda_7}{\lambda_9}$

Finally the confidence interval for the seventh testing hypothesis in model (1) written in the form

$$\overline{Y}_{ijk.} - \overline{Y}_{ijk'.} - \overline{Y}_{ij'k.} + \overline{Y}_{ij'k'.} - \overline{Y}_{i'jk.} + \overline{Y}_{i'jk'.} + \overline{Y}_{i'j'k.} - \overline{Y}_{i'j'k'.} \pm$$

$$(factor) \sqrt{\frac{MSE}{d}} \qquad ...(34)$$

for the transformed model, (34) becomes

$$\overline{Y}^*_{ijk.} - \overline{Y}^*_{ijk'.} - \overline{Y}^*_{ij'k.} + \overline{Y}^*_{ij'k'.} - \overline{Y}^*_{i'jk.} + \overline{Y}^*_{i'jk'.} + \overline{Y}^*_{i'j'k.} - \overline{Y}^*_{i'j'k'.} \pm$$

$$(factor) \sqrt{\frac{MSE^*}{d}} \qquad ...(35)$$

Define

$$w'_{kk'} = \frac{1}{d}(0,...,0,1,0,...,0,-1,0,...,0) : w_{kk'} \perp 1_c$$
$$w'_{jj'} = (0,...,0,1,0,...,0,-1,0,...,0) : w_{jj'} \perp 1_b$$
$$w'_{ii'} = (0,...,0,1,0,...,0,-1,0,...,0) : w_{ii'} \perp 1_a$$

It can be shown that

$$\overline{Y}^*_{ijk.} - \overline{Y}^*_{ijk'.} - \overline{Y}^*_{ij'k.} + \overline{Y}^*_{ij'k'.} - \overline{Y}^*_{i'jk.} + \overline{Y}^*_{i'jk'.} + \overline{Y}^*_{i'j'k.} - \overline{Y}^*_{i'j'k'.}$$
$$= (1'_d \otimes w'_{kk'} \otimes w'_{jj'} \otimes w'_{ii'})Y^*$$
$$= \left(\frac{1}{\sqrt{\lambda_8}}\right)(\overline{Y}_{ijk.} - \overline{Y}_{ijk'.} - \overline{Y}_{ij'k.} + \overline{Y}_{ij'k'.} - \overline{Y}_{i'jk.} + \overline{Y}_{i'jk'.} + \overline{Y}_{i'j'k.} - \overline{Y}_{i'j'k'.}) \quad ...(36)$$

By substituting (14) and (36) in (35), we get

$$\overline{Y}_{ijk.} - \overline{Y}_{ijk'.} - \overline{Y}_{ij'k.} + \overline{Y}_{ij'k'.} - \overline{Y}_{i'jk.} + \overline{Y}_{i'jk'.} + \overline{Y}_{i'j'k.} - \overline{Y}_{i'j'k'.}$$
$$\pm \sqrt{c_7} \quad factor \quad \sqrt{\frac{MSE}{d}} \quad ...(37)$$

where $c_7 = \frac{\lambda_8}{\lambda_9}$

5. MULTIPLE COMPARISON PROCEDURES

Table (1) show the factor terms used in several MCPs for the hypothesis (15) , (16) , (17) , (18) , (19) , (20) , and (21).
Where V equal the error degrees of freedom ; a , b and c as given before ; df_1 being equal the treatment degree of freedom , r is equal to the distance between ranked means; the function q (α , r , v) represents the student range function with significance level α and α_{cp} (α_{ew}) represents comparisonwise (experimentalwise) error rate .

The performance of standard MCPs , such as LSD, HSD , SSD and SNKr have studied by many researcher , including carmar and swanson (1973) , Einot and Gabriel (1975) and Dunnett (1980) . Kirk (1982)stated that each of the standard MCPs has been recommended by one or more statisticians .

In order to compare α_{ew} and α_{cp} it is assumed that the MCPs are used even if a preliminary F test is non significant .The SNKr method is a sequential testing procedure and its error rate is neither a α_{cp} nor α_{ew} . The constant relationship between α_{ew} and α_{cp} was given by kirk (1982) as

$$1 - \alpha_{ew} \geq (1 - \alpha_{cp})^S \quad ...(38)$$

where s is the number of comparisons being made.

6. EFFECT OF SMALL CORRELATIONS

The correction constants C_h : h = 1,2,3,4,5,6,7 may be =(or > or <) one . If the correction constant equal to 1 , then no correction is needed to the MCPs, where as if the correction constant > (or <) 1 , then the standard confidence Interval will be < (or >) the confidence Interval that takes into account the effect of correlation. For 3-way interaction experiment, where a = 4, b = 3 , c = 5 , d = 2 and α = 0.05 the factor terms as given (22) , without correction , would 2.83 , 3.74 and 4.07 for the LSD , HSD and SSD procedures , respectively . The SNKr method would have factor terms varying from 3.74 for r = 4 to 2.83 for r = 2 . To correct for correlation these factor term would need to be multiplied by $\sqrt{C_h}$: h= 1,2,3,4,5,6,7 as shown in formula (25) , (26) , (27) , (31), (32) , (33) and (37). If one ignores the correlation and uses an α = 0.05 , then the true alpha levels are found as show in the tables (2),(3),(4),(5),(6),(7) and (8) respectively:

7. CONCLUSION

1. Using formula (38) , it can be seen that these error rates will affect both α_{cp} and α_{ew} . For example , if one takes α_{cp} = 0.070 for the LSD from table (3) , then by formula (38) with S = 10, we have $\alpha_{ew} \leq$ 0.52 where as under the case of independence , by formula (38) with S = 10, we would have $\alpha_{ew} \leq$ 0.40 .
2. Table (2) ,(3) , (4) , (5) and (8) show that the true alpha inflate (deflate) when the correction constant >(or <)1 .
3. Table (6) and (7) show that the true alpha of the LSD Method deflate when the correction constant >(or <)1.
4. Small correlation can be amplified by the number of treatments d and these correlation can thus easily inflate the type I error rate . For example $\rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = \rho_7 = \rho_8 = 0,$ *then* $C_1 = 1 + d\,\frac{\rho_1}{1-\rho_1}$ and if a small correlation value is assigned to ρ_1 (say 0.02) , then the researcher may assume that the effect of this small correlation can be ignored , however , the value of C_1 for d = 7 is 1.13 .

Table (1) : Factor terms in MCPs

MCPs	Factor						
	$H_0:\alpha_i=\alpha_{i'}$	$H_0:B_J=B_J'$	$H_0:\gamma_k=\gamma_{k'}$	$H_0:(\alpha\beta)_{ij}=(\alpha\beta)_{i'j'}$	$H_0:(\alpha\gamma)_{ik}=(\alpha\gamma)_{i'k'}$	$H_0:(\beta\gamma)_{jk}=(\beta\gamma)_{j'k'}$	$H_0:(\alpha\beta\gamma)_{ijk}=(\alpha\beta\gamma)_{i'j'k'}$
LSD	$\sqrt{2}t(\alpha_{cp},V)$	$\sqrt{2}t(\alpha_{cp},V)$	$\sqrt{2}t(\alpha_{cp},V)$	$2t(\alpha_{cp},V)$	$2t(\alpha_{cp},V)$	$2t(\alpha_{cp},V)$	$2\sqrt{2}t(\alpha_{cp},V)$
HSD	$q(\alpha_{ew},a,V)$	$q(\alpha_{ew},b,V)$	$q(\alpha_{ew},c,V)$	$q(\alpha_{ew},ab,V)$	$q(\alpha_{ew},ac,V)$	$q(\alpha_{ew},bc,V)$	$q(\alpha_{ew},abc,V)$
SSD	$\sqrt{2df_1F(\alpha_{ew},df_1,V)}$	$\sqrt{2df_1F(\alpha_{ew},df_1,V)}$	$\sqrt{2df_1F(\alpha_{ew},df_1,V)}$	$2\sqrt{df_1F(\alpha_{ew},df_1,V)}$	$2\sqrt{df_1F(\alpha_{ew},df_1,V)}$	$2\sqrt{df_1F(\alpha_{ew},df_1,V)}$	$2\sqrt{2df_1F(\alpha_{ew},df_1,V)}$
SNKr	$q(\alpha,r,v)$ $r=2,\ldots\ldots,a$	$q(\alpha,r,v)$ $r=2,\ldots\ldots,b$	$q(\alpha,r,v)$ $r=2,\ldots\ldots,c$	$q(\alpha,r,v)$ $r=4,\ldots\ldots,ab$	$q(\alpha,r,v)$ $r=4,\ldots\ldots,ac$	$q(\alpha,r,v)$ $r=4,\ldots\ldots,bc$	$q(\alpha,r,v)$ $r=8,\ldots\ldots,abc$

Table(2):True alpha level of LSD,HSD,SSD and SNKr

C_2	r	LSD	HSD	SSD	SNKr
1.72	3	0.070	0.088	0.132	0.088
	2	0.070	0.088	0.132	0.079
0.75	3	0.015	0.026	0.039	0.026
	2	0.015	0.026	0.039	0.031

Note: α =0.05,a=4,b=3,c=5,d=2,v=60

C_1	r	LSD	HSD	SSD	SNKr
1.9	4	0.079	0.098	0.115	0.098
	3	0.079	0.098	0.115	0.092
	2	0.079	0.098	0.115	0.083
0.9	4	0.026	0.041	0.044	0.041
	3	0.026	0.041	0.044	0.042
	2	0.026	0.041	0.044	0.044

Table (3): True alpha level of LSD,HSD,SSD and SNKr

Note: α =0.05,a=4,b=3,c=5,d=2,v=60

Table(4):True alpha level of LSD,HSD,SSD and SNKr

C_3	r	LSD	HSD	SSD	SNKr
1.8	5	0.074	0.097	0.116	0.098
	4	0.074	0.097	0.116	0.094
	3	0.074	0.097	0.116	0.09
	2	0.074	0.097	0.116	0.081
0.7	5	0.017	0.013	0.032	0.013
	4	0.017	0.013	0.032	0.016
	3	0.017	0.013	0.032	0.020
	2	0.017	0.013	0.032	0.026

Note: α =0.05,a=4,b=3,c=5,d=2,v=60

Table (5): True alpha level of LSD,HSD,SSD and SNKr

C_4	r	LSD	HSD	SSD	SNKr
1.95	6	0.08	0.099	0.106	0.099
	5	0.08	0.099	0.106	0.097
	4	0.08	0.099	0.106	0.094
0.68	6	0.017	0.013	0.037	0.013
	5	0.017	0.013	0.037	0.015
	4	0.017	0.013	0.037	0.017

Note: α =0.05,a=3,b=2,c=2,d=3,v=24

Table(6):True alpha level of LSD,HSD,SSD and SNKr

C_5	r	LSD	HSD	SSD	SNKr
1.38	6	0.047	0.076	0.079	0.076
	5	0.047	0.076	0.079	0.075
	4	0.047	0.076	0.079	0.073
0.86	6	0.024	0.036	0.045	0.036
	5	0.024	0.036	0.045	0.037
	4	0.024	0.036	0.045	0.038

Note: α =0.05,a=3,b=2,c=2,d=3,v=24

Table (7): True alpha level of LSD,HSD,SSD and SNKr

C_6	r	LSD	HSD	SSD	SNKr
1.185	4	0.037	0.063	0.063	0.063
0.97	4	0.030	0.048	0.049	0.048

Note: α =0.05,a=3,b=2,c=2,d=3,v=24

Table(8):True alpha level of LSD,HSD,SSD and SNKr

C_7	r	LSD	HSD	SSD	SNKr
1.75	9	0.064	0.082	0.102	0.082
	8	0.064	0.082	0.102	0.081
0.75	9	0.005	0.030	0.022	0.030
	8	0.005	0.030	0.022	0.0304

Note: α =0.05,a=3,b=3c=3,d=2,v=9

REFERENCES

AL-Mouel, A. S. , AL-Abdulla , K. J. and AL-Rabeaa , Z. A. (1999) " Type I Error Rates for Multiple Comparison Procedures with Dependent Data for 3-Way Nested ANOVA Model " . J.Basrah Researches , VOL.20, part 1, 39-48.

AL- Rabeaa , Z. A. (1995) , " The Effect of Dependent Data on Type I Error Rates for Multiple Comparison Procedures and Correction Procedures for Tests " . M. Sc. Thesis , University of Basrah , Basrah , Iraq .

Arnold, S. F. (1981) , " The Theory of Linear Models and Multivariate Analysis " . Wiley , New york .

Carmer , S. G. and Swanson , M. R. (1973) , " An Evaluation of Ten Pairwise Comparison Procedures by Montocarlo Methods " , JASA , 68 , 66-74.

Dunnett, C. W. (1980), " Pairwise Multiple Comparisons in the Homogeneous Variance , Unequal Sampie Size Case", JASA , 75 , 789-795 .

Einot, I. and Gabriel , K. R. (1975) , " A Study of the Powers of Several Methods of Multiple Comparisons " , JASA, 70, 574 – 583

Gabbara,S.D. and AL-Rabeaa,Z.A.(1996a),"The Effect of Dependent Data on Type on I Error Rates for Multiple Comparison Procedures".To appear in the Journal of Iraqi Statistical Association, Proceding of the 8the Conference.

Gabbara,S.D. and AL-Rabeaa,Z.A.(1996b),"The Effect of Dependent Data in ANOVA and Correction Procedures for F Test".To appear in the Basrah Journal of Science.

Gastwirth , J. L. And Rubin , H. (1971) , " Effect of Dependence of the Level of Some One – Sample Tests" , JASA , 66, 816 – 820.

Kirk, R. E. (1982) , " Experimental Design : Procedures for the Behavioral Sciences (2 nd ed.) , Belmont, C. A : Brookslcole.

Pavur, R. J. (1988) , " Type I Error Rates for Multiple Comparison Procedures with Dependent Data " , the American Statistician , VOL. 42 , 3 , 171 – 173 .

Pavur, R. J. And Davenport , J. M. (1985) , " The (Large) Effect of (Small) Correlations in ANOVA and Correction Procedures " , American Journal of Mathematical and Management Sciences , VOL. 5, 1 and 2, 77-92.

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