(k, n; f) – Arcs of Type (1, n) in PG(2, q), with

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(k, n; f) – Arcs of Type (1, n) in PG(2, q), with $q \leq 8$

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Abstract

In this paper we discussed the existence of (k, n; f) – arcs of type (1, n) in the projective plane of order $q \leq 8$; with Im(f) = { 0, 1, ω } and $\omega \in \{2, 3, \dots, n-1\}$, that different from G. Raguso and L. Rella (k, n; f) – arcs [12] and we deduced that there are no such (k, n; f) – arcs of type (1, n) with Im(f) = { 0, 1, ω } in PG(2, q) for all $\omega \in \{2, 3, \dots, n-1\}$, $q \leq 7$ and for all n satisfy $(n-1)|q$. But when $q = 8$, we have only (19, 9; f) – arc of type (1, 9), with Im(f) = { 0, 1, 4 }, such that the points of weight 4 form an oval and the points of weight 1 are the points of some 0 – secant of this oval.

Keywords: (k, n; f) – arcs, weighted arcs, PG(2, q), weighting line.

Introduction

In 1978, A. Barlotti [2] presented the notion of a $(k, n; \{w_i\}) -$ set of kind s. The $(k, n; \{w_i\}) -$ set of kind 2 in a projective plane, also called $(k, n; \{w_i\}) -$ arcs, where studied by M. Barnabei [3]. The (k, n; f) – arcs of type $(n-2,\ n)$ in a finite projective plane was developed by E. D'Agostini [5] in 1979. B. J. Wilson [13] gives studying to the (k, n; f) – arcs of type $(n-3,\ n)$ in a finite projective plane and continue in this study the authors F. K. Hameed [7] and M. Y. Abass [1]. Also the (k, n; f) – arcs of type $(n-5,\ n)$ was developed by R. D. Mahmood [11] and completed by F. K. Hameed and others [9]. The notion of (k, n; f) – arcs of type $(1,\ n)$ introduced by G. Raguso and L. Rella [12]. Also, F. K. Hameed [8] generalize the results of the monoidal arcs in PG(2, q).

In this paper we investigated the (k, n; f) – arcs of type $(1, n)$ and its properties in the finite projective plane of order $q \leq 8$.

1. Preliminaries

We will denote by $\mathrm{PG}(2, \mathrm{q})$ the projective desarguesian plane of order $q = p^h$, by $\mathcal{P}$ the set of all points of the plane and by $\mathcal{R}$ the set of all lines of the plane. Then $\mathrm{PG}(2, \mathrm{q})$ have $q^2 + q + 1$ points and $q^2 + q + 1$ lines. On each line lie $q + 1$ points and through every point there pass $q + 1$ lines.

Definition 1.1.[10]
(k, n) – arc in PG(2, q) is a set of k points no $n + 1$ of which are collinear, where $n \geq 2$. Write simply k – arc for (k, 2) – arc.

Definition 1.2. [10]
A line ℓ in PG(2, q) is an i – secant of a (k, n) – arc $\mathcal{K}$ if $|\ell \cap \mathcal{K}| = i$. Let τ_i denote the total number of i – secants to $\mathcal{K}$ in PG(2, q), then the type of $\mathcal{K}$ is defined by $(\tau_0, \tau_1, \ldots, \tau_n)$.

Lemma1.1.[10]
For a $(k,n)-$ arc $\mathcal{K}$, the following equations hold :
(1) $\sum_{i=0}^{n}\tau_i = \mathrm{q}^2+\mathrm{q}+1$; (2) $\sum_{i=1}^{n} i\tau_i = k(q+1)$;
(3) $\sum_{i=2}^{n}\frac{i(i-1)}{2}\tau_i = \frac{k(k-1)}{2}$.

For any function f from $\mathcal{P}$ to the set of the natural numbers N we will say that $f(P)$ is the weight of the point P. From such f we may define a function F from $\mathcal{R}$ to N in the following way:

$$F(r) = \sum_{P\in r} f(P)$$

and we will say that $F(r)$ is the weight of the line r. Moreover, if $F(r) = j$ we will also say that r is a " – weighting " line.

Definition 1.3.[6]
A $(k,n;f\,)$– arc K in $PG(2,q)$ is a function $f:\mathcal{P}\longrightarrow N$ such that $k =$ |support of f (the points of non – zero weight) | and $n = \max F$.
Let us remark that an ordinary $(k,n)-$ arc is a $(k,n;f\,)$– arc with $\mathrm{Im}(f\,) = \{0,1\}$.

Let us use the following notation:
$\omega = \max f$, $W = \sum_{P\in\mathcal{P}} f(P)$ and W will be called the weight of K.
$L_i = f^{-1}(i)$ and $l_i = |L_i|$, $i = 0,1,\dots,\omega$.
[M] indicates the set of all lines through the point M .

Definition 1.5.[8]
A $(k,n;f\,)$– arc K is called monoidal if $\mathrm{Im} f = \{ 0, 1, \omega \}$ and $l_\omega = 1$.

Definition 1.6.[6]
The characters of a $(k,n;f\,)$– arc K are the integers $t_j = |F^{-1}(\,j\,)\,|\; j = 0,1,\dots,n$.

Definition 1.7.[6]
The type of a $(k,n;f\,)$– arc K is the set of Im F . To write explicitly the type of K we can use the sequence $(n_1,\dots,n_\rho)$ where $n_\lambda \in Im\,F$, $\lambda = 1,\dots,\rho$ and $n_1 < n_2 < \cdots < n_\rho = n$.

It is well known from [6] that:

$$k = \sum_{i=1}^{\omega} l_i \,. \tag{1.1}$$

$W = \sum_{i=1}^{\omega} i\, l_i$. (1.2)
$\sum_{r \in [M]} F(r) = W + q\, f(M)$. (1.3)
$|\mathrm{Im}\, F| \geq 2$. (1.4)

A useful result, mentioned in [6], is the following:
If there exists a point P of a $(k, n; f)$– arc K such that every line through it is a n – weighting line, then
$P \in L_{\omega}$ (1.5)
If $M \in L_{\omega}$ and $u \in [M]$, then $F(u) = n$.
Hence $W \leq (n - \omega)q + n.$ (1.6)
An arc with weight such that the equality holds, is called maximal. Of course, a maximal arc is also such that through a point of maximal weight there pass only n – weighting lines.

Finally we shall recall [6] the following relations concerning the characters of a $(k, n; f)$– arc K:
$\sum_{j=0}^{n} t_j = q^2 + q + 1$ (1.7)

$\sum_{j=1}^{n} j\, t_j = (q + 1)W$ (1.8)
$\sum_{j=2}^{n} \binom{j}{2} t_j = \binom{W}{2} + q \sum_{i=2}^{\omega} \binom{i}{2} l_i$. (1.9)

2. (k, n; f) – arcs of type (m, n)

From now on, K shall denote a $(k, n; f)$– arc of type (m, n), where $|Im\ f| \geq 3$. Let firstly state the following:

Lemma 2.1.[7] The weight W of a $(k, n; f) - arc$ of type (m, n) satisfies:

$$m(q + 1) \leq W \leq (n - \omega)q + n .$$

We call arcs for which the values in lemma (2.1) are attained, maximal and minimal $(k, n; f) -$ arcs of type (m, n) respectively.

Theorem 2.1. [7]
Let K be a $(k, n; f) -$ arc of type (m, n) , $m > 0$ and let v_m^s and v_n^s respectively the number of lines of weight m and the number of lines of weight n passing through a point of weight s . Then

$$(n - m)v_m^s = (n - s)(q + 1) - (W - s) ;$$
$$(n - m)v_n^s = (W - s) - (m - s)(q + 1) .$$

Theorem 2.2.[7]
A necessary conditions for the existence of a $(k,n;f)-$arc K of type $(m,n)\,, m>0$ are that:
(1) $q\equiv 0\ mod(n-m)$; (2) $\omega\leq n-m$; (3) $m\leq n-2$.

3. (k, n; *f*) – arcs of type (1, n)

In this section we discussed (k, n; *f*) – arcs of type $(1,n)$. Then we get the following properties as in [12]:
(a) Im$f\subseteq$ { 0, 1, ω };
(b) The weight W of a (k, n; *f*) – arc of type (1, n) satisfies:

$$q+1\leq W\leq (n-\omega)(q+1)+\omega=(n-\omega)q+n\ .$$

(c) From theorem (2.1) we have:

$$v_1^s=\frac{q(n-s)-W+n}{n-1}$$
$$v_n^s=\frac{q(s-1)+W-1}{n-1}$$

(d) From theorem (2.2) we have: $q\equiv 0\ \text{mod}\ (n-1)$ and $\omega\leq n-1$.
(e) From the equations (1.7) and (1.8), the characters of a (k, n; *f*) – arc of type $(1,n)$ are given by:

$$t_1=\frac{q+1}{n-1}\left(n\ \frac{q^2+q+1}{q+1}-W\right)$$
$$t_n=\frac{q+1}{n-1}\left(W-\frac{q^2+q+1}{q+1}\right)$$

(f) From the equation (1.9), the weight W of the plane must be a root of the following equation of degree two:

$$W^2-W[n(q+1)+1]+n(q^2+q+1)+q\omega(\omega-1)l_\omega=0\ . \qquad (3.1)$$

Proposition 3.1.[12]
A necessary condition for the existence in a projective plane π of order q of a (k, n; *f*) – arc of type $(1,n)$ with $\omega\geq 2$ is that the total weight of the plane is $W=(n-\omega)q+n$.

Also from [12], we have the following:

$$\left.\begin{array}{c} v_1^0 = \frac{q\omega}{n-1} \quad , \qquad v_n^0 = q - \frac{q\omega}{n-1} + 1 \\ v_1^1 = \frac{q(\omega-1)}{n-1} \quad , \quad v_n^1 = \frac{q(n-\omega)}{n-1} + 1 \ , \quad v_n^\omega = q+1 \\ t_1 = \frac{q}{n-1}(q\omega + \omega - n) \ , \ t_n = \frac{q}{n-1}[(n-\omega-1)(q+1)+n] + 1 \end{array}\right\} \quad (3.2)$$

$$\left.\begin{array}{c} l_\omega = \frac{q(n\omega - n - \omega^2) + \omega(n-1)}{\omega(\omega-1)} \\ \text{implies that} \\ l_1 = (n-\omega)q + n - \omega l_\omega = \frac{q\omega - n + 1}{\omega - 1} + 1 \\ \text{Also} \\ l_0 = q^2 + q + 1 - l_1 - l_\omega = q^2 + q - \frac{nq}{\omega} \end{array}\right\} \quad (3.3)$$

From (3.3) it follows that:
$\omega | nq$, $\quad (\omega - 1) | (\omega q - n + 1)$ and so $(\omega - 1) | (q - n + 1)$.

4. (k, n; f) – arcs of type (1, n) in PG(2, q), with $q = 2, 3, 4, 5, 7$ and 8

In this section we test if there exist a (k, n; f) – arc of type (1, n) in PG(2, q), when $q = 2, 3, 4, 5, 7$ and 8, that distinct from the arcs of G. Raguso and L. Rella [11]. Then we partition this section into the following cases:

4.1. (k, n; f) – arcs of type (1, n) in PG(2, 2)

From section (3), part (d) we have $q \equiv 0 \mod (n-1)$ and $\omega \leq n - 1$. Then $n = 3$ and $\omega = 2$. But this case leads to the arcs of G. Raguso and L. Rella as in the following theorem:

Theorem 4.1.1. [12]

The (k, n; f) – arcs of PG(2, q), of type (1, n) with $\omega = n - 1$, $n \geq 3$ are precisely the monoidal (k, n; f) – arcs having as points of weight 1 those of a line.

4.2. (k, n; f) – arcs of type (1, n) in PG(2, 3)

In this case we have $n = 4$ and $\omega \leq 3$. Since $\omega > 1$, then we can take $\omega = 2$ and for this value we have the following theorem:

Theorem 4.2.1.[12]
In a projective plane π of odd order q the (k, n; f) – arcs of type (1, n) with $n = q + 1$ and $\omega = 2$ are precisely those which have as points of weight 1 the points of an oval C and as points of weight ω the interior points of C.

Also we can take $\omega = 3$ and also we obtain the G. Raguso and L. Rella arc as in the Theorem (4.1.1).

4.3. (k, n; f) – arcs of type (1, n) in PG(2, 4)
In this case we have $n = 3$ and $n = 5$, but $n = 3$ cannot be occur in PG(2, 4) according to the following theorem:

Theorem 4.3.1.[12]
There do not exist (k, 3; f) – arcs of type (1, 3) in PG(2, q), $q \neq 2$.

Then remains $n = 5$ and this implies that $\omega \leq 4$.

$\Rightarrow \omega = 2 \Rightarrow \text{Im}(f) = \{0,1,2\}$; this case discussed in [12] as in the following theorem:

Theorem 4.3.2.[12]
In a projective plane π of even order q ($\neq$ 2) the (k, n; f) – arcs of type (1, n) with $n = q + 1$ and $\omega = 2$ are precisely those which have as points of weight 1 the points of a line r and as points of weight ω the points of $\left(\frac{q(q-1)}{2}, \frac{q}{2}\right)$ –arc of type $\left(0, \frac{q}{2}\right)$.

$\Rightarrow \omega = 3 \Rightarrow \text{Im}(f) = \{0,1,3\}$; this case impossible because $\omega \nmid nq$, i.e. $3 \nmid 5 * 4$.

$\Rightarrow \omega = 4 \Rightarrow \text{Im}(f) = \{0,1,4\}$; this case discussed in [12] as in the theorem (4.1.1).

4.4. (k, n; f) – arcs of type (1, n) in PG(2, 5)
In this case we have $n = 6$ and $\omega \leq 5$.

$\Rightarrow \omega = 2 \Rightarrow \text{Im}(f) = \{0,1,2\}$; this case discussed in [12] as in theorem (4.2.1).

$\Rightarrow \omega = 3 \Rightarrow \text{Im}(f) = \{0,1,3\}$; in this case we have the following:

The equations in (3.2), become:

$$\left.\begin{array}{l} v_1^0 = 3 \;,\quad v_6^0 = 3 \\ v_1^1 = 2 \;,\quad v_6^1 = 4 \;,\quad v_6^3 = 6 \\ t_1 = 12 \;,\; t_6 = 19 \end{array}\right\} \qquad (4.4.1)$$

Also the equations in (3.3) become:

$$l_3 = 5 \Rightarrow l_1 = 6 \Rightarrow l_0 = 20 \qquad (4.4.2)$$

Since $n = 6$ then there is no more than two points of weight 3 on a line. Then the points of weight 3 form 5 – arc in PG(2, 5). This 5 – arc must have $\tau_1 + \tau_2 \leq t_6$, but $\tau_1 = 10$, $\tau_2 = 10 \Rightarrow \tau_1 + \tau_2 > t_6 = 19$ and this contradiction. Then we have the following theorem:

Theorem 4.4.1. There is no (11, 6; f) – arc of type (1, 6) in PG(2, 5), when $\mathrm{Im}(f) = \{0, 1, 3\}$.

$\Rightarrow \omega = 4 \Rightarrow \mathrm{Im}(f) = \{0,1,4\}$; this case impossible because $\omega \nmid nq$, i.e. $4 \nmid 6 * 5$.

$\Rightarrow \omega = 5 \Rightarrow \mathrm{Im}(f) = \{0,1,5\}$; this case discussed in [12] as in the theorem (4.1.1).

4.5. (k, n; *f*) – arcs of type (1, n) in PG(2, 7)

In this case we have $n = 8$ and $\omega \leq 7$.

$\Rightarrow \omega = 2 \Rightarrow \mathrm{Im}(f) = \{0,1,2\}$; this case discussed in [12] as in theorem (4.2.1).

$\Rightarrow \omega = 3 \Rightarrow \mathrm{Im}(f) = \{0,1,3\}$; this case impossible because $\omega \nmid nq$, i.e. $3 \nmid 8 * 7$.

$\Rightarrow \omega = 4 \Rightarrow \mathrm{Im}(f) = \{0, 1, 4\}$; in this case we have the following:

The equations in (3.2), become:

$$\left.\begin{array}{lll} v_1^0 = 4 \ , & v_8^0 = 4 & \\ v_1^1 = 3 \ , & v_8^1 = 5 \ , & v_8^4 = 8 \\ t_1 = 24 \ , & t_8 = 33 & \end{array}\right\} \qquad (4.5.1)$$

Also the equations in (3.3) become:

$$l_4 = 7 \Rightarrow l_1 = 8 \Rightarrow l_0 = 42 \qquad (4.5.2)$$

Since $n = 8$ then there is no more than two points of weight 4 on a line. Then the points of weight 4 form 7 – arc in PG(2, 7). This 7 – arc must have $\tau_1 + \tau_2 \leq t_8$, because 1 – secants and 2 – secants of the points of weight 4 are 8 – weighting lines but $\tau_1 = 14$, $\tau_2 = 21 \Rightarrow \tau_1 + \tau_2 > t_8 = 33$ and this contradiction. Then we have the following theorem:

Theorem 4.5.1. There is no (15, 8; f) – arc of type (1, 8) in PG(2, 7), when $\mathrm{Im}(f) = \{0, 1, 4\}$.

$\Rightarrow \omega = 5 \Rightarrow \mathrm{Im}(f) = \{0,1,5\}$; this case impossible because $\omega \nmid nq$, i.e. $5 \nmid 8 * 7$.

$\Rightarrow \omega = 6 \Rightarrow \mathrm{Im}(f) = \{0,1,6\}$; this case impossible because $\omega \nmid nq$, i.e. $6 \nmid 8 * 7$.

$\Rightarrow \omega = 7 \Rightarrow \mathrm{Im}(f) = \{0,1,7\}$; this case discussed in [11] as in the theorem (4.1.1).

4.6. (k, n; *f*) – arcs of type (1, n) in PG(2, 8)

In this case we have $n = 5$ and $n = 9$. For $n = 5$ we have $\omega \leq 4$, then

$\Rightarrow \omega = 2 \Rightarrow \mathrm{Im}(f) = \{0,\ 1,\ 2\}$; in this case we have the following:

The equations in (3.2), become:

$$\left.\begin{array}{lll} v_1^0 = 4\ , & v_5^0 = 5 & \\ v_1^1 = 2\ , & v_5^1 = 7\ , & v_5^2 = 9 \\ t_1 = 26\ , & t_5 = 47 & \end{array}\right\} \qquad (4.6.1)$$

Also the equations in (3.3) become:

$$l_2 = 8 \Rightarrow l_1 = 13 \Rightarrow l_0 = 52 \qquad (4.6.2)$$

Since $n = 5$ then there is no more than two points of weight 2 on a line. Then the points of weight 2 form 8 – arc in PG(2, 8). Every 1 – secant and 2 – secant of 8 – arc (the points of weight 2) are 5 – weighting lines. Let $S = \{P_1\ ,\ P_2\ , \dots\ ,\ P_8\}$ be the points of weight 2 and let O be an oval in PG(2, 8), then the points of $O \backslash S = \{P_9\ ,\ P_{10}\}$ are not of weight 2, then $v_5^{f(P_9)}$ and $v_5^{f(P_{10})}$ are 5 or 7 as in equation (4.6.1), but through them there pass exactly eight 1 – secants of S that is $v_5^{f(P_9)} = v_5^{f(P_{10})} = 8$ and this is contradiction. Then we have the following theorem:

Theorem 4.6.1. There is no (21, 5; *f*) – arc of type (1, 5) in PG(2, 8), when $\mathrm{Im}(f) = \{0,\ 1,\ 2\}$.

$\Rightarrow \omega = 3 \Rightarrow \mathrm{Im}(f) = \{0,1,3\}$; this case impossible because $\omega \nmid nq$, i.e. $3 \nmid 5 * 8$.

$\Rightarrow \omega = 4 \Rightarrow \mathrm{Im}(f) = \{0,1,4\}$; this case impossible because $(\omega - 1) \nmid (q - n + 1)$ i.e. $3 \nmid (8 - 5 + 1 = 4)$.

For $n = 9$ we have $\omega \leq 8$, then

$\Rightarrow \omega = 2 \Rightarrow \mathrm{Im}(f) = \{0,1,2\}$; this case discussed in [11] as in the theorem (4.3.2).

$\Rightarrow \omega = 3 \Rightarrow \mathrm{Im}(f) = \{0,\ 1,\ 3\}$; in this case we have the following:

The equations in (3.2), become:

$$\left.\begin{array}{lll} v_1^0 = 3\ , & v_9^0 = 6 & \\ v_1^1 = 2\ , & v_9^1 = 7\ , & v_9^3 = 9 \\ t_1 = 18\ , & t_9 = 55 & \end{array}\right\} \qquad (4.6.3)$$

Also the equations in (3.3) become:

$$l_3 = 16 \Rightarrow l_1 = 9 \ \Rightarrow \ l_0 = 48 \qquad (4.6.4)$$

Since $n = 9$ then there is no more than three points of weight 3 on a line. Then the points of weight 3 form (16, 3) – arc in PG(2, 8). This is contradiction because the maximum size of (u, 3) – arc in PG(2, 8) is $u = 15$, as in [4].

Theorem 4.6.2. There is no (25, 9; f) – arc of type (1, 9) in PG(2, 8), when $\mathrm{Im}(f) = \{0,\ 1,\ 3\}$.

$\Rightarrow \omega = 4 \ \Rightarrow \ \mathrm{Im}(f) = \{0,\ 1,\ 4\}$; in this case we have the following:
The equations in (3.2), become:

$$\left.\begin{array}{lll} v_1^0 = 4\ , & v_9^0 = 5 & \\ v_1^1 = 3\ , & v_9^1 = 6\ , & v_9^4 = 9 \\ t_1 = 27\ , & t_9 = 46 & \end{array}\right\} \qquad (4.6.5)$$

Also the equations in (3.3) become:

$$l_4 = 10 \Rightarrow l_1 = 9 \ \Rightarrow \ l_0 = 54 \qquad (4.6.6)$$

Since $n = 9$ then there is no more than two points of weight 4 on a line. Then the points of weight 4 form 10 – arc (oval) in PG(2, 8). The oval in PG(2, 8) has no 1 – secant. It has only 2 – secants and 0 – secants and every 2 – secant is a 9 – weighting line, then we have 45 9 – weighting lines having two points of weight 4 but from (4.6.5) we have 46 9 – weighting lines ($t_9 = 46$), then we have a 9 – weighting line on which there is no points of weight 4, then the points of weight 1 are collinear on this line which is a 0 – secant of the oval.

Theorem 4.6.3. There is (19, 9; f) – arc of type (1, 9) in PG(2, 8), when $\mathrm{Im}(f) = \{0,\ 1,\ 4\}$, in which the points of weight 4 form an oval and the points of weight 1 are the points of some 0 – secant of this oval.

$\Rightarrow \omega = 5 \ \Rightarrow \ \mathrm{Im}(f) = \{0,1,5\}$; this case impossible because $\omega \nmid nq$, i.e. $5 \nmid 9 * 8$.

$\Rightarrow \omega = 6 \Rightarrow \mathrm{Im}(f) = \{0,1,6\}$; in this case we have the following:
The equations in (3.2), become:

$$\left.\begin{array}{l} v_1^0 = 6 \;, \quad v_9^0 = 3 \\ v_1^1 = 5 \;, \quad v_9^1 = 4 \;, \quad v_9^6 = 9 \\ t_1 = 45 \;, \; t_9 = 28 \end{array}\right\} \qquad (4.6.7)$$

Also the equations in (3.3) become:

$$l_6 = 4 \Rightarrow l_1 = 9 \Rightarrow l_0 = 60 \qquad (4.6.8)$$

Since $n = 9$ then there is no two points of weight 6 on a line. This is contradiction with the axiom of the projective geometry.

Theorem 4.6.4. There is no (13, 9; *f*) – arc of type (1, 9) in PG(2, 8), when $\mathrm{Im}(f) = \{0,\ 1,\ 6\}$.

$\Rightarrow \omega = 7 \Rightarrow \mathrm{Im}(f) = \{0,1,7\}$; this case impossible because $\omega \nmid nq$, i.e. $7 \nmid 9 * 8$.
$\Rightarrow \omega = 8 \Rightarrow \mathrm{Im}(f) = \{0,1,8\}$; this case discussed in [12] as in the theorem (4.1.1).

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