

Inherited Symmetries and Conserved Form

Inaam A.Malloki

Qusay S.Al-Zamel

ABSTRACT

In this paper, we present a theorem in which the point symmetries of auxiliary system $S\{x_1, x_2, u, v\}$ corresponding to the conserved form of the PDE, $R\{x_1, x_2, u\}$ is inherited to an induced system of ODEs, $S_x\{z, U, V\}$. The proof of our theorem can be considered as an algorithm to derive the inherited symmetries of the system of ODEs. Moreover, we apply our theorem to the nonlinear diffusion equation with convection, and obtain an exact solution to the equation.

1-INTRODUCTION.

The most famous and established method for finding exact solutions of differential equations is Classical Symmetries method (CSM), also called group analysis which originated in 1881 from the pioneering work of Sophus Lie [1],[9],[11]. Many good books have been dedicated to this subject and its generalizations [1],[2],[5],[6],[12].

In the last few years we have observed a significant progress in the application of symmetries to the study of the nonlinear PDEs of physical importance, as well as in looking for exact solutions for such equation.

The symmetries (infinitesimal generators) admitted by nonlinear PDEs are useful for finding similarity solutions, as well as to discover whether or not the equations can be linearized by an inevitable mapping and to construct an explicit linearization when one exists [7]. Many DEs do not have any symmetries (infinitesimal generators). Gandarias et al. in 2002 [7] are interested in ODEs which do not admit any symmetry group. They gave an example of a second order ODE that does not admit any symmetry group.

When constructing similarity solutions for second order PDE, Dresner in 1980 observed that the second order ODE is invariant to a stretching group G' related to the family of group G to which the PDE is invariant [11]. At this point, we