

Dynamical Delay Normalize of Master Equations Model of Semiconductor QD Lasers

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Abstract

By re-expressing of the master equations model of quantum dots (QDs) laser theory by C. Gies et.al (2007), we have added a dynamical delay factor to the s-shell population's equations to take into account the retardation procedure. This addition has led to theoretical results nearly in isomorphism with experimental data . We present a theoretical simulation of characteristics and the turn-on dynamics of InGaAs/GaAs semiconductor QD laser output lasing with CW wavelength of 1.3 μ m at room-temperature including the photon-assisted polarization contribution.

Key words: Quantum dot, Dynamical delay, Luminescence.

1. Introduction

One can consider the first two confined shells which are denoted by s and p according to their in-plane symmetry. The s-shell is only spin degenerate, while the p-shell has additional angular- momentum two-fold degeneracy [1]. The spectrum of the potential well introduces a splitting into sub bands with a spacing that depends on the strength of the axial confinement, although the term sub band is somewhat misleading for the QD case, as these possess only a discrete spectrum due to the additional in-plane confinement [2].

The discrete states are located energetically below a quasi – continuum of delocalized states, corresponding to the two-

dimensional motion of carries in a *wetting layer* (WL). While, the localized states exist only below the quasi-continuum states of the WL[3-5]. In Fig.1 a schematic diagram of the energy levels of the coupled QD–WL system is shown. Further details of the QD model are discussed in Ref.[1]. Strictly speaking, the localized states and the WL states are solutions of the single-particle problem for one common confinement potential and must, therefore, form an orthogonal basis [6,7]. In this work, we focus on modulation of the laser theory model for semiconductor QDs in microcavities in Refs.[8,9], which express a site of master equations of semiconductor QDs lasers.

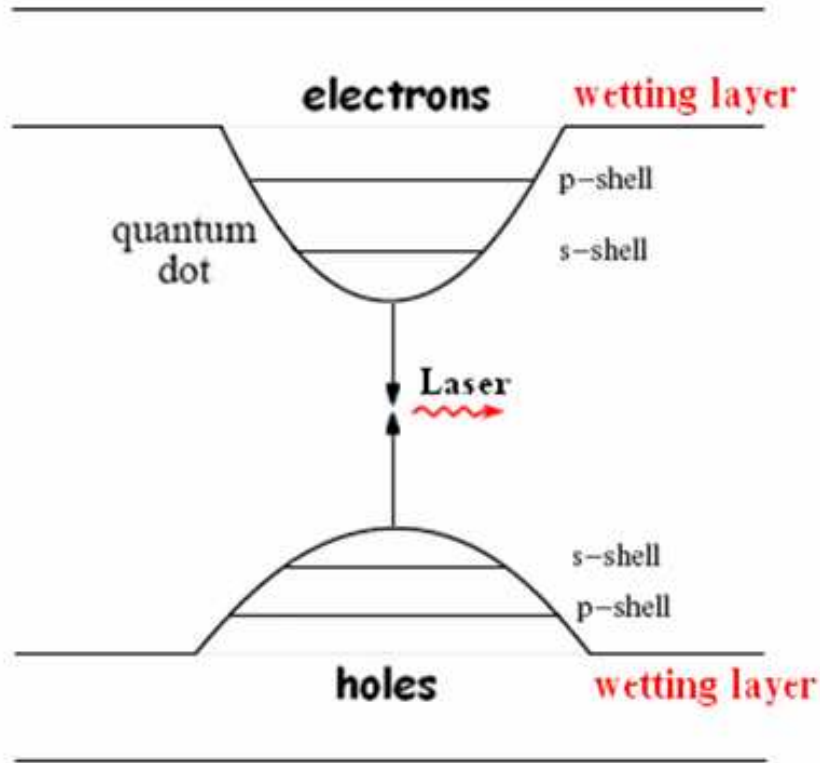


Figure 1: Schematic of the coupled QD-WL system.

2. QD System Hamiltonian

For the recombination dynamics due to carrier-photon interaction, the unpopulated WL states are of negligible importance [8]. Furthermore, if the WL states are mainly unpopulated, Coulomb correlations between the discrete QD states and the energetically displaced quasi-continuum of the WL are much weaker than those between QD states. The driving process for photoluminescence is the

spontaneous recombination of carriers in the valence and conduction band [10,11]. Spontaneous emission is a quantum electro dynamical process that is caused by field fluctuations. It is, however, possible to describe spontaneous emission with a classical light field in the formalism of quantum mechanics or quantum statistics by considering external fluctuations [1,12].

The total Hamiltonian for the system has the following contributions [2]:

$$\mathcal{H} = \mathcal{H}_{Ca}^0 + \mathcal{H}_{Ph}^0 + \mathcal{H}_{Co} + \mathcal{H}_{Di} \quad \dots (1)$$

where

$$\mathcal{H}_{Ca}^0 = \sum_{\varphi} \mathcal{E}_{\varphi}^c e_{\varphi}^{\dagger} e_{\varphi} + \sum_{\varphi} \mathcal{E}_{\varphi}^v h_{\varphi}^{\dagger} h_{\varphi} \quad \dots (2a)$$

$$\mathcal{H}_{Ph}^0 = \sum_{\zeta} \hbar \omega_{\zeta} \left(a_{\zeta}^{\dagger} a_{\zeta} + \frac{1}{2} \right) \quad \dots (2b)$$

$$\mathcal{H}_{Co} = \frac{1}{2} \sum_{\phi' \varphi \phi} [P_{\phi' \varphi, \phi}^{cc} e_{\phi'}^{\dagger} e_{\varphi}^{\dagger} e_{\phi} + P_{\phi' \varphi, \phi}^{vv} h_{\phi'}^{\dagger} h_{\varphi}^{\dagger} h_{\phi}] + \sum_{\phi' \varphi \phi} [P_{\phi' \varphi, \phi}^{cv} e_{\phi'}^{\dagger} h_{\varphi}^{\dagger} h_{\phi} e_{\phi}] \quad \dots (2c)$$

$$\mathcal{H}_{Di} = -i \sum_{\zeta, \phi \varphi} (C_{\zeta \phi \varphi} e_{\phi}^{\dagger} h_{\varphi} a_{\zeta} + C_{\zeta \phi \varphi} h_{\phi}^{\dagger} e_{\varphi} a_{\zeta}) - c.c \quad \dots (2d)$$

Where $\mathcal{H}_{Co}^0$ is the *Free Carrier Hamiltonian*, which contains information about the *single-particle spectrum* $\mathcal{E}_{\varphi}^{c,v}$ and describes a system of non-interacting charge carriers [13,14]. The operators $e_{\varphi} (e_{\varphi}^{\dagger})$ or $h_{\varphi} (h_{\varphi}^{\dagger})$ of the one-particle states $|\varphi\rangle$ with energies $\mathcal{E}_{\varphi}^c$ and $\mathcal{E}_{\varphi}^v$ respectively.

$\mathcal{H}_{Di}^0$ is the *total energy of the free electromagnetic field Hamiltonian* [12]. To obtain expressions for the quantized Hamiltonian involving the electromagnetic field, the transverse electric field and the magnetic field are expanded into modes, where each mode ζ is

associated with a quantum mechanical harmonic oscillator with the mode energy $\hbar\omega_{\zeta}$. In the usual fashion, operators are introduced that create or destroy a photon in the mode ζ , denoted as $a_{\zeta}^{\dagger}$ and a_{ζ} , respectively [6,3].

$\mathcal{H}_{Co}$ accounted for the *Coulomb Interaction between the Carriers Hamiltonian* [1,5,8,14,]. The explicit form of the single-particle wave function $\langle r|\varphi, x\rangle = \psi_{\varphi}^x(r)$ enters the description via the Coulomb matrix elements [1]

$$P_{\phi' \varphi, \phi}^{xx'} = \int d^3r \int d^3r' \psi_{\phi'}^x(r) \psi_{\varphi}^{x'}(r') P(r - r') \psi_{\varphi}^{x'}(r') \psi_{\phi}^x(r) \quad \dots (3)$$

with the band index ($x = c, v$) and $P(r)$ is the *Coulomb potential*, and via the light-matter interaction. The *dielectric constants of the vacuum and the background material* are given by ϵ_0 and ϵ , respectively. In free space, the mode label ζ contains the wave vector and the polarization vector of the electromagnetic field.

$\mathcal{H}_{Di}$ is the *light-mater interaction Hamiltonian* in dipole approximation [8,14,15]. The transition of an electron from the valence into the conduction band (or vice versa) by absorption (emission) of a photon is associated with resonant elementary process. The non-resonant terms contained in Eq.(2d) are neglected

in the *Rotating Wave Approximation* (RWA) [12]. The matrix elements $C_{\zeta \phi \varphi}$ is a *Mode Transition Coupling* (MTC) (coupling between the mode ζ of the electromagnetic field and the carrier transition between states $|\phi\rangle$ and $|\varphi\rangle$), with an electromagnetic field is approximately constant over the extent of a QD and, for simplicity, has an equal envelopes for the conduction and valence band electrons. These elements are given by [8]

$$C_{\zeta\phi\phi} = C_{\zeta} \delta_{\phi\phi} = \sqrt{\frac{\hbar\omega_{\zeta}}{2\epsilon\epsilon_0 V}} M_{cv} W_{\zeta}(r_0) \delta_{\phi\phi} \quad \dots (4)$$

Where ω_{ζ} is the resonance frequency of the optical mode ζ , V is the normalization volume, M_{cv} are the interband matrix elements,

$W(r)$ is the rapidly oscillating Bloch-factor and r_0 is the position of the QD [8,16].

3. The Truncation of Correlations

The time evolution of the single carrier and photon operators are obtained by using Heisenberg's equations of motion together with

the Hamiltonian of the interacting system .For any operator $\mathcal{O}$ it is given by

$$i\hbar \frac{d}{dt} \mathcal{O} = [\mathcal{O}, \mathcal{H}] \quad \dots (5)$$

from this one can derive coupled equations for operator averages, like the carrier/ photon population in a cavity mode or in a continuum mode of free space. The many-body problem is caused by the interaction parts of the Hamiltonian [17, 18]. In order to achieve a consistent formulation of this problem, the classification and truncation of correlation functions will be addressed before the semiconductor equations are introduced.

The appearing operator averages are classified into singles, doubles, triplets, quadruplets, etc.[19], according to the number of

particles they involve. Considering interband transitions and it must be borne in mind that the excitation of one electron is described as the destruction of a valence band carrier and the creation of a conduction band carrier. For the corresponding interaction processes, a photon operator is connected to two carrier operators [8,14]. Formally, this can be seen from integrating the time evolution of a photon operator $a_{\zeta}^{\dagger}$, readily obtained from Eqs. (1) and (5)

$$i\hbar \frac{d}{dt} a_{\zeta}^{\dagger}(t) = -\hbar\omega_{\zeta} a_{\zeta}^{\dagger}(t) + i \sum_{\phi} C_{\zeta} e_{\phi}^{\dagger}(t) h_{\phi}(t) \quad \dots (6)$$

Schematically, N – particle averages denoted as $\langle N \rangle$ and containing $2N$ carrier operators or an equivalent replacement by photon operators, are factorized into all possible combinations of averages involving one up to $N-1$ particle averages. In this work, we introduce a correlation function of order N denoted as $\langle\langle N \rangle\rangle$ (for more details see Res.[1,9, 20]).

The correlation functions defined according to Eqs.(15)-(17) in Ref.[1] show peculiarities when algebraic manipulations are performed on them. This is illustrated in the following example, where one considers the identity operator $e_{\phi}^{\dagger} e_{\phi} e_{\phi}^{\dagger} e_{\phi} = e_{\phi}^{\dagger} e_{\phi}$.

However, that is different for a correlation function

$$\langle\langle e_{\phi}^{\dagger} e_{\phi} e_{\phi}^{\dagger} e_{\phi} \rangle\rangle \neq \langle\langle e_{\phi}^{\dagger} e_{\phi} \rangle\rangle.$$

Exemplary situations include incoherent carrier excitations or coherent excitation of higher states (barrier, wetting layer or higher localized states) with rapid dephasing [21,22] and carrier relaxation [23-26], leading to a quasi-equilibrium distribution of the carriers at the lattice temperature. The absence of a coherent external field is expressed in the vanishing expectation values of the field operator $\langle a_{\zeta} \rangle = 0$ and the coherent polarization $\langle h_{\phi}^{\dagger} e_{\phi} \rangle = 0$. True at the initial time

$t = 0$, it can shown to be preserved during the time evolution taking the operator average [27]

$$a_{\zeta}^{\dagger}(t) = a_{\zeta}^{\dagger}(0) \text{Exp}[i\omega_{\zeta} t] + \frac{1}{\hbar} \sum_{\varphi} \int_0^t dt' C_{\zeta} e_{\varphi}^{\dagger}(t') h_{\varphi}(t') \text{Exp}[i\omega_{\zeta} (t - t')] \dots (7)$$

One can see that the evolution of the field operator is driven by the coherent polarization . whereas, the coherent polarization remains zero at all times without a driving coherent field [5,27]. However, for situations where a coherent

polarization is present, like resonance fluorescence [12,21,28,29], these additional terms must be considered in all equations of motion.

4. Semiconductor luminescence Equations

The semiconductor equations could be included on a microscopic level by adding a Hamiltonian analogous to Eq.(2d) for the carrier-phonon interaction [30,31]. Predominantly, it was interested in the luminescence and effects introduced by the Coulomb and light-matter interaction [8]. Where the theory is given the

time evolution of the carrier population of conduction-or valence-state (n_{φ}^c and n_{φ}^v , respectively), which obtained from Eqs.(1) and (5) that leads to:

$$i\hbar \frac{d}{dt} n_{\varphi}^c = -2i \text{Re} \sum_{\zeta} C_{\zeta}^* \mathcal{O}_{\varphi}^{\zeta} + 2i \text{Im} \sum_{\phi \phi' \varphi'} P_{\phi \phi' \phi \varphi'} (f_{\phi \phi' \phi \varphi'}^c - f_{\phi' \varphi \phi \phi'}^{cv}) \dots (8)$$

$$i\hbar \frac{d}{dt} n_{\varphi}^v = 2i \text{Re} \sum_{\zeta} C_{\zeta}^* \mathcal{O}_{\varphi}^{\zeta} - 2i \text{Im} \sum_{\phi \phi' \varphi'} P_{\phi \phi' \phi \varphi'} (f_{\phi \phi' \phi \varphi'}^v - f_{\phi' \varphi \phi \phi'}^{cv}) \dots (9)$$

Where $n_{\varphi}^c = \langle e_{\varphi}^{\dagger} e_{\varphi} \rangle$, $n_{\varphi}^v = \langle h_{\varphi}^{\dagger} h_{\varphi} \rangle$, and $\mathcal{O}_{\varphi}^{\zeta} = \langle a_{\varphi}^{\dagger} h_{\varphi}^{\dagger} e_{\varphi} \rangle$ is the Photon-Assisted Polarization (PAP) [2,27,32], here additional intraband correlation functions appear:

$$f_{\phi' \varphi \varphi' \phi}^c = \langle\langle e_{\phi'}^{\dagger} e_{\varphi}^{\dagger} e_{\varphi'} e_{\phi} \rangle\rangle$$

$$f_{\phi' \varphi \varphi' \phi}^v = \langle\langle h_{\phi'}^{\dagger} h_{\varphi}^{\dagger} h_{\varphi'} h_{\phi} \rangle\rangle$$

$$f_{\phi' \varphi \varphi' \phi}^{cv} = \langle\langle e_{\phi'}^{\dagger} h_{\varphi}^{\dagger} e_{\varphi'} h_{\phi} \rangle\rangle$$

The rotational symmetry of the system and the resulting conservation of angular momentum ensures that all off-diagonal terms $\langle\langle \mathbb{O}_{\varphi} \mathbb{O}_{\varphi'} \rangle\rangle$ with $\varphi \neq \varphi'$ describe

forbidden transitions and remain zero during the time evolution [8]. An inclusion of higher angular momentum states is straight forward, but unnecessary at low temperatures and left out for transparency [33]. Also remember that

polarization like averages of the form $\langle h_{\varphi}^{\dagger} e_{\varphi} \rangle$ vanish in the incoherent regime [4].

From Heisenberg's equation of motion for the Population of Photon (PP) n_{ph}^{ζ} , it can be seen by integrating the time evolution of a photon number, readily obtained from Eqs.(1) and (5)

$$i\hbar \frac{d}{dt} n_{ph}^{\zeta} = 2i \operatorname{Re} \sum_{\varphi} C_{\zeta}^* O_{\varphi}^{\zeta} \quad \dots (10)$$

where $n_{ph}^{\zeta} = \langle a_{\zeta}^{\dagger} a_{\zeta} \rangle$ which couples to O_{φ}^{ζ} in the incoherent regime $\langle a_{\zeta}^{\dagger} \rangle = 0$ and

thus, according to Eq.(15) in Ref.[1] which is obtained from Eqs.(1) and (5), the corresponding equation of motion is given by

$$i\hbar \frac{d}{dt} O_{\varphi}^{\zeta} = (\Delta \tilde{\mathcal{E}}_{\varphi} - \hbar \omega_{\zeta} - i\mathcal{Q}) O_{\varphi}^{\zeta} + (n_{\varphi}^c - n_{\varphi}^v) \sum_{\phi} P_{\varphi\phi\varphi\phi} O_{\phi}^{\zeta} + iC_{\zeta} n_{\varphi}^c (1 - n_{\varphi}^v) + i \sum_{\phi} C_{\zeta} f_{\phi\varphi\varphi\phi}^{cv} \quad \dots (11)$$

the evolution is determined by the Coulomb- renormalized energies:

$$\begin{aligned} \Delta \tilde{\mathcal{E}}_{\varphi} &= \tilde{\mathcal{E}}_{\varphi}^c - \tilde{\mathcal{E}}_{\varphi}^v \\ \tilde{\mathcal{E}}_{\varphi}^c &= \mathcal{E}_{\varphi}^c - \sum_{\phi} P_{\phi\varphi\varphi\phi} n_{\phi}^c \\ \tilde{\mathcal{E}}_{\varphi}^v &= \mathcal{E}_{\varphi}^v - \sum_{\phi} P_{\phi\varphi\varphi\phi} n_{\phi}^v \end{aligned} \quad \dots (12)$$

Where $\square$ is a *dephasing of phenomenological* (DP) causing a broadening of the spectral lines [2,27]. With the electron-hole correlation function $f_{\varphi'\varphi\varphi'\phi}^{cv}$ the spontaneous emission

source term is of particular interest, as it deviates from the source term obtained in atomic models [8].

5. Dynamic Laser Equations

By the fact that polarization- like averages of the form $\langle h_{\varphi}^{\dagger} e_{\varphi} \rangle$ vanish due to the incoherent carrier generation, and so does the expectation value of the photon operators, $\langle a_{\zeta} \rangle = 0$. Therefore, for the dynamical evolution of the photon population ($n_{ph}^{\zeta} = \langle a_{\zeta}^{\dagger} a_{\zeta} \rangle$) in the mode ζ and the

carrier populations of electron ($n_{\varphi}^e = \langle e_{\varphi}^{\dagger} e_{\varphi} \rangle$) and hole ($n_{\varphi}^h = \langle h_{\varphi}^{\dagger} h_{\varphi} \rangle$), the contribution of the light-matter interaction $\mathcal{H}_{Di}$ in the Heisenberg equations of motion, can be obtained from Eqs.(1) and (5) as

$$\left(\hbar \frac{d}{dt} + 2k_{\zeta} \right) n_{ph}^{\zeta} = 2 \operatorname{Re} \sum_{\varphi'} \tilde{C}_{\zeta} O_{\varphi'}^{\zeta} \quad \dots (13)$$

$$\hbar \frac{d}{dt} n_{\varphi}^{e,h} |_{opt} = -2 \operatorname{Re} \sum_{\zeta} \tilde{C}_{\zeta} O_{\varphi}^{\zeta} \quad \dots (14)$$

where $\tilde{C}_{\zeta} = |C_{\zeta}|^2$ and k is the *total cavity loss* (TCL) or *Cavity Damping*. In Eq.(13), we have introduced the *loss rate* $2k_{\zeta}/\hbar$. The mode index ζ labels cavity modes as well as leaky modes. The dynamics of the PP in a given mode is determined by PAP that describes the expectation value for a correlated event, where a photon in the mode ζ is created in connection

$$\left[\hbar \frac{d}{dt} + k_{\zeta} + Q + i(\tilde{\mathcal{E}}_{\varphi}^e + \tilde{\mathcal{E}}_{\varphi}^h - \hbar\omega_{\zeta}) \right] O_{\varphi}^{\zeta} = n_{\varphi}^e n_{\varphi}^h - (1 - n_{\varphi}^e - n_{\varphi}^h) n_{ph}^{\zeta} + i(1 - n_{\varphi}^e - n_{\varphi}^h) \sum_{\phi} P_{\varphi\phi\varphi\phi} O_{\phi}^{\zeta} + \sum_{\phi} f_{\phi\varphi\varphi\phi}^{\zeta v} + \mathcal{D}_{\varphi}^{e\zeta} - \mathcal{D}_{\varphi}^{h\zeta} \quad \dots (15)$$

Where $\mathcal{D}_{\varphi}^{e\zeta} = \langle\langle a_{\zeta}^{\dagger} a_{\zeta} e_{\varphi}^{\dagger} e_{\varphi} \rangle\rangle$ and $\mathcal{D}_{\varphi}^{h\zeta} = \langle\langle a_{\zeta}^{\dagger} a_{\zeta} h_{\varphi}^{\dagger} h_{\varphi} \rangle\rangle$. The evolution of O_{φ}^{ζ} is determined by the detuning of the QD transitions from the optical modes.

with the transition of an electron from the conduction to the valence band.

By using Heisenberg's equations of motion together with the Hamiltonian of the interacting system, the equation of motion for the PAP contains additional contributions beyond the doublet level

Then equation of motion (15) for the resonant s-shell / laser mode transition, for the PAP takes the form

$$\left(\hbar \frac{d}{dt} + k + Q \right) O_s = n_s^e n_s^h - (1 - n_s^e - n_s^h) n_{ph} + \mathcal{D}_s^e - \mathcal{D}_s^h \quad \dots (16)$$

In the equation of motion for the PAP of the non-lasing modes, the negligible photon population and the short lifetime of these modes

allows for the omission of the feedback term and carrier-photon correlations

$$\left[\hbar \frac{d}{dt} + k_{\zeta} + Q + i(\tilde{\mathcal{E}}_s^e + \tilde{\mathcal{E}}_s^h - \hbar\omega_{\zeta}) \right] O_s = n_s^e n_s^h \quad \dots (17)$$

the index $\zeta = \zeta_{\perp}$ is omitted for the laser mode.

The part $\zeta \neq \zeta_{\perp}$ of the sum in Eq.(14) can be evaluated, yielding a time constant $\tau_{\perp}$ for the spontaneous emission into non-lasing modes according to [12],

$$\gamma_{\parallel} = \operatorname{Re} \sum_{\zeta \neq \zeta_{\perp}} \frac{2\tilde{C}_{\zeta}}{\hbar[k_{\zeta} + Q - i(\Delta\tilde{\mathcal{E}}_s - \hbar\omega_{\zeta})]} \quad \dots (18)$$

In a laser theory, one typically distinguishes between the rate of spontaneous emission into

lasing and non-lasing modes, $\gamma_{\perp} = \frac{1}{\tau_{\perp}}$ and $\gamma_{\parallel} = \frac{1}{\tau_{\parallel}}$, respectively. Both rates add up to the

total spontaneous emission rate (SER), γ in relation $\frac{1}{\gamma} = \frac{1}{\gamma_{\perp}} + \frac{1}{\gamma_{\parallel}}$. It is convenient to introduce the spontaneous emission coupling

$$\frac{\gamma_{\parallel}}{\gamma} = (1 - \beta) \quad \dots (19)$$

With Eq.(14) one can now determine the population dynamics in the s-shell. For the spontaneous emission into non-lasing modes, the solution of Eq.(17) is used according to Eqs.(18) and (19). Furthermore, one can include a

factor (SEC), β and to express the rate of spontaneous emission into non-lasing modes in terms of this β factor [34]:

transition rate of carriers from the p- to the s-shell in an approximation where $C \equiv C_{\zeta\perp}$ to obtain the carrier dynamics for both the s-shell and p-shell as:

$$\frac{d}{dt} n_s^{e,h} = -\frac{2\tilde{C}}{\hbar} \text{Re}[O_s] - \gamma_s(1 - \beta)n_s^e n_s^h + \gamma_{\perp}^{e,h}(1 - n_s^{e,h})n_p^{e,h} \quad \dots (20)$$

$$\begin{aligned} \frac{d}{dt} n_p^{e,h} = & (1 - n_p^e - n_p^h) \sum \gamma_{\perp}^{e,h}(1 - n_s^{e,h})n_p^{e,h} - \gamma_p n_p^e n_p^h \\ & - \gamma_{\perp}^{e,h}(1 - n_s^{e,h})n_p^{e,h} \end{aligned} \quad \dots (21)$$

In Eq.(20), the first term describes the carrier dynamics due to the interaction with the laser mode, while the second term represents the loss of carriers due to recombination into non-lasing modes [9,35]. In the last term, the quantity $(1 - n_s^{e,h})$ is the blocking factor [36,37] ensures that the populations cannot exceed unity. In first term of Eq.(21) a carrier generation rate is included together with $(1 - n_p^e - n_p^h)$, which is called Pauli-Blocking factor [8].The

second term describes spontaneous recombination of p-shell carriers and the third contribution is the above-discussed carrier relaxation.

Only photons from the laser mode are assumed to build up correlations. Therefore, in all correlation functions only photon operators for the laser mode are considered. By means of Eq. (17) in Ref.[1], one can introduce [9]

$$\langle\langle a^{\dagger} a^{\dagger} a a \rangle\rangle = \langle a^{\dagger} a^{\dagger} a a \rangle - 2 \langle a^{\dagger} a \rangle^2 \quad \dots (22)$$

since $\langle a \rangle = \langle a^{\dagger} \rangle = 0$ for a system without coherent excitation, only a factorization into doublets is possible.

The following time evolution of the intensity correlation function $\mathcal{J}$, where driven from Eqs.(1) and (22) and is given by:

$$\left(\hbar \frac{d}{dt} + 4k \right) \mathcal{J} = 4 \tilde{C}_{\zeta} \text{Re} \sum_{\varphi'} \mathcal{D}_{\varphi'}^a \quad \dots (23)$$

Where $\mathcal{D}_{\varphi}^a = \langle\langle a^{\dagger} a^{\dagger} a h_{\varphi}^{\dagger} e_{\varphi} \rangle\rangle$ and the sum involves all resonant laser transitions from various QDs. In this equation another quadruplet

function enters, which represents a correlation between the PAP and the PP. For the corresponding equation of motion we obtain

$$\left(\hbar \frac{d}{dt} + 3k + q\right) \mathcal{D}_{\varphi}^a = -2 \tilde{C}_{\zeta} \mathcal{O}_{\varphi}^2 - (1 - n_{\varphi}^e - n_{\varphi}^h) \mathcal{J} + 2n_{\varphi}^h \mathcal{D}_{\varphi}^e - 2n_{\varphi}^e \mathcal{D}_{\varphi}^h - 2 \mathcal{M}_{\varphi\varphi'}^{eh} + 2 \mathcal{M}_{\varphi\varphi'}^{ee} \dots (24)$$

where $\mathcal{D}_{\varphi}^e = \langle\langle a^{\dagger} a e_{\varphi}^{\dagger} e_{\varphi} \rangle\rangle$, $\mathcal{D}_{\varphi}^h = \langle\langle a^{\dagger} a h_{\varphi}^{\dagger} h_{\varphi} \rangle\rangle$, $\mathcal{M}_{\varphi\varphi'}^{ee} = \langle\langle a^{\dagger} a^{\dagger} h_{\varphi}^{\dagger} h_{\varphi'}^{\dagger} e_{\varphi} e_{\varphi'} \rangle\rangle$ and $\mathcal{M}_{\varphi\varphi'}^{eh} = \langle\langle a^{\dagger} a^{\dagger} e_{\varphi}^{\dagger} h_{\varphi'}^{\dagger} e_{\varphi} h_{\varphi'} \rangle\rangle$. The triplet carrier-photon, or population-photon correlations in the second line are the same as in Eq.(16), and their evolution is given by

$$\left(\hbar \frac{d}{dt} + 2k\right) \mathcal{D}_{\varphi}^e = -2 \tilde{C}_{\zeta} \text{Re} \left[\mathcal{D}_{\varphi}^a + \sum_{\varphi'} \mathcal{B}_{\varphi\varphi'}^e + (n_{ph} + n_{\varphi}^h) \mathcal{O}_{\varphi} \right] \dots (25)$$

$$\left(\hbar \frac{d}{dt} + 2k\right) \mathcal{D}_{\varphi}^h = -2 \tilde{C}_{\zeta} \text{Re} \left[\mathcal{D}_{\varphi}^a + \sum_{\varphi'} \mathcal{B}_{\varphi\varphi'}^h + (n_{ph} + n_{\varphi}^h) \mathcal{O}_{\varphi} \right] \dots (26)$$

where $\mathcal{B}_{\varphi\varphi'}^e = \langle\langle a^{\dagger} h_{\varphi}^{\dagger} e_{\varphi}^{\dagger} e_{\varphi'} e_{\varphi} \rangle\rangle$ and $\mathcal{B}_{\varphi\varphi'}^h = \langle\langle a e_{\varphi}^{\dagger} h_{\varphi}^{\dagger} h_{\varphi'} h_{\varphi} \rangle\rangle$, are the correlation functions in the sum which obey the equations of motion

$$\left(\hbar \frac{d}{dt} + k + q\right) \mathcal{B}_{\varphi\varphi'}^h = \left[(1 - n_{\varphi'}^e - n_{\varphi'}^h) \mathcal{D}_{\varphi}^h - \tilde{C}_{\zeta} \mathcal{O}_{\varphi}^* \mathcal{O}_{\varphi'}^* \right] (1 - \delta_{\varphi\varphi'}) \dots (27)$$

$$\left(\hbar \frac{d}{dt} + k + q\right) \mathcal{B}_{\varphi\varphi'}^e = \left[(1 - n_{\varphi'}^e - n_{\varphi'}^h) \mathcal{D}_{\varphi}^e + \tilde{C}_{\zeta} \mathcal{O}_{\varphi} \mathcal{O}_{\varphi'} \right] (1 - \delta_{\varphi\varphi'}) \dots (28)$$

A closer look reveals that the correlation functions determined by Eqs.(27) and (28) only contribute if correlations between different QDs exist; i.e. superradiant coupling plays a role in the system. Specifically, the s-shell states for electrons are spin degenerate and the two spin

states are coupled to different light polarizations [9,38]. If, however, the phenomenon of superradiant coupling itself is to be studied [39-42], the correlation functions must be included via their own equations of motion.

6. Dynamical Delay Factor

When one use the theoretical model of laser theory for semiconductor QDs in microcavities in Refs.[8,9] to study the turn-on dynamics and relaxation oscillations in the InGaAs /GaAs QD laser with a wavelength of

1.3 μm . one gets poor agreement with experimental results, i.e. the theory is not accurate enough and not able to interpret the produced work by other researchers [43, 44].

Since they did not take into account the delay time inherent to occur between pumping process and building the enough population inversion, which leads to the gain then the production of electromagnetic field. This is illustrated in Fig.1. To overcome this problem, we have added a factor to the s-shell populations equations (i.e. Eqs.(20)) to take into account the retardation procedure. This addition has led to theoretical results nearly in isomorphism with experimental data in comparison to what can be gained from Ref.[9].

The first term in Eqs.(20) describes the carrier dynamics due to the interaction with the laser mode, which has the main effect of this poor agreement. We assume here; there are

variations in the absolute value square of MTC ($\tilde{C}$) in this term. We can scale it in order $\tilde{C} \rightarrow \mathcal{G} \tilde{C}$ to have the modulus of the time delay process, where is called a *Dynamical Delay Factor* (DDF) where $\mathcal{G} \ll 1$.

The added new factor with taken into accounts the delaying of carrier dynamics due to the interaction with the laser mode incoherent regime. This is normalizing the model to make a good agreement with experimental results [43,44]. Then we can write this factor in carrier populations equations of s-shell equations as $(-2\tilde{C} \mathcal{G} \mathcal{O}_z / \hbar)$.

7. The Master equations Model of QD Lasers

Finally, before the QD laser *master equations model* (MEM) is presented, it is instructive to study relaxation oscillations in semiconductor QD Lasers. For this purpose we use the stationary limit of Eqs.(13) and (20) – (28). In the following, we take a several assumptions:

- 1st: Considering the resonant s-shell contributions from identical QDs, we can replace $\sum_{\varphi'}$ by the number N of QDs in Eq. (13). This number used in the calculations is increased with decreasing β in order to have the thresholds occur at the same pump rate, i.e. $N = \tilde{N}\beta$, where $\tilde{N}$ is the *number of emitters* [8].
- 2nd: We scale the MTC in the first term in Eq.(20) in order $\tilde{C} \rightarrow \mathcal{G} \tilde{C}$ to have the modulus of the time delay process of the turn-on dynamics of the field in QD Lasers appear. The parameter DDF play a role to determine the delay time by controlling on the dynamical interaction between carrier and the laser mode.
- 3rd: To describe the carrier-generation rate in the laser-transition level, we can use the pumping rate $\mathbb{P} = N \gamma_{\perp}^{e,h} (1 - n_{\tau}^{e,h}) n_p^{e,h}$ [9] in Eq.(21).
- 4th: The PAP from Eq.(20), ignoring spontaneous emission for the above-threshold solution, that will vanish the correlation $\mathcal{D}_{\varphi}^a$ in Eqs.(23) and (24).

- 5th: The remaining correlation functions referring to different QDs $\varphi \neq \varphi'$ are related to superradiant coupling [45]. The same applies to the expectation value $\mathcal{M}_{\varphi\varphi'}^{ee}$, which also vanishes together with the corresponding correlation function for $\varphi = \varphi'$.
- 6th: Coulomb interaction contributions to the carrier dynamics of n_{φ}^e and n_{φ}^h are very weak, that are analogous to those correlations $\mathcal{D}_{\varphi}^e$ and $\mathcal{D}_{\varphi}^h$ in Eqs.(25) and (26).
- 7th: Annihilating two valence-band electrons in case of $\langle a e_{\varphi}^{\dagger} h_{\varphi}^{\dagger} h_{\varphi} h_{\varphi} \rangle$ and two conduction-band electrons in case of $\langle a^{\dagger} h_{\varphi'}^{\dagger} e_{\varphi'}^{\dagger} e_{\varphi'} e_{\varphi} \rangle$ is only possible if these carriers belong to different QDs [2,12,27]. Hence, for $\varphi = \varphi'$ these expectation values vanish altogether. Thus, either their factorization, or the remaining correlation functions $\mathcal{B}_{\varphi'}^e$ and $\mathcal{B}_{\varphi'}^h$ exist in the case $\varphi \neq \varphi'$. Then for $\varphi = \varphi'$ these expectation values vanish altogether in Eqs.(27) and (28).

- 8th: If one consider correlations between photons with the same circular polarization, can find they are linked to states for which only one electron or hole per s-shell and QD are available. Under the assumption that superradiance is weak

in the system [40,41], the discussed correlation functions are neglected.

With correlations inserting in Eq.(13), the PAP from Eq.(20) ignoring spontaneous

$$\frac{d}{dt} n_{ph} = \frac{2}{\hbar} \left[-k n_{ph} + \frac{\tilde{N}}{\beta} \tilde{C} O_s \right] \quad \dots (29)$$

$$\frac{d}{dt} O_s = -\frac{1}{\hbar} \left[(k + \mathcal{Q}) O_s - n_s^e n_s^h + (1 - n_s^e - n_s^h) n_{ph} \right] \quad \dots (30)$$

$$\begin{aligned} \frac{d}{dt} n_s^{e,h} = -2 \frac{\mathcal{G} \tilde{C}}{\hbar} O_s - \gamma_s (1 - \beta) n_s^e n_s^h \\ + \gamma_{\perp}^{e,h} (1 - n_s^{e,h}) n_p^{e,h} \quad \dots (31) \end{aligned}$$

$$\frac{d}{dt} n_p^{e,h} = \mathbb{P} (1 - n_p^e - n_p^h) - \gamma_p n_s^e n_s^h - \gamma_{\perp}^{e,h} (1 - n_s^{e,h}) n_p^{e,h} \quad \dots (32)$$

Where $n_s^{e,h}$ ($n_p^{e,h}$) is electron / hole population of s-shell (p-shell), $\mathbb{P}$ is the pump rate, $\gamma_{s,p}$ is s-shell (p-shell) spontaneous emission rate into non-lasing modes $\gamma_{\perp}^{e,h}$ is electron / hole spontaneous emission rate into lasing modes.

In the following, we shall use the site of QD semiconductor laser MEM (29)-(32) to compute numerically to study the turn-on dynamic of field with contribution of photon-assisted polarization in InGaAs /GaAs QD Lasers.

emission for the above - threshold solution, and expressing the higher-order correlations with the help of Eqs.(13) and (20) - (28), we can write:

The ordinary differential system MEM (29)-(32) includes the time variation of photon population n_{ph} in the QD laser cavity, photon-assisted polarization O_s s-shell / p-shell carriers populations of electron and hole ($n_s^{e,h}$ and $n_p^{e,h}$ respectively). These variables are functions to various parameters (i.e. control parameters). Table (1) constitutes these parameters with their values and units which are used in the present work. The pump rate is one of the important time dependent parameter.

8. QD Laser Characteristics

The MEM (29)-(32) provide convenient methods to fit the input/output characteristics of QD microcavity lasers. The pump rate $\mathbb{P}$ is

$$\mathbb{P}(t) = \mathbb{P}_a \frac{1}{\sqrt{2\pi\Delta t^2}} \text{EXP} \left[\frac{-t^2}{2\Delta t^2} \right] \quad \dots (33)$$

Where $\mathbb{P}_a$ is the pump pulse area, which is directly corresponds to the number of two-level systems excited by the pulse. Δt is a pulse

duration (pulse width). The calculations are based on producing a curve of photon population against pump rate in Fig.2 for pump pulses shorter than the spontaneous emission time, which represents InGaAs/GaAs QD laser

input/output curve. By using this figure especially, the operation threshold pump rate can be obtained at the curve jump, i.e. we have $P_{th} = 3.146 \text{ ps}^{-1}$ in this figure. The reabsorption present in the system modifies the height of the jump. The upper branch of the input/output curve can be masked to an extent where the threshold is not even fully developed.

To control the dynamical interaction between carrier and the laser mode, parameter G (DDF) in Eqs.(31) play an important role to normalize the intensity of laser field and the

delay time. So, we can use this theoretical model to study the turn-on dynamical of InGaAs/GaAs

QD laser to obtain results which are in agreement with experimental data [48,49].

The QD laser dynamics in this model are based on the following; the laser microcavity of QD can be considered as a four level system (see Fig.1) where holes have two levels (s- and p-shell) in valence band and electrons have two levels (s- and p-shell) in conduction band. QD Laser procedure includes the direct pumping between p-shell holes to p-shell electrons, then through equilibrium transition between holes state, i.e. holes rise from p-shell to the s-shell in valence band and electrons relaxes from p-shell to s-shell in conduction band. The laser operation occur through the recombination process between descending captured electrons from s-shell and capture of rising holes from s-shell, then the light is emitted. Each of the six variables in the model under investigation has its own dynamics.

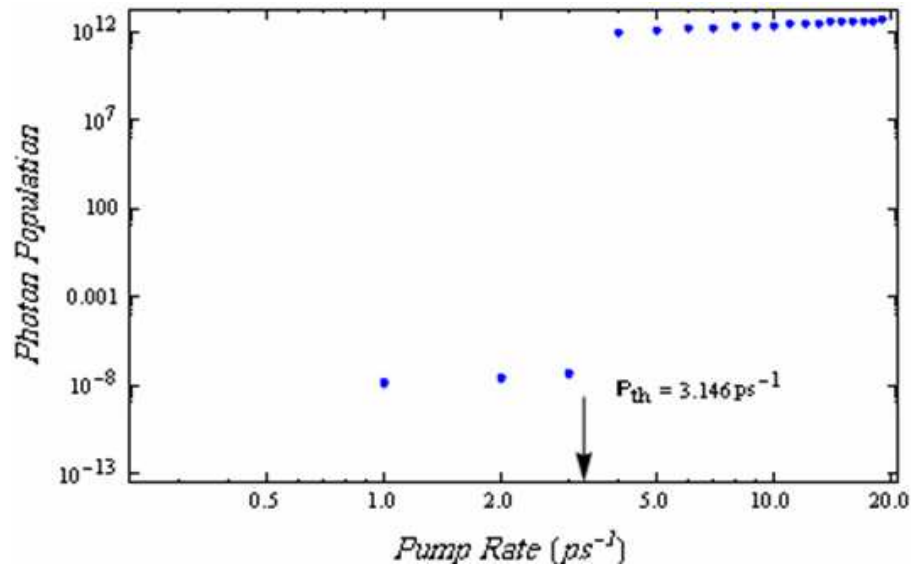
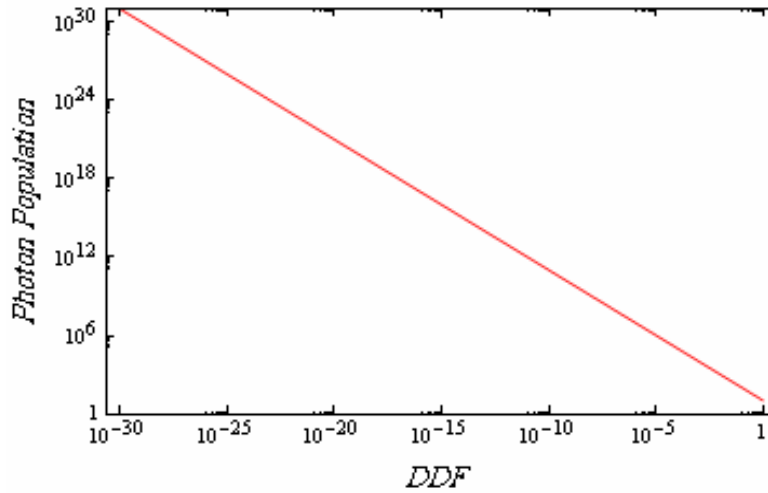


Figure 2: Steady-State input-output characteristic of InGaAs /GaAs QD laser: Simulated PP vs. Pump Rate P . The threshold Pump Rate $P_{th} = 3.146 \text{ ps}^{-1}$ is determined from the extrapolated laser onset if spontaneous emission is neglected. The other parameters[47]: $k=20\mu\text{eV}$, $C=6.$, $\beta=0.02$, $\tau_s^e=1.$ ps, $\tau_s^h=5.$ ps, $\tau_s=50.$ ps, $\tau_p=2.$ fs, $\Delta t=10.$ ps, $r_{th}=10.$, $P_{th}=3.146 \text{ ps}^{-1}$, $N=20$.

The dynamical delay factor play very clear role to determine the characteristics of QD

laser. Such as the field and the delay time of QD laser output are dependent on it. Fig.3. represents

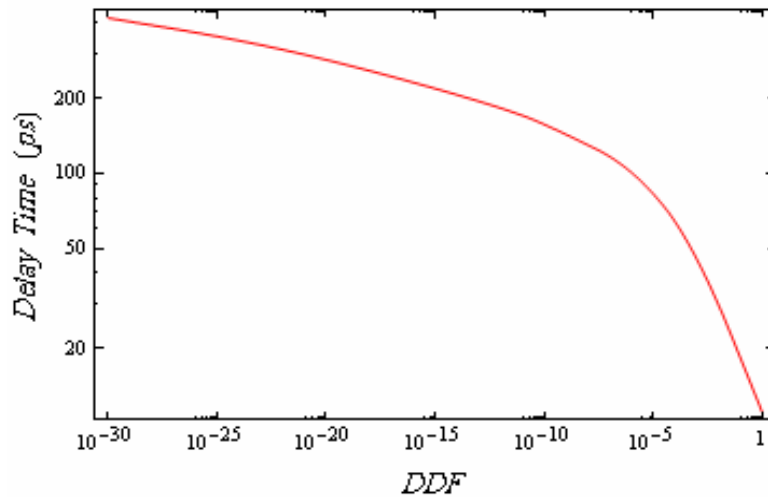
the variation of photon population and the delay time as a function of this factor.



(a)

Figure 3: Characteristic of InGaAs /GaAs QD laser with a wave-length of 1.3 μm varying with DDF change: (a) Steady-State of PP vs. DDF, (b) Delay Time vs. DDF.

Continues



(b)

Continued

So clear, both quantities are dependent to DDF region of variation ($10^{-30} \rightarrow 10^0$). By controlling on the dynamical interaction between lasing state carriers $n_s^{e,h}$ and the laser mode, the parameter DDF play an important role to determent the laser characteristics. From the figure, with decreasing DDF, the output field is

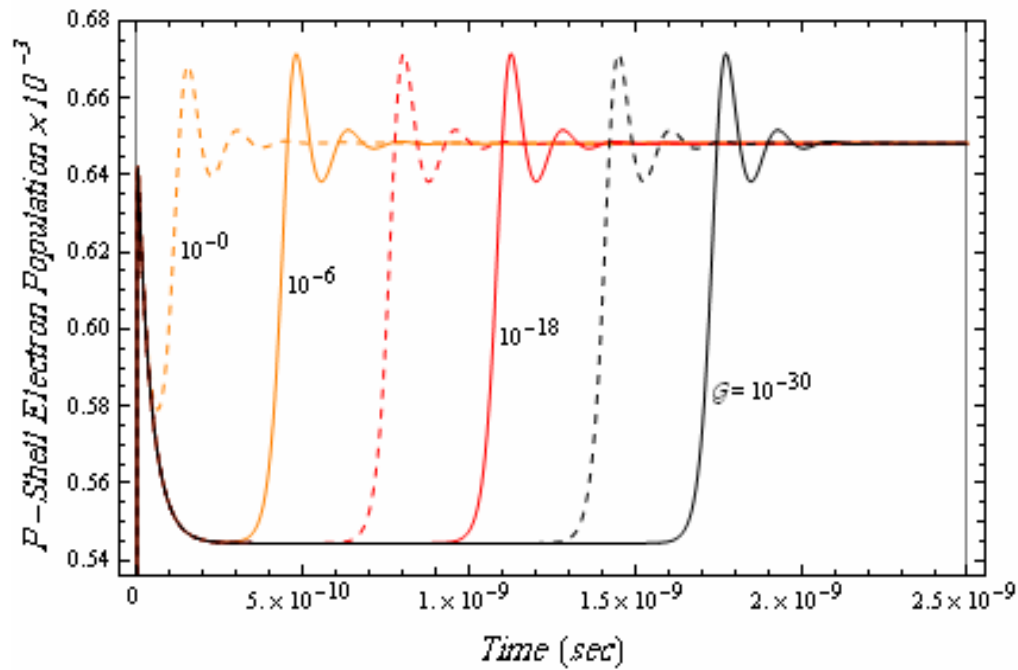
increasing linearly but the delay time is increasing nonlinearly in scale ($10^{-10} \rightarrow 10^0$). According to the obtained results, we have chosen the value $|\hbar|$ for DDF in all our results in this work.

9. Turn-on dynamics of QD lasers

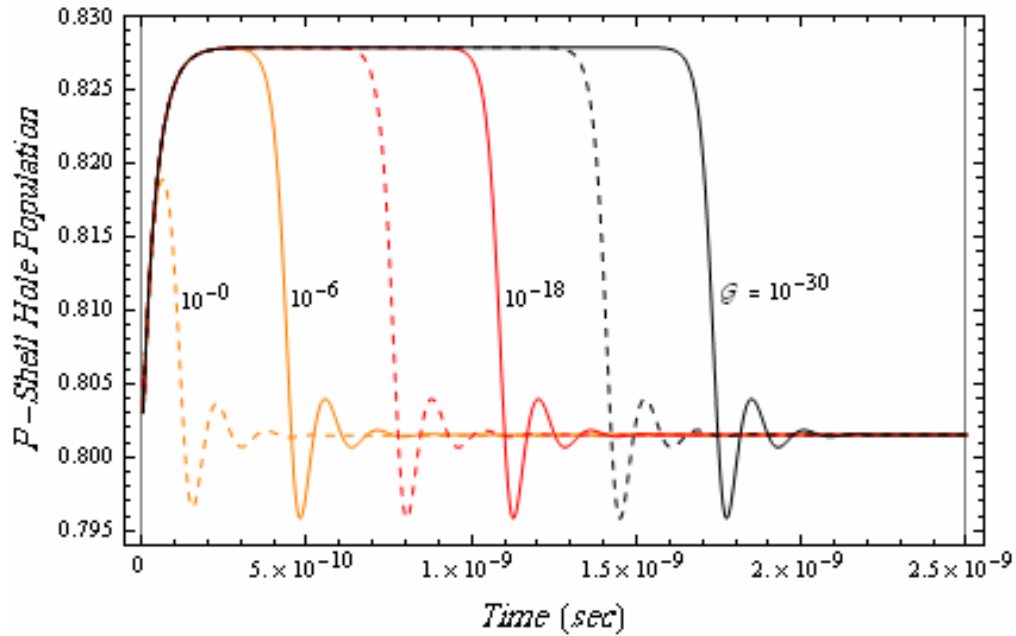
The dynamical delays factor, DDF do attenuate the carrier dynamics due to the interaction with the QD laser mode incoherent regime. By normalizing the theoretical model in Refs.[8,9], we can make the turn-on dynamical results of the model in good agreement with experimental results. So this factor affects the populations of p-s-shell (i.e. upper and lower of pump/lasing QD states) as shown in Fig.4, especially, its effect on turn-on delay time for different DDF values viz ($10^{-30} \rightarrow 10^0$). Clearly

in Fig.4(a) and (b), the DDF decreasing lead only to increase the turn-on dynamical delay time of all model carriers population dynamic. But for the other dynamics, the curve behavior for different value of DDF shows no changed.

While in Fig.5(a) and (b) shows how the number of photons and polarization of carriers, i.e. PP and PAP suffered different effects as the delay time and amplitude of the oscillation in the transient region and time arrival of the steady state output for both field and polarization.



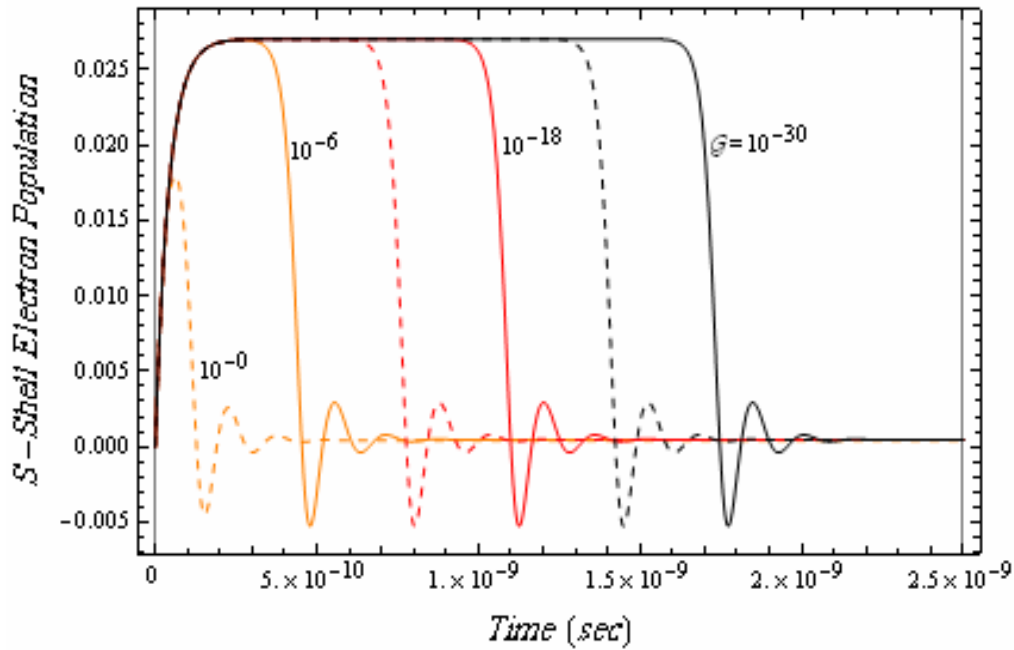
(a)



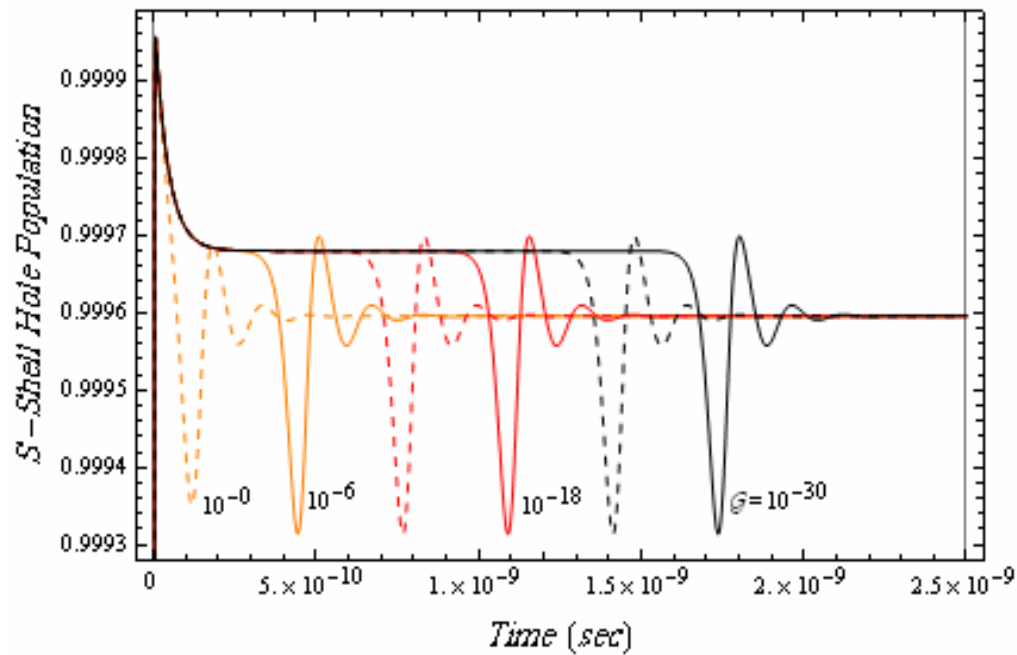
(b)

Figure 4: Turn-on dynamic of InGaAs /GaAs QD laser model carriers in MEM (29)-(32) for different value of DDF $G = 10^{-m}$ (where $m=0,6,12,18,24$ and 30 respectively): (a) and (b) Population of p-shell carriers (pump states) $n_p^{e,h}$, respectively. (c) and (d) Population of s-shell carriers (lasing states) $n_s^{e,h}$, respectively.

Continues

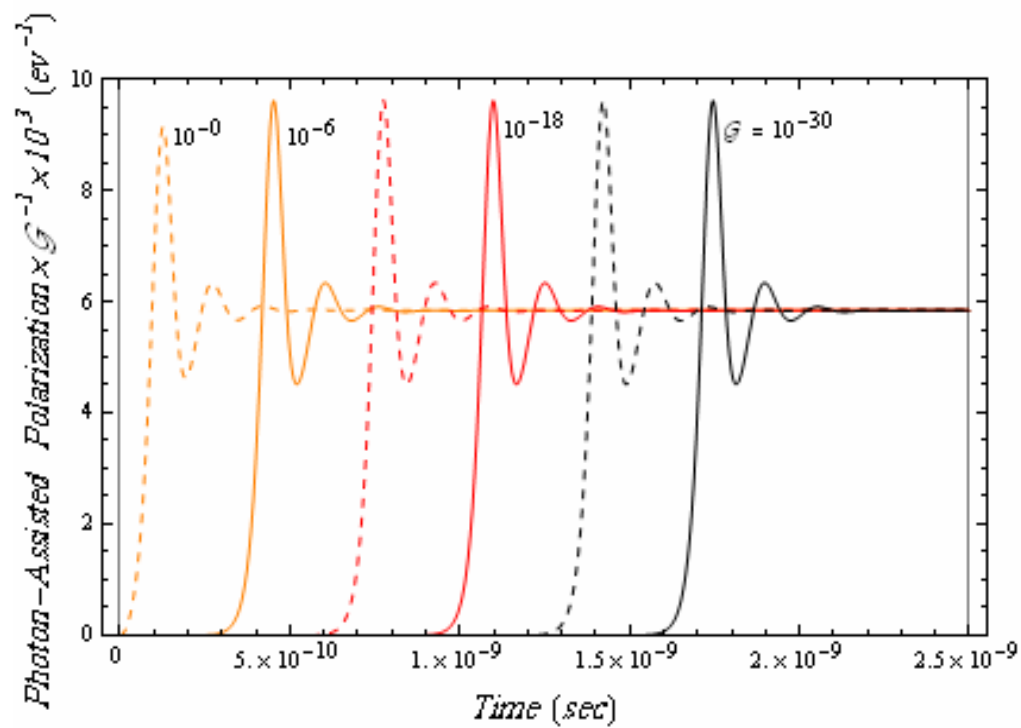


(c)



(d)

Continued



(a)

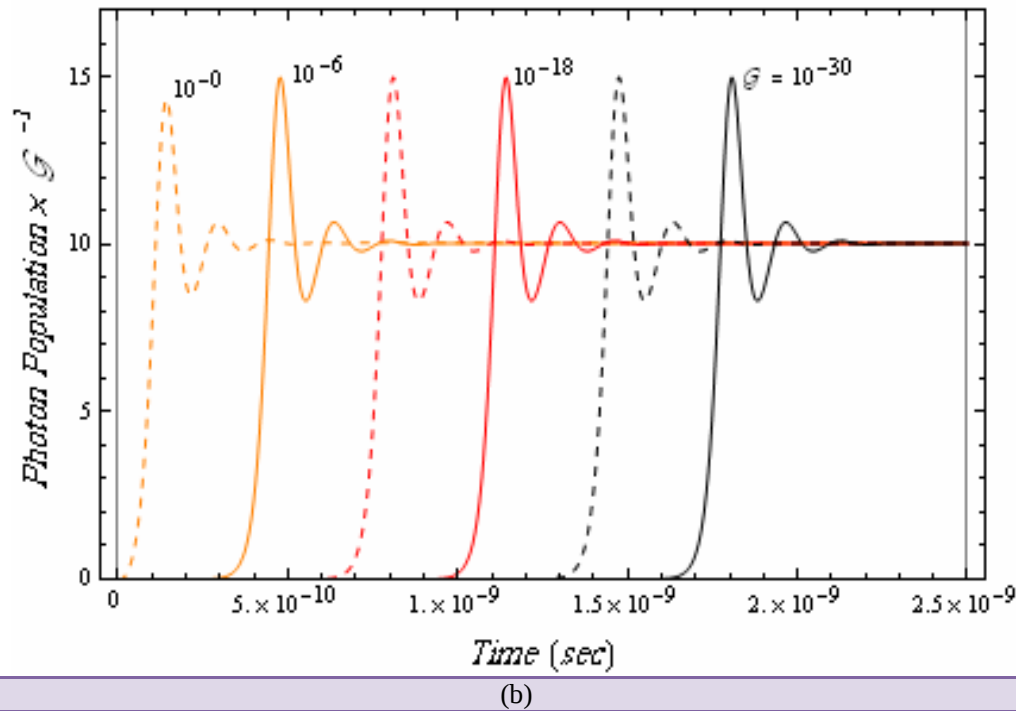


Figure 5: Turn-on dynamic of InGaAs /GaAs QD laser model variables for different value of DDF, $G = 10^{-m}$ (where $m=0,6,12,18,24$ and 30 respectively): (a) PAP and (b) PP.

9. Conclusion

The new addition (DDF) in to the MEM by Gies et.al, making a good normalize of this model to study the turn-on dynamical characteristics of semiconductor QD laser. That is clear with study the QD laser characteristics and the dynamics in delay time and intensity of laser field. Our specific results of InGaAs /GaAs QD semiconductor laser output at room-

temperature CW lasing with a wavelength of $1.3\mu\text{m}$ are studied. That is shown in: delay time to reach steady state, level of output in the steady state. For this model, the PAP and the populations of pumping / lasing states are a function of PP. The system of QD lasers dynamics can be modeled extremely well by this mentioned of model equations.

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معايرة التأخير الدينامي لنموذج المعادلات الرئيسي للليزرات QD لشبه موصل

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الخلاصة:

من خلال إعادة صياغة نموذج المعادلات الرئيسي لنظرية ليزر النقاط الكمّية (QDs) للباحثين C. Gies و مشاركيه (2007), أضفنا عامل تأخير دينامي إلى معادلات تعدادات طبقة S - ليأخذ في الحسبان إجراء التأخر. هذه الإضافة أدت إلى نتائج نظرية تقارب البيانات التجريبية تقريبا. في هذا البحث نقدّم محاكاة نظرية لخصائص و ديناميكيات بدء التشغيل للليزر QD لشبه موصل InGaAs /GaAs يعمل CW بطول الموجة من $1.3 \mu\text{m}$ في درجة الحرارة الغرفة مع تضمين مساهمة الإستقطاب بمساعدة الفوتون.