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ELECTROMAGNETIC TRANSITIONS IN DEFORMED NUCLEI YB^{168,170}

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ABSTRACT: The energy levels and electromagnetic transition probabilities in Yb^{168,170} isotopes have been studied using dynamic deformation model (DDM) and interacting boson model (IBM). This work also included calculation of the M1 matrix element in IBM by using high order terms of the analytic solution of this model. The potential energy surface and deformation parameters reflect to large deformed nuclei properties. The predictions of the two models are compared with available experimental data.

1. INTRODUCTION

1979 and 1982, large of experimental data on the energy levels and electromagnetic transitions of Yb-isotopes have been obtained [1, 2], which suggest that these nuclei are described as a part of well deformed nuclei in nuclear collective motion. At the same time, these nuclei have SU(3) structure in interactiong boson model due to the large number of valence protons and neutrons. This number of nucleons generate a large qudrupole interactions in these nuclei.

In the present work we applied two nuclear models; dynamic deformation model (DDM) and interacting boson model (IBM) in studying energy levels, electromagnetic transitions and other nuclear properties of the Yb^{168,170} isotopes.

2. MODELS

2.1 Dynamic Deformation Model (DDM)

The DDM has been developed depending on the theory of pairing-pulse-quadruple (PPQ) model of Kumar and Baranger [3]. This model is able to describe the nuclear properties of a particular nucleus without using any fitting parameters [4], only A and Z are needed to calculate energy levels and transition probabilities.

The microscopic Hamiltonian used is

$$H = H_{av} + V_{pairing} \qquad \text{.....(1)}$$

where

$$H_{av} = \frac{P^2}{2M} + \frac{M}{2}\sum_{k=1}^{3} W_k^2 X_k^2 + h W_o [V_{ls}\,\bar{l}.\bar{s} + v_{ll}(\bar{l}^2 - <\bar{l}^2>_N)] \qquad \text{.....(2)}$$

2.2 Interacting Boson Model

The IBM [5-7] describe low-lying energy levels in the even-even nuclei, starting from the symmetric coupling of bosons. In this model one can describe collective states by a system of N identical bosons. These bosons are with angular momentum L=0 (s-boson) and L=2 (d-boson). Unitary transformations among the six components in the model (single state of s-boson and five states of d-boson) generate the group (U (6)), which plays the role of dynamical symmetries [8]. The reduction of this group lead to three dynamical symmetries (U (5), SU (3) and O (6)) corresponding to geometrical idea (spherical vibrator, deformed rotor and γ-soft) respectively.

In IBM one can usually use the following Hamiltonian which describes the interactions between the bosons [9].

$$H = E_d n_d + a_0 P.P + a_1 L.L + a_2 Q.Q + a_3 T_3.T_3 + a_4 T_4.T_4 \quad \text{.....(3)}$$

where E_d is the energy of d-boson, n_d is a number of d-boson operator P,l and Q represent pairing, angular momentum and Qudruple operators respectively. T_l (l=3, 4) are octupole and hexadecapole operators.

$$n_d = (d^* \times d).\ P = \frac{1}{2}(d^*.d + s.s),\ L = \sqrt{10}\ (d^* \times d)$$

$$Q = (d^* \times s + s^* \times d) - X(d^* \times d),\ T_\ell = (d^* \times d)^\ell,\ \ell = 3,\ 4$$

The operators (s, s^* and d, d^*) are the annihilation and creat operator for the s and d-bosons.

3. RESULTS AND DISCUSSION

3.1 Energy Levels

For $Yb^{168,170}$ isotopes, IBM calculation were done for N=14 and 15 bosons respectively, where eq.(3) was numerically diagonalized by IBM code. The values of the parameters used in this calculations are chosen to fit the experimental results, where a_1=0.01 Mev and a_2=0.012 MeV for Yb^{168} isotopes; a_1=0.01 MeV and a_2=0.011 MeV for Yb^{170} isotopes. The value of X in the quadrupole operator equal to $-\sqrt{7/2}$ which is the typical value of Su(3) limit [9].

The results of these calculations for ground state, beta and gamma-bands are shown in figs. (1 and 2). From these figs. one can see a good agreement between theory and experiment for ground state band. For Yb^{168} the O_2^+ (ß-head band) state at 1.155 MeV in experimental data is not included in fig.(1) in column of DDM results because of higher in energy of 2_3^+ (1.23 MeV) which is a member of same band, the energy of this state in DDM results equal to 1.62 MeV so the energy difference with experimental data explain that this level is not collective and, perhaps, it is thought to be outside this model space. One the other hand, in IBM results the energy of O_2^+ state comply with experimental. After we use a_0=0.0045 MeV which is important in O(6) limit [10], the energy of this state become 1.023 MeV.

In general the agreement between IBM and experimental is better than that between DDM and

this band is found in the results of the two models. Fig.(2) shows the identicalness between the perimental with two model in ground state band for Yb^{170} isotope. The energy difference of 0_2^+ state tween the experimental and DDM results is lowest than that for Yb^{168} isotope, and it lie lower the 2_3^+ .139 MeV). The IBM results is quite equal to experimental data fro ß-band specially after uses =0.0045 MeV. Finally, the energy ratio $E(4_1^+)/E(2_1^+)$ remains very close to 3.3 which tend to tational properties.

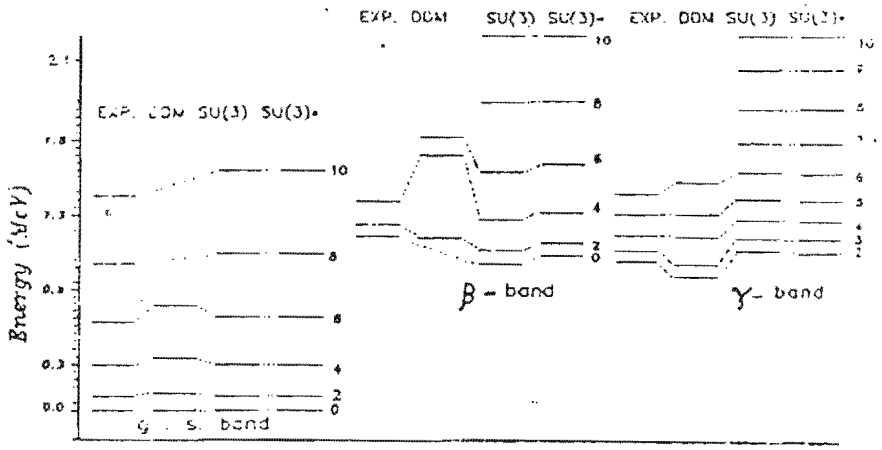


Figure 1: A comparsion between theoretical colculation and experimental results of Yb-168 isotope (SU(3))* denotes $SU(3)+a_0=0.0045$ MeV).

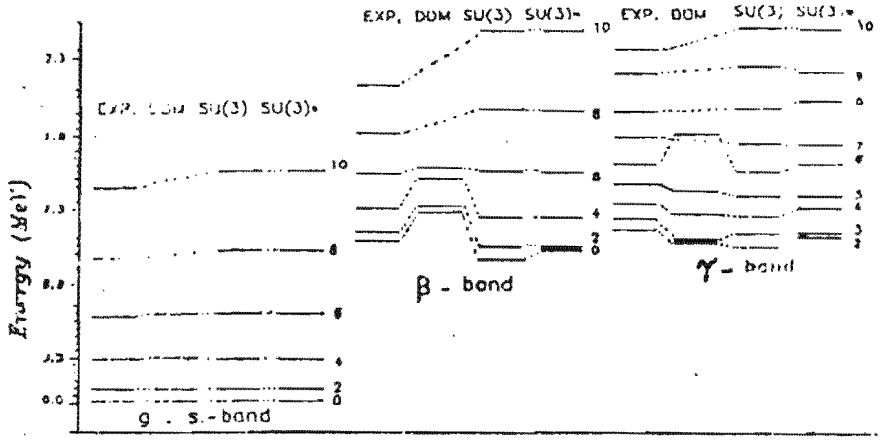


Figure 2: A comparsion between theoretical calculation and experimental results of Yb-170 isotope (SU(3)* denotes $SU(3)+a_0=0.0045$ MeV).

.2 B(E2) and Branching Ratios

To calculate E2 transitions probability in IBM we use the quadrupole transition operator

B(E2) values for transitions within the ground state bans an γ-band. Further, this table include some transitions for ß-band levels to ground state band. Both models give the same trend for the interband transitions. A comparision of B(E2) branching ratios is given in Table (2). The harmony of DDM with IBM results is good in general. The DDM values for B(E2) ratios predict weak branch from the decay of 2_g to ground state but stronger to the 4_g state. The first excited O_2 (ß- head band) in DDM results decays predominantly to γ- head band.

The ratio B(E2; $2_\beta^+ \text{----} O_g^+$)/B(E2; $2_\gamma^+ \text{----} O_g^+$) values in IBM results equal to 0.19 for Yb^{168} and 0.23 for Yb^{170}, where it is close to value of 1/6 predicted in the SU(3) limit. These features reflec the SU(3) limit in IBM (deformed rotor).

Table (1):B(E2) values of $YB^{168,170}$ isotopes, given in $e^2.b^2$ unit.

		Yb-168			Yb-170		
I_i	I_f	EXP.	IBM	DDM	EXP.	IBM	DDM
2_g	0_g	1.155(8)	1.149	0.866	1.14(6)	1.132	0.937
4_g	2_g		1.623	1.253		1.602	1.407
6_g	4_g		1.750	1.440		1.732	1.529
0_g	2_γ	0.132(12)	0.663	0.288	0.077(15)	0.572	0.182
0_g	2_β	0.042(3)	0.125	0.005		0.093	0.011
0_β	2_g	0.08(29)	0.130	0.032	0.057(11)	0.110	0.114
3_γ	2_g		0.238	0.0995		0.101	0.075
3_γ	2_γ		1.851	1.358		1.826	1.346
4_γ	4_g		0.264	0.144		0.221	0.113
6_γ	4_γ		1.186	1.109		1.167	1.217
2_β	0_g	0.010(1)	0.025	0.001		0.018	0.002
0_β	2_γ		0.025	0.034	0.030(6)	0.027	0.038
5_γ	3_γ		0.989	0.818		0.976	0.798

Table (2):Branching ratios values of $Yb^{168,170}$ isotopes.

		Yb-168			Yb-170		
I_i	I_f/I_f	EXP.	IBM	DDM	EXP.	IBM	DDM
2_γ	$0_g/2_g$	0.58(8)	0.612	0.601		0.630	0.529
4_γ	$4_g/2_\gamma$		0.426	0.264		0.361	0.264
3_γ	$2_g/2_\gamma$		0.128	0.036		0.111	0.056
3_γ	$4_g/2_\gamma$		0.065	0.026		0.055	0.020
2_β	$2_g/O_g$		1.101	5.660		1.467	9.424
4_β	$4_g/2_g$		0.685	0.324		0.862	0.126
2_β	$4_g/2_g$		2.840	7.581		2.347	3.394
			0.459	9.013		0.247	3.345

3.3 Ml Transition

The Ml transitions in the simplest interacting boson model are forbidden because the Ml transition operator is proportional of the total angular momentum, (which is a good quntum number for all nuclear states). In this model the complete Ml operator though second order depend on the E2 and E0 operators, where the matrix element can be written [11].

$$< I_f \mid T(Ml) \mid I_i > = -B\, f(I_i, I_f) < I_f \mid T(E2) \mid I_i > + C[I_i (I_i + 1)(2I_i + 1)] < I_f \mid n \mid I_i > \quad \ldots\ldots(5)$$

For transition I----I + 1 and I----I the $f(I_i, I_f)$ given by [9]

$$f(I_i, I_f) = [(1/40)(I_i + I_f + 3)(I_f - I_i + 2)(I_i + I_f + 2)(I_i + I_f - 1)]^{\frac{1}{2}} \quad \ldots\ldots(6)$$

The second term in eq.(5) only contributes to transition I-----> I. Thus for I---> I + 1 one can write using the reduced E2/M1 mixing ratio

$$(E2/M1) = < I_f \mid T(E2) \mid I_i > / < I_f \mid T(M1) \mid I_i > \quad \ldots\ldots(7)$$

Table (3) shows the theoretical M1 transitions matrix elements. The value of the constant B of M1 operator has been extracted according to eqs.(5, 6 and 7) depended of experimental data, where estimated at 0.0028 for Yb isotopes.

Finally from this Table general feature γ--> g transitions are largely E2 (or the calculated M1 components are too small). The results reflect that all states in these tables are symmetric.

Table (3): Ml transitions matrix elements of $Yb^{168,170}$ isotopes, given in Un.

		Yb-168		Yb-170	
I	I	IBM	DDM	IBM	DDM
3_γ	2_g	0.0051	-0.0144	0.0056	0.0024
3_γ	4_g	0.0050	-0.0122	0.0054	0.0025
3_γ	2_γ	0.0155	-0.0076	0.0106	-0.0452
4_γ	3_γ	0.0205	-0.0104	0.0206	-0.0591
5_γ	4_γ	0.0240	-0.0234	0.0242	-0.0806
5_γ	6_g	0.0109	-0.0216	0.0120	0.0075
5_γ	4_β	0.0016	---	0.0012	---
5_γ	4_g	0.0101	-0.0279	0.0108	0.0071
4_β	3_γ	0.0017	-0.0171	0.0023	-0.0240

3.4 Potential Energy Surface

The potential energy surface $V(\gamma, \beta)$ is calculated by using the following eq. [12].

$$V(\gamma, \beta) = \frac{N E_d \beta^2}{1 + B^2} + \frac{N(N-1)}{(1 + \beta^2)} (\alpha_1 \beta^2 + \alpha_2 \beta^2 \cos 3\gamma + \alpha_3 \beta^2 + \alpha_4)$$

where α_i, s are simply related to IBM Hamiltonian parameters. From figs.(3-5) one can see that the potentials of $Yb^{168,170}$ are different from these of a spherical vibrator and γ-unstable which would have minimums at $\beta=0$ and $\beta=1$ (γ-independent) respectively [13]. The minimum potential occurs at $\beta=1.3$ for prolate side ($\gamma=0$) hence the prolate oblate energy deference equal to 3 MeV. It is clear

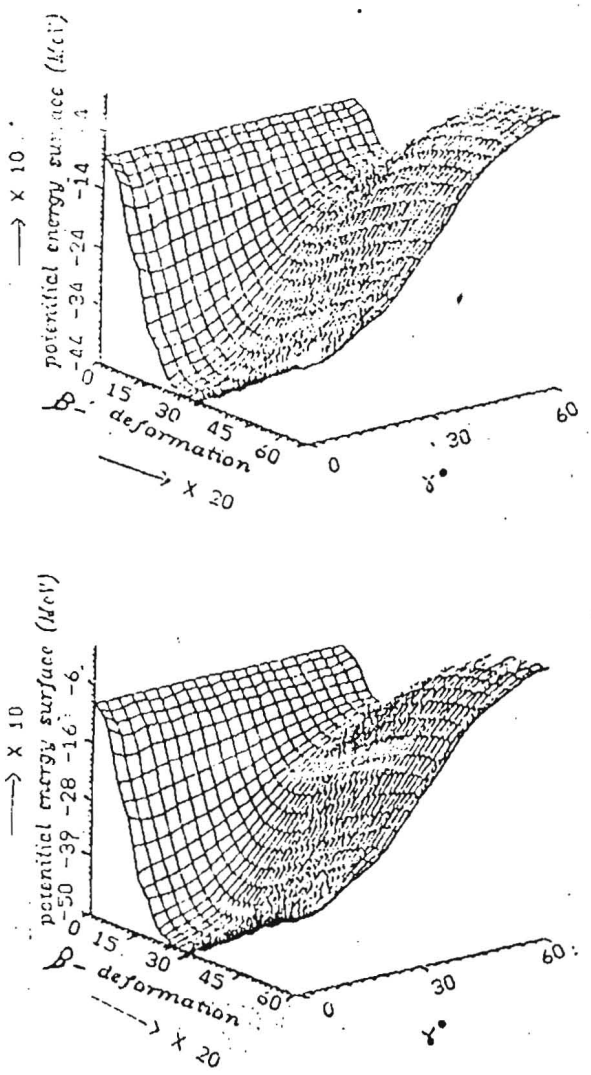
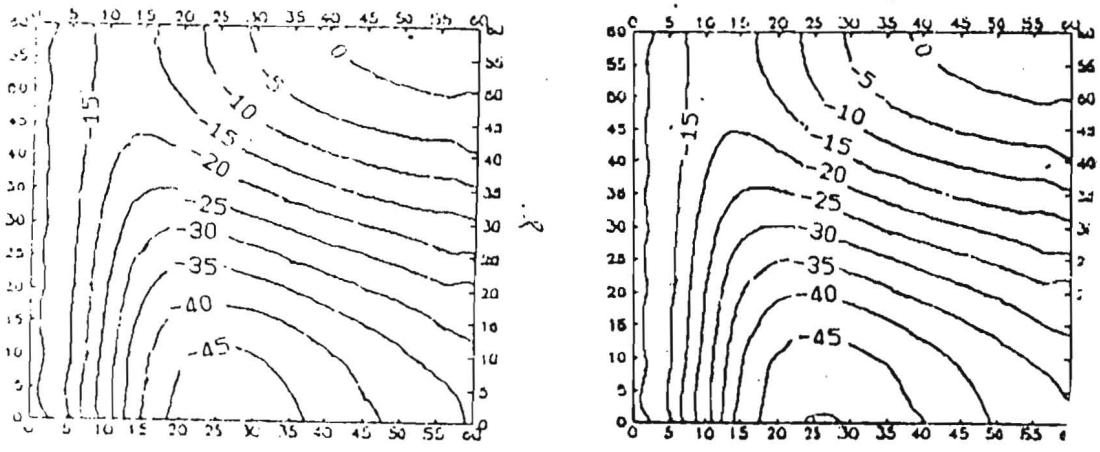


Figure 3: Potential energy surface of $Yb^{168,170}$ isotopes.



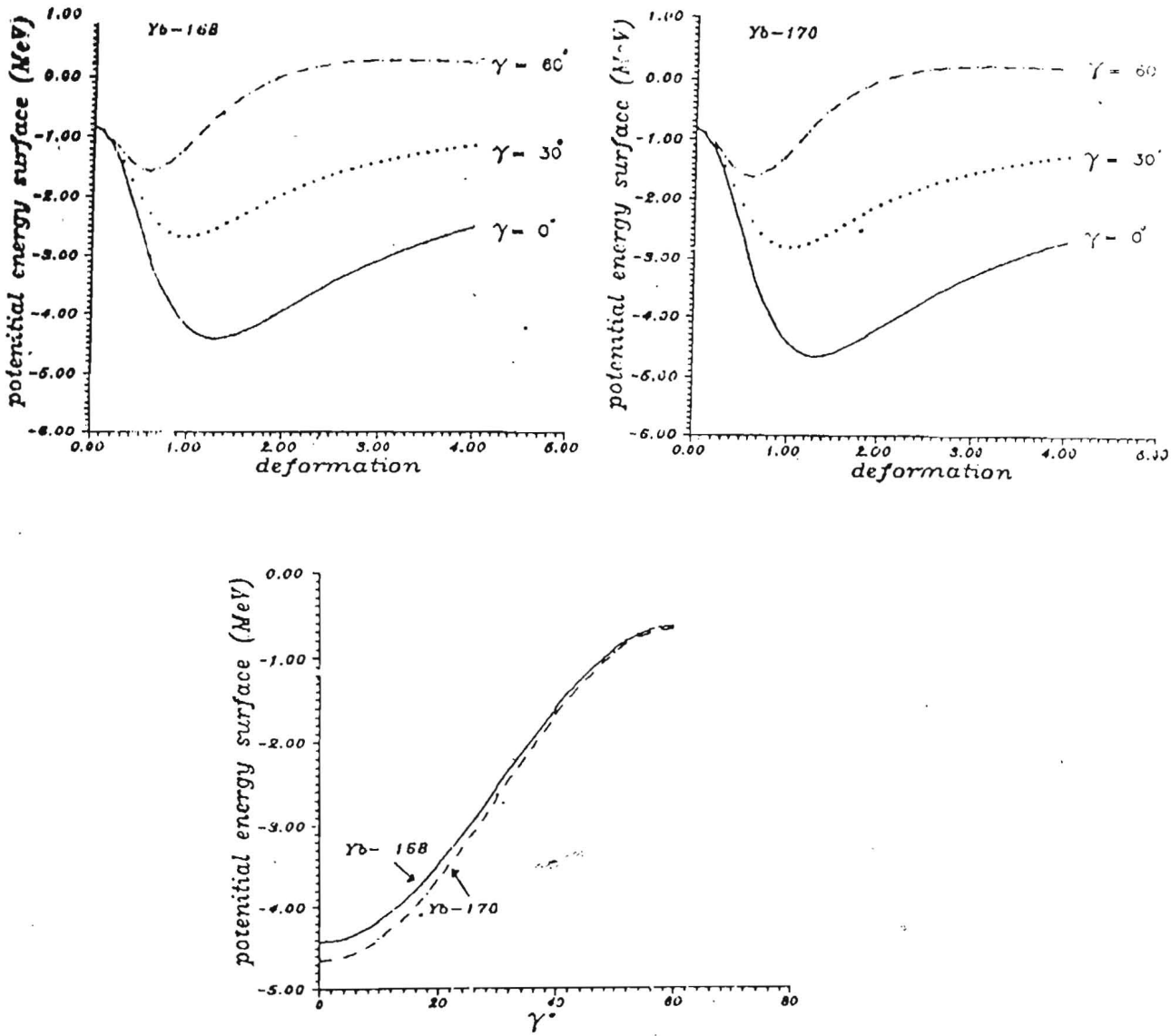


Figure (5): Potential energy surface a, b the V (γ =0, 30, 60), ß) (c) the V (γ, $\bar{B}$=1.3) for Yb (A=68, 170).

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المستخلص

لقد تم دراسة مستويات الطاقة واحتمالية الانتقالات الكهرومغناطسية بين مستويات الطاقة لأثنين من نظائر البترييوم ا
تحتوي على 168 و 170 نيكلون باستخدام اثنين من النماذج النووية (نموذج الانحراف الديناميكي ونموذج البوزونات المتفاع
وشملت هذه الدراسة حساب عناصر مصفوفة الانتقال ثنائي القطب المغناطسي باستخدام الحدود ذات المراتب العليا لحلول نم
البوزونات المتفاعلة. كما تم دراسة طاقة سطح الجهد وعوامل التشوه لهذه الأنوية ومن خلالها تبين أنها تمتلك خصائص الأ
دائمة التشوه. وقورنت النتائج المستحصلة مع القيم العملية المتوفرة.