Level structure of the Ge, Se, and Kr ($N = 52, 53$) isotopes within the framework of the interacting boson model

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The levels structure and electromagnetic transitions of ^{84,85}Ge, ^{86,87}Se, and ^{88,89}Kr isotopes have been investigated in the interacting boson model (IBM-2), including the interacting boson-fermion model (IBFM-2), as a first application of the model. In the calculation, the fermions are allowed to excite to the $2d_{5/2}, 1g_{7/2}, 2d_{3/2}, 3s_{1/2}$, and $1h_{11/2}$ single-particle orbitals. The results of the model calculation have been found to be in good agreement with both shell-model and available experimental data. Also the strong staggering of the yrast band has been found to be well described.

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I. INTRODUCTION

The nuclei in the vicinity of the waiting-point ^{78}Ni isotope present interesting aspects of nuclear structure, namely, the understanding of the mechanism of single-particle excitations. They have been extensively investigated both theoretically and experimentally [1–11]. Due to their nearness to the doubly closed shells (28, 50) and $Z = 38$ and 40 and $N = 56$ subshell closures, much interest has continued to be shown in the $28 < Z < 50$ nuclei. The various nuclear models have allowed researchers to make reasonable predictions of nuclear structure within small regions of nuclei and to calculate some nuclear qualities. Among the most successful of these models is the shell model which finds applicability near closed shell nuclei [12–18]. The Ge, Se, and Kr isotopes having 4, 6, and 8 protons outside the full $1f_{7/2}$ shell display some unexpected features. The observed low-lying states and $B(E2)$ do not exhibit features of either good vibrators or good rotors. Recently, for ^{70,72,74,76}Ge isotopes, shell-model calculations have been performed within the $p_{3/2}, p_{1/2}, f_{5/2}, g_{9/2}$ orbital space for both protons and neutrons [19,20]. The energy levels and electromagnetic transitions for Ge isotope chains have been discussed within the interacting boson approximation (IBA-2) model [21]. In the experimental work of Jones *et al.*, they identified the level schemes of $^{84-88}$Se isotopes [22]. On the theoretical side, Speidel *et al.* [23] have attempted a description of the nuclear structure of the $^{74-82}$Se isotopes using the interacting boson model (IBM-2), and newer calculations within the shell model have been performed by Srivastava and Ermamatov [24] for $^{78-84}$Se isotopes. Recently, large-scale shell-model calculations have been undertaken by Sieja *et al.* [25] in the vicinity of ^{78}Ni, where the energy levels and transition rates of $N = 52$–54 isotopes with $Z = 30$–36 have been investigated. The low-lying levels have been identified in $^{88-92}$Kr isotopes from investigations of β-decay properties of neutron-rich Br isotopes [26–28]. Many excited states in even-even Kr isotopes have been identified from the spontaneous fission of ^{248}Cm [29].

The aim of the present study is to apply for the first time the interacting boson model to ^{84,85}Ge, ^{86,87}Se, and ^{88,89}Kr isotopes. The results will be compared with recent experimental data [17,18,25,30].

II. MODEL

The IBM-2 Hamiltonian [31] can be expressed as

$$H_B = \varepsilon_d(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa_{\pi\nu}\hat{Q}_\pi\hat{Q}_\nu + \sum_{\rho=\pi,\nu}\hat{V}_{\rho\rho} + \hat{M}_{\pi\nu}, \quad (1)$$

where ε_d is the single d-boson energy, $n_{d\pi}$ and $n_{d\nu}$ are the number operators of the proton and neutron d bosons, respectively. $\kappa_{\pi\nu}Q_\pi Q_\nu$ is the quadruple interaction between proton and neutron bosons.The quadruple operator is expressed as

$$\hat{Q}_\rho = d_\rho^\dagger s_\rho + s_\rho^\dagger\tilde{d}_\rho + \chi_\rho[d_\rho^\dagger\tilde{d}_\rho]^{(2)}, \quad (2)$$

The $V_{\rho\rho}$ term represents the interaction between like bosons and is given by

$$\hat{V}_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_\rho^{(L)}\left([d_\rho^\dagger d_\rho^\dagger]^{(L)}[\tilde{d}_\rho\tilde{d}_\rho]^{(L)}\right), \quad (3)$$

where $\rho = \pi,\nu$. The last term in Eq. (1) is the Majorana operator which shifts the mixed proton-neutron symmetry states with respect to totally symmetric ones, where

$$\hat{M}_{\pi\nu} = \frac{1}{2}\xi_2[(d_\nu^\dagger s_\pi^\dagger - d_\pi^\dagger s_\nu^\dagger)(\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)] + \sum_{k=1,3}\xi_k[d_\nu^\dagger d_\pi^\dagger]^{(k)}[\tilde{d}_\pi\tilde{d}_\nu]^{(k)}. \quad (4)$$

By using the model wave function obtained by diagonalization of the IBM-2 Hamiltonian, the electromagnetic transition probabilities can be calculated. The $E2$ transition operator is expressed [32] as

$$T^{E2} = e_\pi Q_\pi + e_\nu Q_\nu, \quad (5)$$

where the quadrupole operators Q_ρ has the same definition as in Hamiltonian (2), and e_π and e_ν are the proton and neutron

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