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Triaxiality And Quadrupole Interactions Effect On The Structure Of $Gd^{154-158}$ Isotopes using interacting boson model

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ABSTRACT

The energy spectra, electric quadrupole transition probability and branching ratios of even-even Gd $^{154-158}$ nuclei were studied by using interacting boson model (IBM). The study of influence of cubic [d x d x d] and quadrupole [Q×Q×Q] interactions on the nuclear structure of these nuclei are undertaken. The analysis of the obtained results and the parameter values predict that these nuclei have large deformed nuclei properties. A good agreement between theoretical and available experimental results is obtained.

INTRODUCTION

The nuclear properties of even - even nuclei in the interacting boson model (Arima and Iachell, 1976,1978,1979) are obtained from a system of a fixed number of bosons. The neutron (proton) boson in this model is identified with a pair of valence neutrons (protons) outside the nearest closed shells. The bosons may have angular momentum L=0 (s-boson) and L=2 (d-boson). The interactions between substates of single state of s-boson and five substates of d-bosons lead to a group structure in six dimensions U(6) , which is broken to three dynamical symmetries U(5),S(3) and O(6) corresponding to the geometrical idea ; spherical vibrator , deformed rotor and Y-soft respectively (Lipas, 1984).

which gives a good agreement between the experimental and the model results (Barrett it al . 1983;Warner and Casten 1985; Warner 1991 ; Harder and Krusche 1994).

In the present work energy levels, B (E2) and branching ratios in $Gd^{154-158}$ isotopes are studied first, and most important the influence of cubic and quadrupole interactions on these properties are studied as well.

MODEL

In calculating the energy levels we use the Hamiltonian which is written in the language of second quantization (s, s+, d, d+), where s, s+ and d, d+ are the annihilation and creation operators of s and d-boson respectively. The general form within one body and tow body terms is written (Chianng et.al 1985):

$$H=E_s(s^{+}.s)+E_d(d^{+}.d)+\sum_{L=0,2,4}1/2\ (2L+1)^{1/2}C_L\{[d^{+}xd^{+}]^{(L)}x[dxd]^{(L)}\}^{(0)}$$

$$+1/\sqrt{2}\,V_2\{[d^{+}xd^{+}]^{(2)}x[dxs]^{(2)}+[d^{+}xs^{+}]^{(2)}x[dxd]^{(2)}\}^{(0)}$$

$$+1/2\ v_0\{[dxd^{+}]^{(0)}x[dxs]^{(0)}\quad +[s^{+}xs^{+}]^{(0)}x[dxd]^{(0)}\}^{(0)}$$

$$+U_2\{[d^{+}xs^{+}]^{(2)}x[dxs]^{(2)}\}^{(0)}\quad +U_0\{[s^{+}xs^{+}]^{(0)}x[sxs]^{(0)}\}^{(0)}\qquad \text{-----------------(1)}$$

Another form often used which is (Casten and Warner 1988):

$$H = E_d\,n_d + a_0\,P.P + a_1\,L.L + a_2\,Q.Q + a_3\,T_3.T_3 + a_4\,T_4.T_4 \qquad \text{-------------(2)}$$

Where the coefficients a_i (I=0----4) are a linear components for the coefficients in equation (1), and;

$n_d=(d^{+}.d)$: the number of d-boson operator

$P=(d.d)+1/2(s.s)$: the paring operator

$L = \sqrt{10}\,[d'xd']^{(1)}$: the angular momentum operator

$Q = [d'xs+s'xd]^{(2)} - X[d'xd']^{(2)}$: the quadrpouole operator

$T_l = [d'xd']^{(l)}(l=3,4)$: the octopole and hexadropole operator

The electric quadrupole transition operator in IBM has the form

$$T(E2) = \alpha\,[d'.s+s'.d]^{(2)} - \beta\,[d'.d]^{(2)} \qquad \text{----------------(3)}$$

Where α and β are two parameters one can vary them to fit the experimental data

THE RESULTS

Choice Of The Parameters

The Gd-isotopes A=154 to A=158 with Z=64 to 98 have a number of boson which vary form 11 to 13 respectively, where the closed shells z=50 and N=82 are taken as the core. This number gives a good range to study the energy levels because it generate a large system space. The values of the parameters in eq. (2) which are used in this calculations are chosen to fit the experimental results of low lying states . After several iterations we found the best values of the parameters as shown in Table(1).

The Energy Levels

The theoretical and experimental (Mitsuo 1984) energy levels of Gd-154 nucleus are given in fig.(1) , where the levels $I<10^+$ have been grouped into different bands. From this fig. It could be seen that a good agreement in ground state, beta and gamma-bands. The energy and deformation parameters $<\beta>$ and $<\gamma>$of 2^+_2 state show that this state is predominantly $K^+=0^+$ Band and 2^+_3 state mainly $K^+=2^+$ band. Further more the 0^+_2 state lie lower than 2^+_2 state (beta head band lower gamma head band).

The K- characters and deformations parameters of the bands comply with the known level spectrum of SU (3)limit. Of IBM (Warner and Casten 1982).

Fig.(1) also includes the $\beta\beta$ and $\gamma\gamma$ -bands, where 0_3 ($\beta\beta$-head band)have (1.295,1.242)MeV in IBM and Exp. . Results, while 4_4 is a member of same the band which has (1.698,1.595) MeV in IBM and Exp. results respectively. The 2_5 state ($\gamma\gamma$-head band)has (1.531,1.489) MeV in IBM and Exp. results. At the same time ,3_2 which belongs to the same band has 1.661 and 1.614 MeV in IBM and Exp. results. These energy values show the ability of the IBM to fit this band. Finally, the energy ratio $E4_1/E2_1$ equal to 3.31 and 3.01 in IBM and Exp. Results respectively which is due to the rotational properties of this nucleus (Warner and Casten 1988).

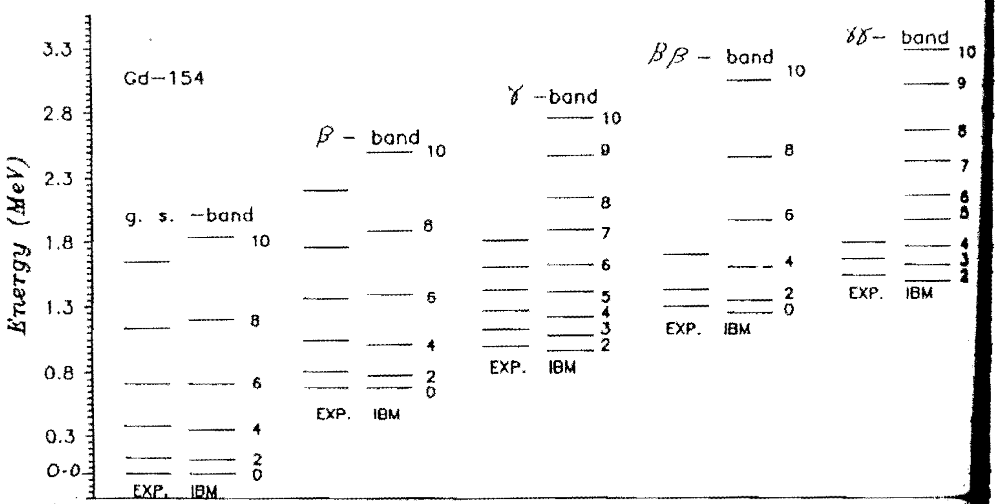

Fig.(1) Comparison of IBM calculations with experimental results of Gd-15 nucleus

Fig.(2) shows the comparison between the Exp. and calculated energy levels of Gd-156 nucleus. In general, there is a good agreement between experimental and model result for low-lying states in ground state beta and gamma-bands. However a poor agreement was obtained for odd higher spin, the different energy between IBM and Exp. results which varies from 0.1 to 0.4 MeV in gamma band .From this fig . One can see two columns in $\beta\beta$-band region which represent, First Exp. data for $\beta\beta$-band, where 0^+ (1.168),2^+(1.258) and 4^+ (1.4627) MeV , second :Exp*. data for $\beta\beta\beta$ -band 0^+ (1.715),2^+(1.779).Meanwhile, the IBM column 0_3^+(1.675),2_4^+(1.769) and 4_4^+(1.980) MeV etc. from comparison between theoretical and experimental results for tow bands one can conclude that the IBM results are in harmony with $\beta\beta\beta$-band , but it is not able to determine the $\beta\beta$-band . This result reflects that the $\beta\beta$-band is not collective or , perhaps , it is thought to be outside that model space (Mixed symmetry band outside one filed of boson system). The last region in this fig. represents the $\gamma\gamma$-band , where 2_5^+(1.827,1.907)MeV 3_2^+ (1.916,2.008) and 4_5^+ (2.020,2.135) MeV in IBM and Exp. results respectively where these states are lower members of this band . From the above values one can see the identification between IBM and Exp. results, the energy ratio $E4_1/E2_1$ equal to 3.318 and 3.236 in IBM and Exp. result respectively.

The calculated energies of the excitation states in Gd^{158} below 3.5 MeV are compared with experiment results as shown in fig (3). The results of ground state band are give correctly, but beta head and gamma head-bands lied lower than 0.1 MeV. In this nucleus after the analysis of 2_2^+ and 2_3^+states's properties, we found that 2_2^+ state becomes gamma head band and 2_3^+ is a member of beta band.

The energy of 0_2^+ (beta head band) is equal to (1.196,1.058)MeV while 2_2^+ (1.107,1.132)MeV and 2_3^+ (1.259,1.244) MeV in Exp. and IBM.

Results. The 0_3-head band in IBM result nearly to β β β than β β - band . This fig. also included the γ γ-band states . Finally, the energy ratio $E4_1^+/E2_1^+$ is found to be equal to 3.32 and 3.28 in IBM an Exp. results respectively . From these results , we can get the necessarily change of the parameter values range , whenever we study the higher Gd-isotopes.

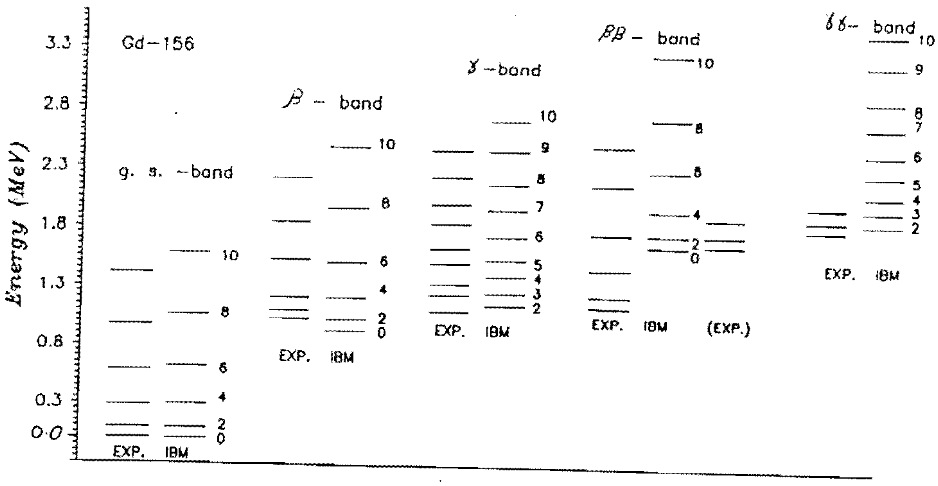

Fig.(2) Comparison of IBM calculations with experimental results of Gd-156 nucleus.

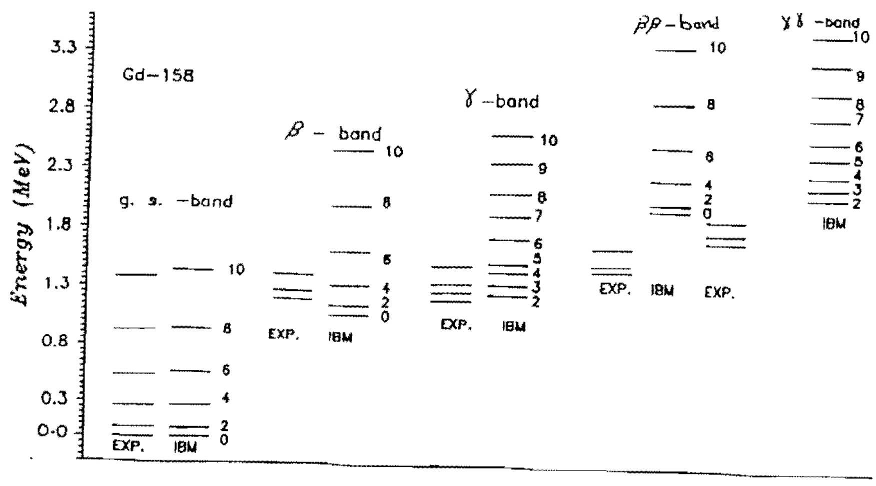

Fig.(3) Comparison of IBM calculations with experimental results of Gd-158 nucleus.

Triaxiality

In this section the influence of the cubic and quadrupole interactions of Gd-isotopes are studied, a modified Haimiltonian system becomes

$$H_{sdC} = H_{sd} + H_C$$

Where H_C is a cubic Haimiltonian (Heyde 1993):

$$H_C = \sum \Theta_3 [d^\dagger d^\dagger d^\dagger]^{(3)} . [ddd]^{(3)}$$

And

$$H_{sdCQ} = H_{sd} + H_C + H_Q$$

Where H_Q is a quadrupole interactions Haimiltonian after using cubic interactions

$$H_Q = V_{QQQ}[QxQxQ]$$

Where Θ_3, V_{QQQ} and H_{sd} represent the cubic interaction parameter , the quadrupole-quadrupole-quadrupole parameter interaction and the Haimiltonian in eq. (2) respectively.

Fig. (4a) shows the influence of H_Q and H_c terms on the ground states band levels for Gd ^{154} isotope. At θ_3 =0.01 only (H_{sdC} uses) the values of energy difference are equal to 0.0002 --- 0.001 MeV , which is very small in comparison with energy values. In using the H_{sdCQ} Haimiltonian (θ_3=0.01 and V_{QQQ} = 0.005) there is an increase change in energy equal to 0.01 ---0.15 MeV. From fig. (4b) one can see the effect of H_c and H_Q terms on the energy of β -band , when H_C is used the energy of any level becomes lower by 0.1 ---0.04 MeV to ones that are in used H_{sd} Hamiltonain . At the same time , if we use H_{sdCQ} Hamiltonain the energy up to 0.2---0.3 MeV for any level , although , the theoretical results are in harmony with experimental data. From fig.(4c) it is clearly seen that the γ - band levels are affected :where the energy decreases 0.1---0.02 MeV in using H_{sdC} Hamiltonain and increases 0.04-0.2 MeV by using H_{sdCQ} Hamiltonain . The $\beta\beta$ -band levels effect is shown in fig. (4d) where :the energies increase is equal 0.25 -0.4 MeV in using H_{sdCQ} Hamiltonain, Finally, from these figures one can conclude that large effect results from cubic and quadrupole interactions in high-energy bands. Figs (5a-5d) show the effect of cubic and quadrupole interactions of the energy bands in the Gd ^{156} isotope. From these figs .it could be seen that the exact effect was in Gd ^{154} nucleus. Fig.(6a) show the influence of H_c and H_Q terms on the ground state band levels of Gd ^{158} isotope . However the β-band levels were effected by the use of these terms. For example the, 0_2^+ energy level increase 0.35 MeV as well as the 2_3^+have the same effect for 0_2^+ level. From fig. (6c) one can see the large effect of cubic interactions on the γ-band specially, the odd angular momentum. At the same time, the V_{QQQ} term effect was observed on the high energy levels of this band. Briefly ,the H_{sdCQ} results reflect the importance of these interactions in high bands energy.

B (E2) And The Branching Ratios

The calculation of the E2 transition probabilities requires the use of eq. (3). The α and β values used for Gd-isotopes were determined by fitting the experimental data of B(E2), 2_1 -----> 0_1),as shown in Table (1) . Table (2) shows some B(E2) values for transitions . From this Table one can find a good agreement between IBM and experimental (Kumar et al. 1979; Gupta et al.1980) results for transitions within ground state band for Gd$^{154-158}$.Moreover, this Table includes some transitions for beta band levels to ground state band levels .

In table [3] some B(E2) branching ratios for Gd-isotopes are tabulate. The B(E2) ratio 2_γ ------> $0_g/2_\gamma$ ------->2_g has a small values in all isotopes ,which means 2_γ predominantly decays in to 2_g state. This qualitative behavior is well reproduced by experiment data for deformed nuclei .The B(E2) ratio 2_γ ------>$0_\beta/2_\gamma$ ------>0_g increases with increasing of the neutron number. The ratio $3_\gamma \rightarrow 2_g / 3_\gamma \rightarrow 2_\gamma$ is small, this means that the rotational transition to the 2_γ state is preferred. The ratio $4_\gamma \rightarrow 2_g / 4_\gamma \rightarrow 2_\gamma$ is very small and shows that the 4_3 state is really a vibrational state. it belongs clearly to the 2_γ band. Finally, the B(E2) ratio $2_\beta \rightarrow 0_g / 2_\gamma \rightarrow 0_g$ is equal to 00.1428 and 0.1203 in the Gd^{154} and Gd^{156}, while this ratio is equal to 7.407 in Gd^{158} when H_{sd} Hamiltonian is used. This means that the cubic interactions must be incorporated in determining beta band energies, because this ratio is close to the value of 1/6 predicted in SU(3) limit (Warner and Casten 1988).

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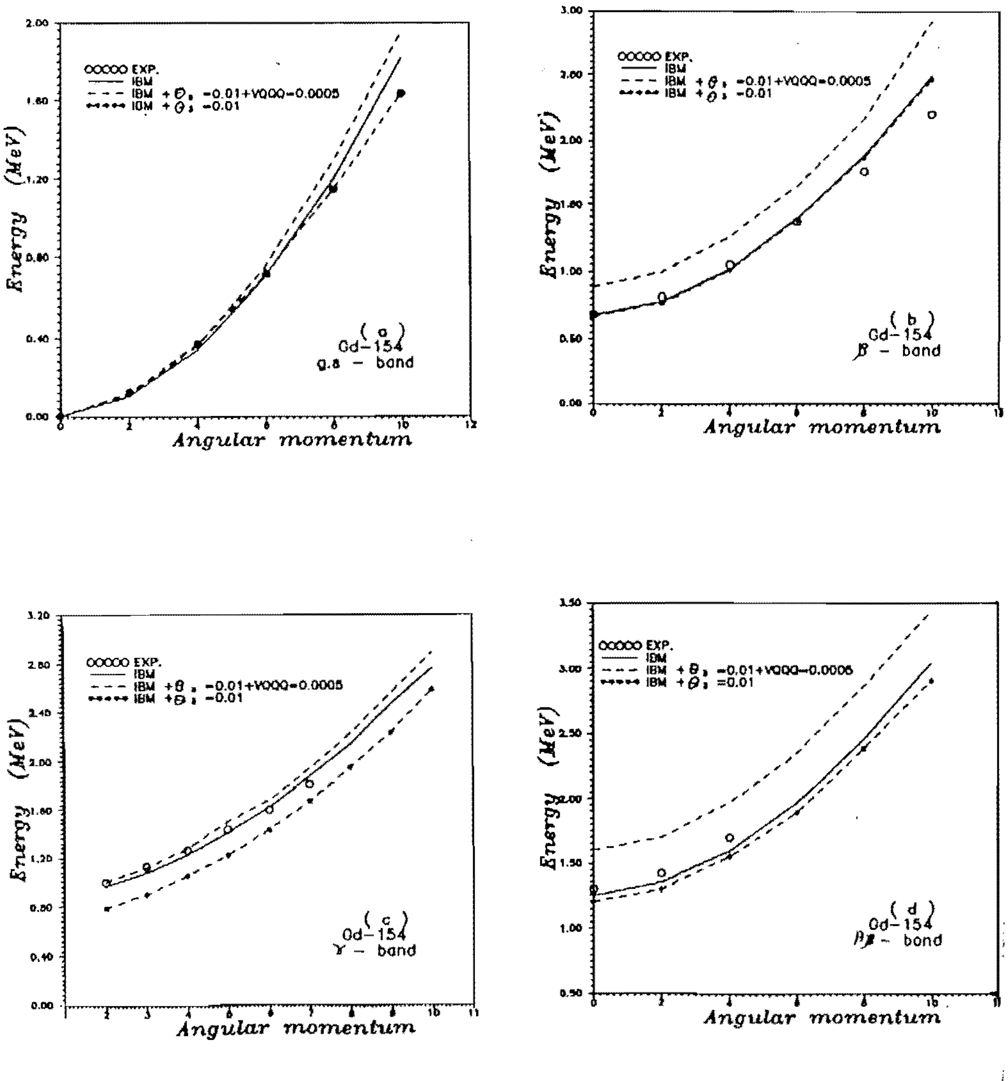

Fig.(4a-4d) The change in energy levels in the bands of Gd-154 as a function of angular momentum.

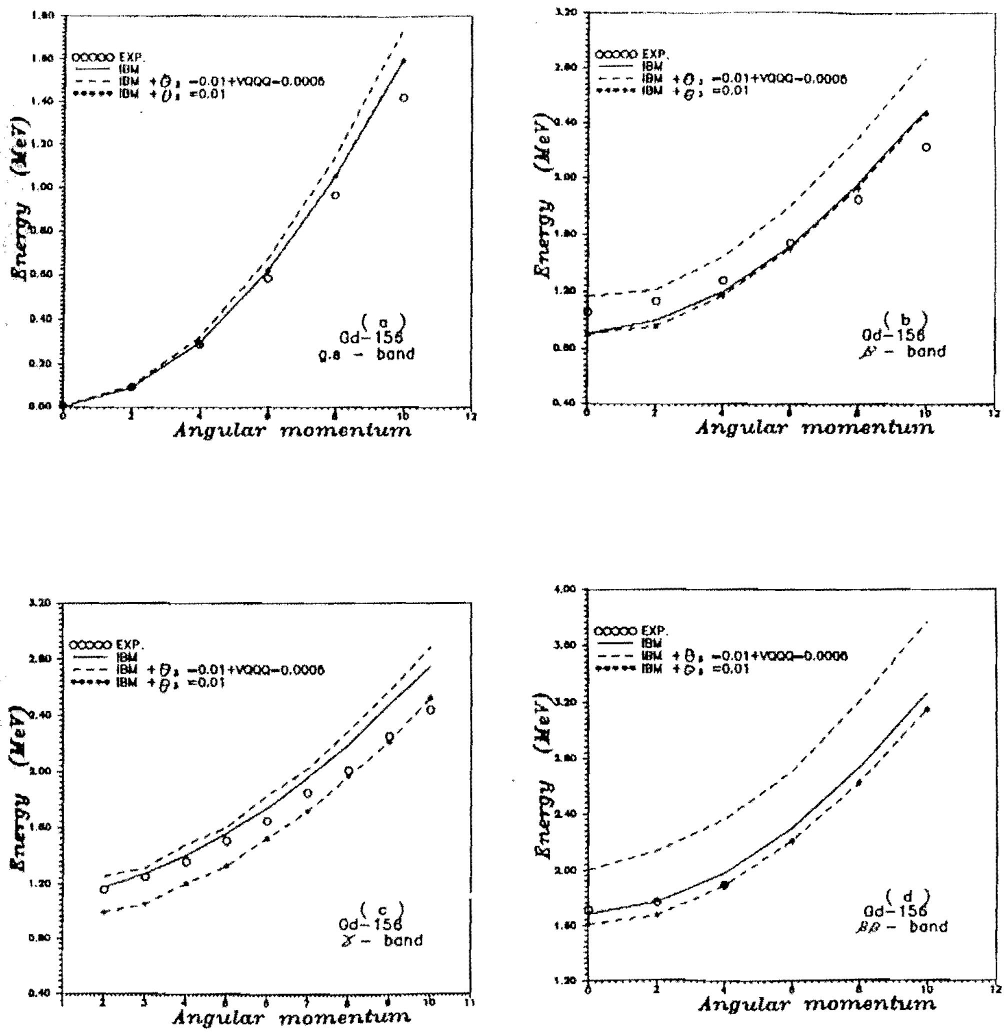

Fig.(5a-5d) The change in energy levels in the bands of Gd-156 as a function of angular momentum.

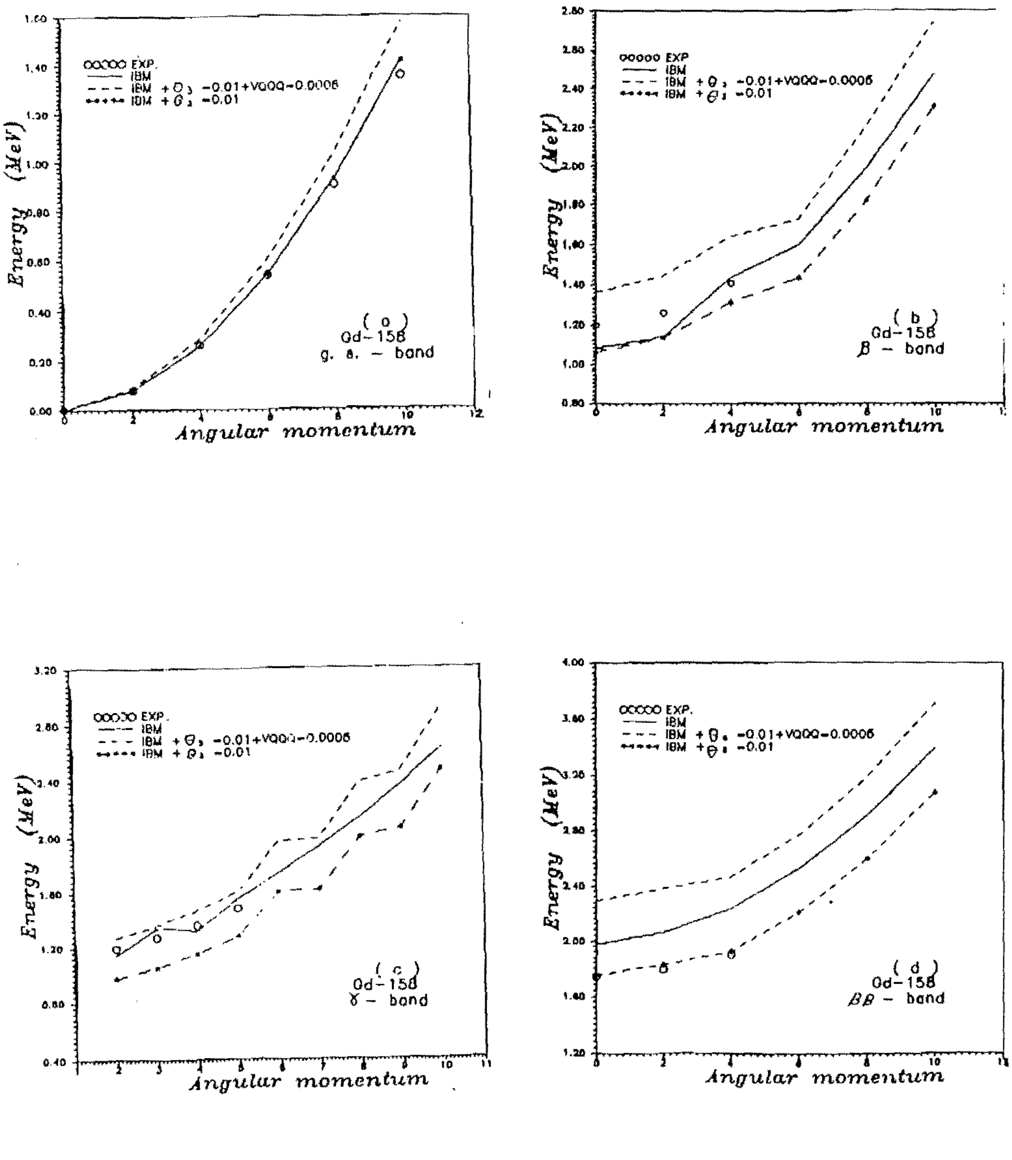

Fig.(6a-6d) The change in energy levels in the bands of Gd-158 as a function of angular momentum.

Table [1] best parameter values using in this work.

A	E_d	a_0	a_1	a_2	a_3	a_4	X	α	β	V_{QQQ}	θ_3
154	0.0	0.0	0.01	-0.011	0.03	0.0	-1.323	0.16	-0.037	0.0005	0.01
156	0.0	0.0	0.007	-0.013	0.026	0.0	-1.223	0.16	-0.037	0.0005	0.01
158	0.0	0.0	0.006	-0.014	0.014	0.0	-1.323	0.16	-0.037	0.0005	0.01

Table [2] B(E2) values of $Gd^{154-158}$ isotopes given in $e^2.b^2$

		Gd-145		Gd-156		Gd-158	
I_i	I_f	Exp.	IBM	Exp.	IBM	Exp.	IBM
2_g	0_g	0.773	0.7778	0.928	0.9102	1.004	1.0516
4_g	2_g	1.178	1.0962		1.2840		1.4842
6_g	4_g	1.390	1.1765	1.970	1.3812	2.120	1.5986
8_g	6_g		1.1823	2.170	1.3941	2.320	1.6184
0_β	2_g	0.210	0.0219		0.0250		0.0284
2_β	0_g	0.0048	0.0038		0.0042		0.0047
2	2_g	0.4000	0.0047		0.0058		0.0059
4_β	2_g	0.0035	0.0048		0.0053		0.0053
4_β	4_g	0.0380	0.0030		0.0040		0.0059
6_β	4_g	0.0027	0.0043		0.0049		0.0056
3_γ	2_g		0.0590		0.0613		0.0613
3_γ	2_β		0.0099		0.0110		0.0535
3_γ	2_y		1.1317		1.3544		1.1587
3_γ	4_g		0.0466		0.0418		0.0360
4_γ	2_γ		0.3988		0.4620		0.5330
5_γ	3_γ		0.6202		0.7288		0.8439
6_γ	4_γ		0.7561		0.8852		1.0227

Table [3] Branching ratios values of Gd [154-158] isotopes

		Gd-154		Gd-156		Gd-158	
I_1	I_f/I_f	Exp.	IBM	Exp.	IBM	Exp.	IBM
2_γ	$0_g/2_g$	0.460	0.489		0.538		0.681
2_γ	$0\ /0_g$	0.141	0.190		0.189		0.006
2_γ	$2_\beta/2_g$	1.030	0.262		0.239		2.043
3_γ	$2_\beta/2_\gamma$		0.052		0.045		0.038
4_γ	$2_\gamma/4_\beta$		0.026		0.028		0.029
5_γ	$4_g/6_g$		0.740		0.771		0.961
6_β	$4_g/6_g$		2.300		1.750		1.400

المستخلص

لقد تم دراسة مستويات الطاقة و احتمالية الانتقالات رباعية القطب الكهربائي و نسب التفرع لنظائر Gadolinium التي تحتوي على ١٥٤،١٥٦و١٥٨ نيكلون باستخدام نموذج البوزونات المتفاعلة. وقد تم في هذه الدراسة التركيز على تأثير إضافة حدود جديدة إلى هاملتوني النظام تمثل توسع في مدى التفاعلات بين البوزونات .ومن خلال النتائج المستحصلة تبين أهمية مثل هذه التفاعلات في تحديد خواص هذه الانوية. وعند تحليل نتائج هذا النموذج إضافة إلى قيم المعاملات المستخدمة تبين امتلاك هذه الانوية إلى خصائص الانوية دائمة التشوه. وقد وجد تطابقا جيدا بين نتائج هذا النموذج مع القيم العملية المتوفرة.