## Detailed Description of Mixed Symmetry States in $^{94}\mathrm{Mo}$ Using Interacting Boson Model\*

F.H. Al-Khudair<sup>1</sup> and LONG Gui-Lu<sup>1,2,3,4</sup>

<sup>1</sup>Department of Physics, Tsinghua University, Beijing 100084, China

<sup>2</sup>Institute of Theoretical Physics, the Chinese Academy of Sciences, Beijing 100080, China

 $^3\mathrm{Key}$  Laboratory for Quantum Information and Measurements, MOE 100084, China

<sup>4</sup>Center of Atomic and Molecular NanoSciences, Tsinghua University, Beijing 100084, China

(Received December 6, 2001)

**Abstract** We have investigated the low-lying collective states and electromagnetic transitions in <sup>94</sup>Mo within the framework of the interacting boson model. The influence of model parameters on the energy levels and electromagnetic properties has been investigated. The analysis of the obtained results and the parameter values predict that the  $2_3^+$  state is the lowest mixed symmetry state with pure  $F = F_{\text{max}} - 1$  in this nucleus. The calculated results predicate that the  $2_5^+$  (two-Q-phonon) mixed symmetry state is closed to the  $J = 2^+$  at 2.870 MeV in the experimental data, and the 2.965 MeV state is the lowest mixed symmetry with  $J = 3^+$ .

**PACS numbers:** 21.60.Fw, 27.60.+j

Key words: interacting boson model, mixed symmetry states, <sup>94</sup>Mo nucleus

## 1 Introduction

The interacting boson model (IBM) has been applied to the description of quadruple collective phenomena observed in the even-even medium heavy nuclei.<sup>[1-3]</sup> In the simplest version of this model (IBM-1), the nuclear properties of nuclei are obtained from a system of a fixed number of boson, and in this version no distinction is made between proton boson and neutron boson, therefore all states in IBM-1 are symmetric.<sup>[4]</sup> Bosons are represented by creation and destruction operators  $(s^{\dagger}, s)$  for s-boson and  $(d^{\dagger}, d)$  for d-boson. The states in IBM-2 include all the symmetry states of the IBM-1 as well as the mixed symmetry states which are outside the IBM-1 spaces, i.e., states are not completely symmetric with respect to the proton-neutron boson exchange. An important property of this new version is that the proton-neutron symmetry character of each state is specified in terms of a new quantum number called F-spin. The F-spin raising, lowering, and Z-component operators given in Ref. [5] are

$$F_{+} = s_{\pi}^{\dagger} s_{\nu} + (d_{\pi}^{\dagger} \cdot \tilde{d}_{\nu}) ,$$
  

$$F_{-} = s_{\nu}^{\dagger} s_{\pi} + (d_{\nu}^{\dagger} \cdot \tilde{d}_{\pi}) ,$$
  

$$F_{z} = (N_{\pi} - N_{\nu})/2 . \qquad (1)$$

For the proton boson,  $F_z = 1/2$ , while for the neutron boson  $F_z = -1/2$ . The two kinds of bosons form an *F*-spin multiplet  $|\pi\rangle = |1/2, 1/2\rangle$  and  $|\nu\rangle = |1/2, -1/2\rangle$ . The total symmetric states have the maximum value of *F*-spin  $F_{\text{max}} = (N_{\pi} + N_{\nu})/2$  while the mixed symmetry states characterized by decreasing *F*-spin value (*F* =  $F_{\text{max}} - 1, F_{\text{max}} - 2, \dots, F_{\text{min}} = |N_{\pi} - N_{\nu}|/2)$ . There are U(5), SU(3), and O(6) dynamical symmetries in this model, which correspond to spherical vibrator, deformed rotor and  $\gamma$ -unstable motion respectively.<sup>[9]</sup>

The usual form of the IBM-2 Hamiltonian used in most calculations can be written as  $^{[6]}$ 

$$H = \epsilon (n_{d\pi} + n_{d\nu}) + \kappa_{\pi\nu} Q_{\pi} \cdot Q_{\nu} + \sum_{\rho = \pi, \nu} \kappa_{\rho\rho} Q_{\rho} \cdot Q_{\rho} + \sum_{\rho = \pi, \nu} V_{\rho\rho} + M_{\pi\nu}$$
(2)  
$$+ \sum_{L=0}^{4} G_{\pi\nu}^{(L)} ([d^{\dagger}\tilde{d}]_{\pi}^{(L)} [d^{\dagger}\tilde{d}]_{\nu}^{(L)})$$

where  $\epsilon$  is the single-boson excitation energy,  $n_{d\pi}$  the number operator of proton *d*-boson, and  $n_{d\nu}$  the number operator of neutron *d*-boson.  $\kappa_{\pi\nu}Q_{\pi} \cdot Q_{\nu}$  is the quadruple interaction between proton and neutron bosons,  $\kappa_{\rho\rho}Q_{\rho} \cdot Q_{\rho}$ are the quadruple interactions between like-bosons, where  $Q_{\rho}$  quadruple operator is given by the usual expression

$$Q_{\rho} = (s_{\rho}^{\dagger} \vec{d}_{\rho} + s^{\dagger} d_{\rho}^{\dagger})^2 + \chi_{\rho} (d^{\dagger} \vec{d}\,)^2 \,, \tag{3}$$

 $\operatorname{and}$ 

$$M_{\pi\nu} = \xi_2 (d_{\nu}^{\dagger} s_{\pi}^{\dagger} - d_{\pi}^{\dagger} s_{\nu}^{\dagger}) (\tilde{d}_{\nu} s_{\pi} - \tilde{d}_{\pi} s_{\nu}) + \frac{1}{2} \sum_{k=1,3} \xi_k [d_{\nu}^{\dagger} d_{\pi}^{\dagger}]^{(k)} [\tilde{d}_{\nu} \tilde{d}_{\pi}]^k$$
(4)

is the Majorana operator, and it separates the full symmetric states from mixed symmetry states. The  $V_{\rho\rho}$  rep-

<sup>\*</sup>The project supported in part by National Natural Science Foundation of China under Grant No. 10047001 and Major State Basic Research Development Program under Contract No. G200077400