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Structure of $2p_{3/2}$ to $1f_{5/2}$ Light Germanium Isotopes in the Framework of the IBM-3

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Abstract

The nuclear level structure of Ge^{64-70} is analyzed taking into account the experimental information available, with respect to symmetry of IBM-3. The level energies, electric quadrupole reduced transition probability B(E2), magnetic dipole reduced transition probability B(M1), interband B(E2) ratios and multipole mixing ratios $\delta(E2/M1)$are compared with available data. The adopted level schemes and the mixed symmetry states properties are discussed in the framework of isospin symmetry.

Keywords:
Energy levels, mixed symmetry states, transitions probability, mixing ratio, interacting boson model (IBM-3)

Introduction

The Ge element has 32 protons and 40 isotopes with neutron number starting from 29 in Ge^{61} with half-life 40 ms. Five of those isotopes are stable with an enrichment varied from 7.61%, for Ge^{76}, to 36.28%, for Ge^{74}. The interest of this work is the even-even isotopes with A=64-70, due to the existence of the experimental data to compare with and the restriction of the bosons space of the model used (no more than a total of 7 bosons). For the other isotopes, in which the neutron number pass mid shell, the bosons are counted as the one-half number of the neutron holes, and this is not accepted in the model we used. An interesting feature in Ge isotopes is that the nucleus with N=Z does not exist in nature, while nuclei with N-Z= 6 to 16 are the most stable.Nuclei in the region of Ge, Se and Kr are situated between both of the proton and neutron shell-closures of 28 and 50 and were used to consider as being near spherical, so that their structure may be described by vibrational models, at least in the low energy region. Many experiments and theoretical works were performed on the nuclei in this region and found that the low lying level structure of those nuclei is not a simple vibrator [1-4]. One of the peculiarities is the existence of the unusually low-lying excited 0^+ state, just above and just below the first excited 2^+ state, which can not be understood simply as the 0^+ member of the two- phonon triplet (0^+, 2^+ and 4^+) states. This explained [5] as a rotational band member built on the excited 0^+ state which coexists with the vibrational band members built on the ground state in the same nucleus. The deformation in the Germanium nuclei seems to be true due to the existence of the low lying positive and negative parity states of $I = 1$ and 3 in some isotopes. This means that there is a coexistence of spherical and deformed state in the nuclei in this region supported by the existence of the electric quadruple moment of the first excited 2^+ state. The experimental data from references [6-7] has been taken as evidence of the coexistence of two different shapes, vibrational and rotational, and there is a shape transition between them [8]. Investigation of the even mass Ge isotopes by means of the interacting boson model with fermium pair model has been done by Hsieh et.al.[9]. They took Ni^{56} nucleus as a core for their study and counting boson numbers and then assumed that one of the bosons can be broken to form a fermions pair which may occupy the $f_{5/2}$ or $g_{9/2}$ orbital. In this study a suggestion was made, that the complex shape coexistence is in the Ge^{68} nucleus. More complicity in the structure of these nuclei appears when the reduced transition probabilities were studied. It is found that, in spite of the fact that energies of $0_2^+, 2_2^+$ and 4_1^+ in some isotopes support a vibrational character, the B(E2) value and their ratios do not justify such an interpretation.

In the present work the IBM-3 was used to calculate the energy levels and other nuclear properties of Ge isotopes from neutron number 32 to 38.This is a restricted choice, because this model is relevant to lighter nuclei in which neutron and proton fill the same set of shell, $2p_{3/2}$ for protons and $2p_{3/2}$ to $1f_{7/2}$ for neutrons, which is just above the major closed shell at magic number 28.

The Model Hamiltonian

In the early version of the Interacting Boson Approximation Model (IBA), or (IBM-1), were there are no distinction made between proton and neutron bosons, and number of bosons taken to be the number of nucleons outside the closed shell divided by two, and the most general Hamiltonian written as [10]

$$H = \varepsilon n_d + a_0 P.P + a_1 L.L + a_2 Q.Q + a_3 T.T + a_4 T.T \ \ldots\ldots \qquad (1)$$

where n_d is the d-boson number operator, P and Q represent pairing and quadrupole operators which are written in the language of second quantization (s, s^+,d,d^+), where s, s^+, d, d^+ are the annihilation and creation operators of s and d-bosons respectively as

$$Q = (s^\dagger \tilde{d} + d^\dagger \tilde{s})^{(2)} + \chi (d^\dagger \tilde{d})^{(2)}, \ P = \frac{1}{2}(\tilde{d}.\tilde{d} + \tilde{s}.\tilde{s}) \qquad (2)$$

and L and T are given by

$$L = \sqrt{10}(d^\dagger \tilde{d})^{(1)} \qquad , T_l = (d^\dagger \tilde{d})^{(l)}, l = 3,4$$

The model space states consistence only the fully symmetric F-spin states.

The IBM-2 version [11], which statuses that: for a given nucleus, the number N_ν and N_π is found by counting neutrons and protons from the nearest closed shell. The model Hamiltonian is

$$H = \varepsilon_d (n_{d\nu} + n_{d\pi}) + \kappa (Q_\nu . Q_\pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \qquad (3)$$

wher Q is

$$Q_\rho = [d_\rho{}^\dagger s_\rho + s_\rho{}^\dagger d_\rho]^{(2)} + \chi_\rho [d_\rho{}^\dagger d_\rho]^{(2)} \qquad (4)$$

The terms $V_{\pi\pi}$ and $V_{\nu\nu}$ which correspond to interaction between like-boson, are sometimes included in order to improve the fit to experimental energy spectra.

The Majorana term, $M_{\nu\pi}$, which contains three parameters ξ_1, ξ_2 and ξ_3 may be written as

$$M_{\nu\pi} = \frac{1}{2}\xi_2 ([s_\nu^\dagger d_\pi^\dagger - d_\nu^\dagger s_\pi^\dagger]^{(2)}.[s_\nu d_\pi - d_\nu s_\pi]^{(2)}) - \sum_{k=1,3} \xi_k ([d_\nu^\dagger d_\pi^\dagger]^{(k)}.[d_\nu d_\pi]^{(k)}). \qquad (5)$$

The Majorana term played a great role in producing the M1 matrix elements and the mixed symmetry states.

In the present work the IBM-3 Hamiltonian has been used to produce the energy levels and the transition matrix elements. This model considers three types of bosons: proton-proton boson (π), neutron- neutron boson (ν), and proton-neutron bosons (δ). The π, ν and δ bosons are the three members of a T=1 triplet and their inclusion is necessary to obtain an isospin invariant formulation of the IBM. This means that the Hamiltonian does not depend only on the total number of boson N, also the isospins T as well. The model Hamiltonian is of the form [12-14]

$$H = \varepsilon_s \tilde{n}_s + \varepsilon_d \tilde{n}_d + H_2, \qquad (6)$$

where

$$H_2 = \frac{1}{2}\sum_{L_2T_2} C_{L_2T_2}((d^\dagger d^\dagger)^{L_2T_2}.(\tilde{d}\tilde{d})^{L_2T_2} + \frac{1}{2}\sum_{T_2} B_{0T_2}((s^\dagger s^\dagger)^{0T_2}.(\tilde{s}\tilde{s})^{0T_2})$$
$$+\sum_{T_2} A_{2T_2}((s^\dagger d^\dagger)^{2T_2}.(\tilde{d}\tilde{s})^{2T_2}) + \frac{1}{\sqrt{2}}\sum_{T_2} D_{2T_2}((s^\dagger d^\dagger)^{2T_2}.(\tilde{d}\tilde{d})^{2T_2})$$
$$+\frac{1}{2}\sum_{T_2} G_{0T_2}((s^\dagger s^\dagger)^{0T_2}.(\tilde{d}\tilde{d})^{0T_2}).$$

.

The product of angular momentum and isospin is

$$(b_1^\dagger b_2^\dagger)^{L_2T_2}.(b_3b_4)^{L_2T_2} = (-1)^{(L_2+T_2}\sqrt{(2L_2+1)(2T_2+1)}\left[(b_1^\dagger b_2^\dagger)^{L_2T_2}\times(\tilde{b}_3\tilde{b}_4)^{L_2T_2}\right]^{00} \quad (7)$$

where

$$\tilde{b}_{lm,m_z} = (-1)^{(L+m+m_z+1)} b_{l-m-m_z}$$

The symbols T_2 and L_2 represent the two- boson system of the isospin and angular momentum. The parameters A,B,C,D and G are the two-body matrix elements and they have been studied macroscopically by Evans et.al. [15] The parameters A_1, C_{11} and C_{31} are similar to those of Mjorana interaction parameters, ξ_1 ,ξ_2 and ξ_3, in the IBM-2, which have great effect on shifting the energy of the mixed symmetry states with respect with the symmetric states.The fitting parameters were chosen according to the microscopic studied of IBM-3 parameters in reference[15], which shows that the dependence of IBM-3 Hamiltonian on the isospin (T) value, as well as the boson number ($N=2+N_\nu$). The dependence on isospin is more dramatic than that on the boson number. These parameters were chosen according to Table-1.

Table-1. Boson number (N) and isospin (T) of the Ge Isotopes.

	Ge^{64}	Ge^{66}	Ge^{68}	Ge^{70}
N	4	5	6	7
T	0	1	2	3

The IBM-3 Hamiltonian contains sixteen parameters, all possibly function of T and N, so it is hard to find the best fit with experimental data unless one has to follow a guide line, which is in this case the shell model [13].

For the discussion of symmetry of the selected Ge isotopes the Hamiltonian was written in terms of a linear combination of the Casimir operator. So we can rewrite the Hamiltonian as[16]:

$$H_{Casimir} = \lambda C_{2U_{sd}(6)} + \alpha_T T(T+1) + \alpha_1 C_{1U_d(5)} + \alpha_2 C_{2SU_{sd}(3)} + \alpha_3 C_{2U_d(5)}$$
$$\alpha_4 C_{2O_{sd}(6)} + \alpha_5 C_{2O_d(5)} + \alpha_6 C_{2O_d(3)}, \quad (8)$$

where $\hat{C}_{nG}$ denotes the n^{th} order Casimir operator of algebra G. The IBM limits were used to locate the nuclei under consideration. The groups coefficients showed that these nuclei are more close to the U(5) limits and the transitional nuclei O(6). The λ parameter determines the position of the mixed symmetry states as well as the 1^+ state. The a_T parameter was fitted to relative position of $0^+_{T=2}$, i.e the shift between the T=0 ground state in the Z=N Ge^{64} and the first T=2 states. In our case we assumed, that the ground state energy of ^{64}Zn is equal to that of the IBM-3 $0^+_{T=2}$ state in ^{64}Ge. This supported by the following estimation: we estimate the energy of the isospin analogue to state in Zn^{64} by considering the binding energy difference of Ge^{64} and Zn^{64} and then subtracting the Coulomb energy difference. This approximation is rather crude, because Coulomb energy is sensitive which depends on the shape of the nucleus. By using the data in and the following Coulomb energy formula [17]:

$$E_{Coulmob} = 0.70\frac{Z^2}{A^{1/3}}(1-0.76Z^{-2/3}), \quad (9)$$

we obtained the energy of the T=2 isospin analogue state in Zn^{64} to be 6.420 MeV which is close to the energy of $0^+_{T=2}$=6.421MeV in our IBM-3 calculation with a_T=1.070. The $\alpha_i(i=1-6)$ coefficients determined by fitting to the experimental, completely symmetric, ground state band. These coefficients are fixed with respect to the fitting of the experimental low isospin states. So the low lying states for the even Ge isotopes are considered as follows:

$$H_{Ge^{64}} = -0.260C_{2U_{sd}(6)} + 1.07T(T+1) + 0.540C_{1U_d(5)} + 0.010C_{2U_d(5)} + 0.010C_{2O_{sd}(6)}$$
$$0.011C_{2O_d(5)} + 0.035C_{2O_d(3)},$$
$$H_{Ge^{66}} = 0 - 0.160C_{2U_{sd}(6)} + 1.070T(T+1) + 0.550C_{1U_d(5)} + 0.008C_{2U_d(5)} + 0.020C_{2O_{sd}(6)}$$
$$0.013C_{2O_d(5)} + 0.025C_{2O_d(3)},$$
$$H_{Ge^{68}} = -0.130C_{2U_{sd}(6)} + 1.070T(T+1) + 0.690C_{1U_d(5)} + 0.018C_{2U_d(5)} + 0.008C_{2O_{sd}(6)}$$
$$0.002C_{2O_d(5)} + 0.021C_{2O_d(3)},$$
$$H_{Ge^{70}} = -0.133C_{2U_{sd}(6)} + 1.070T(T+1) + 0.220C_{1U_d(5)} + 0.007C_{2U_d(5)} + 0.060C_{2O_{sd}(6)}$$
$$0.068C_{2O_d(5)} + 0.001C_{2O_d(3)}.$$

Corresponding to these coefficients of the group, the model parameters used in the present work are listed in Table-2.

Table-2: The IBM-3 parameters used for calculations of energy levels and transitions matrix elements in Ge^{64-70}.

Nucleus	^{64}Ge	^{66}Ge	^{68}Ge	^{70}Ge
$\varepsilon_{d\rho} - \varepsilon_{s\rho}(\rho=\pi,\nu)$	0.844	0.792	0.914	0.533
$A_i(i=0,1,2)$	-4.780,-1.640, 1.640	-4.560,-1.860, 1.860	-4.520,-1.896, 1.896	-4.420,-1.994, 1.994
$C_{i0}(i=0,2,4)$	-5.368,-4.948, -4.458	-5.148,-4.668, -4.318	-4.836,-4.610, -4.316	-5.560,-4.282, -4.268
$C_{i2}(i=0,2,4)$	1.052,1.472, 1.962	1.272,1.752, 2.102	1.584,1.810, 2.104	0.852,2.138, 2.152
$C_{i1}(i=1,3)$	-2.032,-1.680	-2.152,-1.902	-2.146,-1.936	-2.154,-2.144
$B_i(i=0,2)$	-4.800,1.620	-4.600,1.820	-4.54,1.880	-4.546,1.874
$D_i(i=0,2)$	0.000,0.000	0.000,0.000	0.000,0.000	0.000,0.000
$G_i(i=0,2)$	0.04472,0.04472	0.08944,0.08944	0.03577,0.03577	0.26833,0.26833
$\alpha_0=\beta_0=\alpha_1=\beta_1$	0.05	0.057	0.055	0.09
g_0	0.00	0.00	0.00	0.00
g_1	1.20	1.20	1.20	1.20

Energy levels calculations

The calculated low energy and low spin energy levels, for the even Ge isotopes, are shown in Figures-1-5 and listed in Table-3 together with the available experimental data, taken from references [18-21]. A satisfactory agreement for the entire chain of isotopes is obtained. These isotopes (Z=32) have been chosen with $N_\pi=2$ each relative to Z=28 magic number. The neutron boson (N_ν) numbers goes from 2 to 5, also related to closed shell at 28. Here we notice that all the bosons are particles and this is one of the reasons which stops the calculations, in the present work, at ^{70}Ge. All the energy levels wave function are mixed with three kinds of bosons (π,ν and δ) spaces.

Table-3. Experimental [18-21] and Calculated IBM-3 energy levels, in (MeV), for Ge^{64-70} isotopes

J^+	Exp	Calc.		Exp.	Calc.		Exp.	Calc.		Exp.	Calc.	
	Energy	Energy	Isospin	Energy	Energy	Isospin	Energy	Energy	Isospin	Energy	Energy	Isospin
0_1	0.000	0.000	0.00	0.000	0.000	1.00	0.000	0.000	2.00	0.000	0.000	3.00
0_2		1.215	0.00		1.334	1.00	1.755	1.670	2.00	1.215	1.217	3.00
0_3		2.158	0.00		2.421	1.00	2.617	2.683	2.00	2.306	2.532	3.00
0_4		2.335	0.00		2.454	1.00	3.204	3.132	2.00	2.880	2.577	3.00
1_1		5.102	1.00		3.003	1.00	3.086	3.191	2.00	3.242	3.297	3.00
1_2		5.543	1.00		3.782	1.00		4.069	2.00		4.301	3.00
1_3		6.242	1.00		4.132	1.00		4.837	2.00		4.455	3.00
1_4		6.668	1.00		4.563	1.00		4.972	2.00		5.045	3.00
2_1	0.901	0.907	0.00	0.957	0.957	1.00	1.015	0.995	2.00	1.039	1.004	3.00
2_2	1.578	1.627	0.00	1.647	1.776	1.00	1.777	1.889	2.00	1.707	1.801	3.00
2_3		2.081	0.00		2.185	1.00	2.457	2.462	2.00	2.157	2.224	3.00
2_4		2.753	0.00		2.378	1.00	2.947	2.676	2.00	2.535	2.515	3.00
3_1	2.689	2.578	0.00	2.495	2.754	1.00	2.428	2.935	2.00	2.451	3.307	3.00
3_2		5.452	1.00		3.253	1.00		3.401	2.00	3.040	3.366	3.00
3_3		5.893	0.00		3.684	1.00		4.279	2.00		4.255	3.00
3_4		6.086	1.00		4.032	1.00		4.400	2.00		4.311	3.00
4_1	2.052	2.117	0.00	2.173	2.126	1.00	2.267	2.183	2.00	2.153	2.238	3.00
4_2	2.154	2.858	0.00	2.725	2.954	1.00	2.832	3.103	2.00	2.805	3.374	3.00
4_3		3.243	0.00		3.228	1.00	3.040	3.650	2.00	3.005	3.509	3.00
4_4		3.618	0.00		3.543	1.00	3.186	3.872	2.00	3.190	3.708	3.00
5_1	3.716	3.968	0.00		4.031	1.00		4.256	2.00		4.329	3.00
5_2		6.082	0.00		4.482	1.00		4.657	2.00		5.081	3.00
5_3		7.222	1.00		4.841	1.00		5.222	2.00		5.207	3.00
5_4		7.940	1.00		5.263	1.00		5.560	2.00		5.476	3.00
6_1	3.406	3.628	0.00		3.504	1.00	3.696	3.565	2.00	3.290	3.396	3.00
6_2		4.388	0.00		4.331	1.00		4.508	2.00		4.285	3.00
6_3		7.136	0.00		4.434	1.00		5.030	2.00		5.030	3.00
6_4		7.768	1.00		4.914	1.00		5.259	2.00		5.219	3.00

Fig.-1 contains the calculated (IBM-3) and the experimental ground state energy levels. As one can see from the figure that the model well reproduced this band and this provides no surprise. However, the clear fluctuation of the 6^+ level about the typical behavior of the collective energy spectrum as a function of neutron number could be related to the noncollective feature of this state specially near closed shell at Z=28 and N near 50. This may suggest the presence of some admixture of the wave function of this state from the two quasiparticle configuration, or the high spin which these states own.

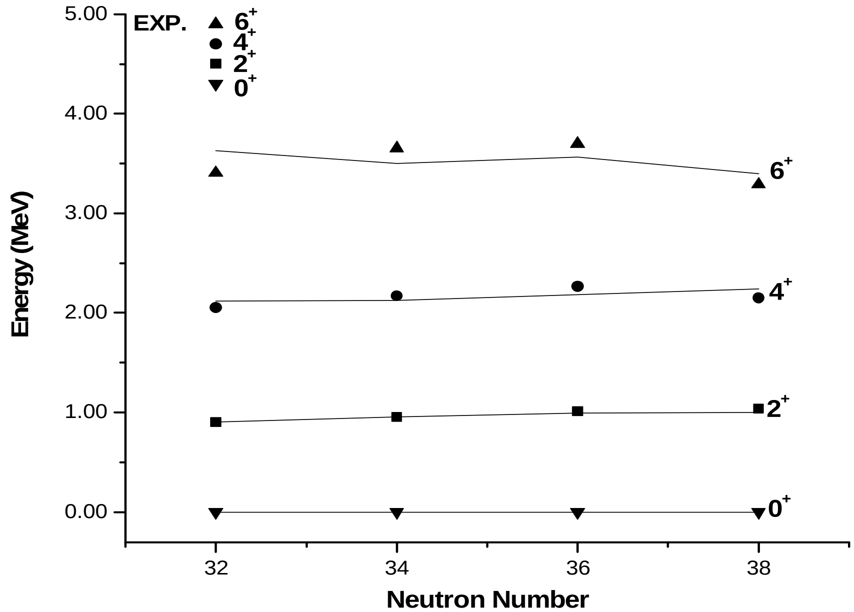

Figure-1: A comparison between experimental [20, 21] and IBM-3 calculated values of the ground band levels in even Ge^{64-70} isotopes.

Fig.-2 shows what is called the β vibrational band, i.e. energy spaces between states are equal, but they really are not. Experimental values are more extreme than theory. The 0_2^+ and 2_2^+ states are interesting cases. Their energies dropped suddenly in Ge^{70}, for both experimental and the IBM-3 predictions, and the 0_2^+ continue falling down in energy for higher neutron numbers isotope(Ge^{72}) in which it is lower than the first excited state in this isotopes. This mechanism described as the behavior of the 0^+ intruder state, which excited as (N-1) s boson [22]. Other characters of these states are their high energy related to the energy of 4_1^+. The three states, $0_1^+, 2_2^+$ and 4_1^+ suppose to be closed in energy, because they represent members of the two phonon triplet states. The energy of 4_1^+ is almost twice the energy of the other, which means that there is more deformation in the character of these states. The prediction of the model is a good agreement with this. However, the model calculation according to that push the energy of 4_2^+ higher, as shown in the figure, in order to generate the rotation behaviors of this band. The behavior of the two phonons triplet is shown in Fig.-3, which shows almost a linear energy dropping of these states as a function of mass number. The 0^+ state, after dropping below the 2_1^+ in ^{72}Ge, suddenly pushed up higher in energy in Ge^{74}. The strange position and behavior of this state are very rare in nuclei, it happened only in five nuclei in the whole chain of even-even isotopes in the nuclear chart, and they are Ge^{72}, $Zr^{90,96,98}$ and in Mo^{98}.

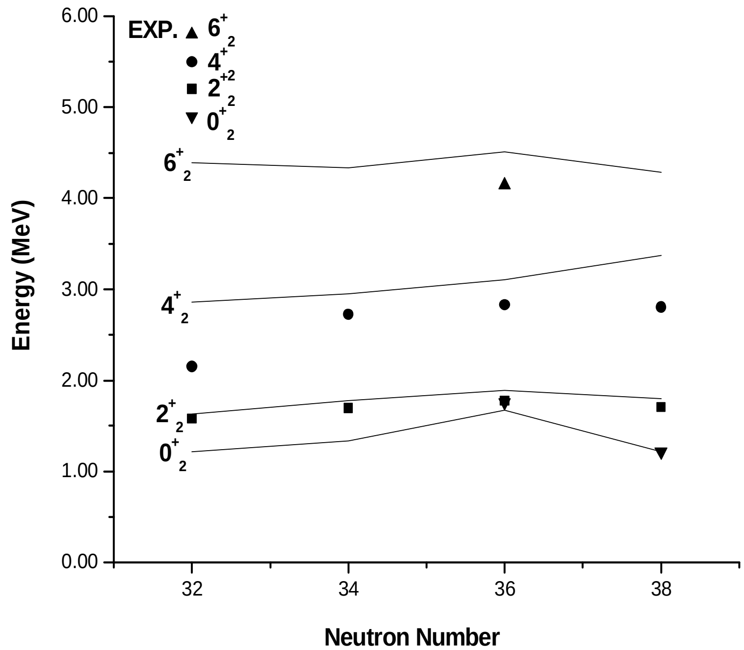

Figure-2: A comparison between experimental [20, 21] and IBM-3 calculated values of the quasi-beta band levels in even Ge$^{64-70}$ isotopes.

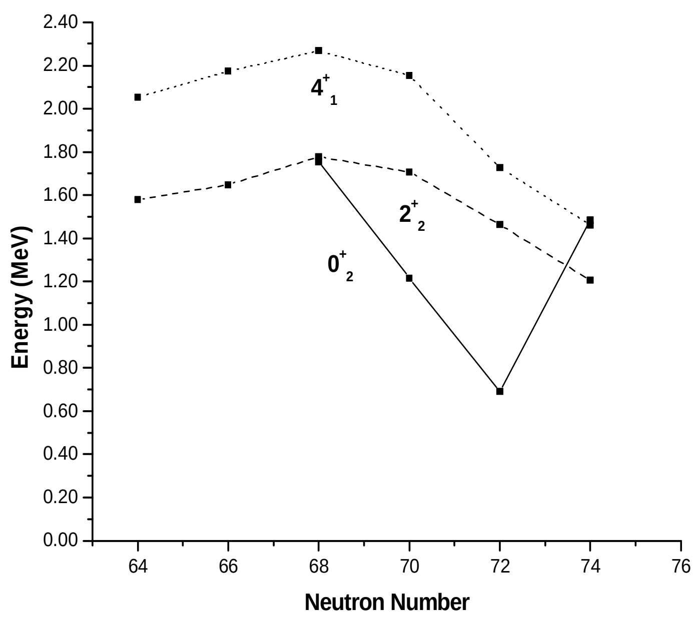

Figure-3: The systematic of experimental values [20, 21] of the two phonons-triplet, $0_2^+, 2_2^+$ *and* 4_1^+**, levels in even Ge$^{64-74}$ isotopes.**

Fig.-4 contains the excitation energy of the levels which are the member of the so called Gamma-Band. Good agreements between experimental data and theory for the 2^+ were produced. As for the 3^+ we have acceptable agreement for N=32 and 34, then the theory predicted a high energy for this state. For the rest of the states we can't compare them due to the lack of experimental data.

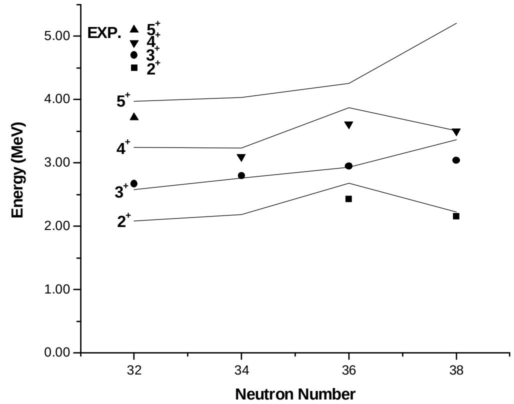

Figure-4: A comparison between experimental [20, 21] and IBM-3 calculated values of the quasi-gamma band levels in even Ge^{64-70} isotopes.

The rest of the levels are listed in Table-3 with their available experimental values as well.

In order to see the shape coexistence in the Ge isotopes one has to calculate the ratios $E_{4_1^+}/E_{2_1^+}$ and $E_{6_1^+}/E_{2_1^+}$ and compare with experimental ratios. This comparison can give indications of the nuclear shape. It is well known that nuclei tend to vary their shape smoothly from spherical near closed shell, $E_{4_1^+}/E_{2_1^+}=2$, to deformed near the mid shell, $E_{4_1^+}/E_{2_1^+}=3.3$, and in between the gamma soft nuclei. As shown in Fig.-5, this ratio starts from 2.277, N=Z=32, and decreases to 2.07 for Ge^{70}, which means that the Ge isotopes change their shape from the gamma-soft to the vibrational like nuclei. Also we can see that there is an agreement between experimental data and the model prediction.

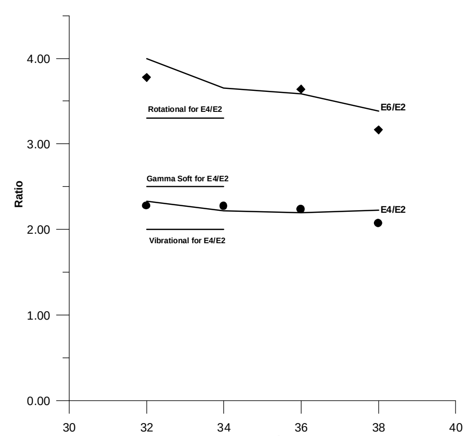

Figure-5: A comparison between experimental values of the ratios E_4/E_2 *and* E_6/E_2 **and the IBM-3 prediction**.

Isospin excitation and mixed symmetry states

One of the most important things of the IBM-3 is the prediction of the energy of isospin excitation states. In the case of Ge^{64} isotope (N=Z=32) for example, T=0 is the lowest isospin value. The excitation energy from the T=0 ground states to the first isospin excitation

T=1 band is well reproduced; where the calculated energy of the $0^+_{T=1}$ state equals to 5.350 MeV which is close to the energy of the T=1 isospin analogue to ground state in Ga^{64} to be 4.756 MeV. Based on isospin analogue state in Zn^{64}, the calculation suggested that the first isospin excitations energy, $J=2^+$, states with T=1 at 5.049 MeV and with [N-1,1] U(6) labeling. The lowest mixed symmetry state is $J^{\pi}=2^+$ comes from [N-2,2] partition with T=0 at 4.629 MeV which is inconsistent with calculated one given in ref.[13] at 4.6 MeV, but up to now no experimental evidence for such conclusions has been discovered. The first scissor mode state in the Ge^{64} is calculated at 5.102 MeV with [N-1,1] partition.

Because IBM-3 has three charge states, for three kinds of boson, it is possible to have U(6) partitions into three rows, namely the [N1,N2,N3] states which are the characteristic of IBM-3. We found that such states produced at high energy, upwards at about 7.5 MeV, and the lowest example being a scissor mode at 7.545 MeV which is predominantly the [2, 1, 1] partition with T = 1 .These suggestions do not contradict the experimental data. In order to identify the lowest mixed symmetry state in the $Ge^{68,70}$ we analyze the wave function of low lying 2^+ states in these nuclei. The main components of the wave function for 2^+_3 and 2^+_4 are given as follows, respectively:

$$\left|2^+_3\right\rangle_{68} = 0.615\left|s_\nu^4 s_\pi d_\pi\right\rangle - 0.435\left|s_\nu^3 s_\pi^2 d_\nu\right\rangle - 0.308\left(\left|s_\nu^3 s_\pi s_\delta d_\delta\right\rangle + \left|s_\nu s_\delta^4 d_\delta\right\rangle\right) + 0.377\left|s_\nu^2 s_\pi s_\delta^2 d_\nu\right\rangle + 0.218\left(\left|s_\nu^2 s_\delta^3 d_\delta\right\rangle - \left|s_\nu^3 s_\delta^2 d_\pi\right\rangle\right) + \ldots\ldots\ldots\ldots,$$

$$\left|2^+_4\right\rangle_{68} = -0.459\left|s_\nu^2 s_\pi d_\nu^2 d_\pi\right\rangle - 0.265\left|s_\nu^3 d_\nu d_\pi^2\right\rangle - 0.388\left|s_\nu s_\pi^2 d_\nu^3\right\rangle - 0.393\left|s_\nu^2 s_\pi d_\nu^2 d_\pi\right\rangle - 0.293\left|s_\nu^2 s_\pi d_\nu^2 d_\pi\right\rangle + 0.164\left(\left|s_\nu s_\pi s_\delta d_\nu^2 d_\delta\right\rangle + \left|s_\nu^2 s_\delta d_\nu d_\pi d_\delta\right\rangle\right) + 0.115\left(\left|s_\nu s_\delta^2 d_\nu d_\delta^2\right\rangle - \left|s_\nu^2 s_\pi d_\nu d_\delta^2\right\rangle\right) + \ldots,$$

for Ge^{68} isotope, and

$$\left|2^+_3\right\rangle_{70} = -0.355\left|s_\nu^2 s_\pi d_\pi d_\nu^3\right\rangle - 0.2215\left|s_\nu s_\pi^2 d_\nu^4\right\rangle + 3294\left(\left|s_\nu^3 s_\pi^2 d_\nu^2\right\rangle + \left|s_\nu^4 s_\pi d_\delta d_\nu\right\rangle\right) + 0.222\left|s_\nu d_\pi^2 d_\nu^4\right\rangle + 0.1808\left|s_\nu^2 s_\pi d_\pi d_\nu^3\right\rangle + \ldots\ldots\ldots\ldots,$$

$$\left|2^+_4\right\rangle_{70} = 0.304\left|s_\nu^4 s_\pi^2 d_\nu\right\rangle - 0.4800\left|s_\nu^5 s_\pi d_\pi\right\rangle - 0.227\left|s_\nu^3 s_\pi s_\delta^2 d_\nu\right\rangle + 0.215\left|s_\nu^4 s_\pi s_\delta d_\delta\right\rangle + 0.158\left|s_\nu^4 d_\pi^2 d_\nu\right\rangle + 0.116\left|s_\nu s_\pi s_\delta^2 d_\nu^3\right\rangle + \ldots\ldots\ldots\ldots,$$

for Ge^{70} isotope.

The wave functions show that the 2^+_3 state at 2.462 MeV closed to experimental one at 2.457 MeV is the one d-boson mixed symmetry in Ge^{68}, while the calculated 2^+_4 at 2.515 MeV closed to experimental one at 2.535 MeV is the one d-boson mixed symmetry state in Ge^{70}, and the two states generated from the [N-1,1] U(6) partition. For the other 2^+ states, large mixed symmetry components are included in the calculated 2^+_5 state at 3.356 MeV closed to 3.027 MeV in the experimental data in Ge^{68} , 2^+_6 state at 3.406 MeV and 3.187 MeV in the IBM-3 and experimental results , respectively in Ge^{70} (i.e. 2^+_{2m}) state. Fig.-6 shows the mixed symmetry states $2_{1m}, 1^+_{1m}$ 3^+_{1m} and 4^+_{1m} band members as a function of neutron number. The agreement between available experimental data and IBM-3 is good despite of the high energy of these sates. The existences of more experimental data give us opportunity to test the model prediction in this region.

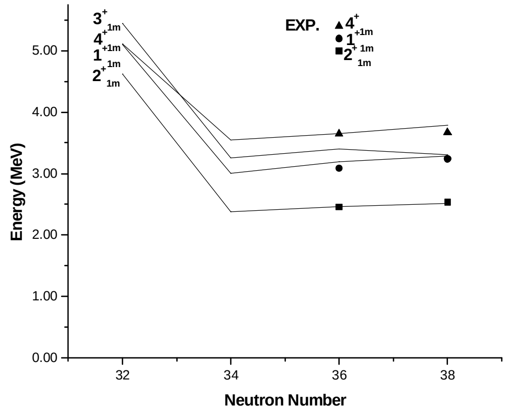

Figure-6: A comparison between experimental [20, 21] and IBM-3 calculated values of the mixed symmetry states in even Ge^{64-70} isotopes.

Electromagnetic transition

The electric quadruple transition (E2) operator, in the IBM-3, written as [23]

$$Q = Q^0 + Q^1 \qquad (11)$$

where

$$Q^o = \alpha_o \sqrt{3}\left[\left(s^\dagger \tilde{d}\right)^{20} + \left(d^\dagger \tilde{s}\right)^{20}\right] + \beta_0 \sqrt{3}\left[\left(d^\dagger \tilde{d}\right)\right]^{20}$$

$$Q^1 = \alpha_1 \sqrt{2}\left[\left(s^\dagger \tilde{d}\right)^{21} + \left(d^\dagger \tilde{s}\right)^{21}\right] + \beta_1 \sqrt{2}\left[\left(d^\dagger \tilde{d}\right)\right]^{21} \qquad (12)$$

where α_n *and* $\beta_n, n = 0,1$ have linear combinations with the usual parameters of the E2 operator[24], proton (e_π) and neutron (e_ν) effective charges, in the IBM-2. This can be written as:

$$e_\nu = \alpha_0 + \alpha_1, \qquad e_\nu \chi_\nu = \beta_0 + \beta_1,$$

$$e_\pi = \alpha_0 + \alpha_1, \qquad e_\pi \chi_\pi = \beta_0 - \beta_1 \qquad (13)$$

Since we have third kind of boson (δ) with T=1 in this model (IBM-3), we should create an effective charge (e_δ), which can be related to the model parameters as:

$$e_\delta = \alpha_0, \qquad e_\delta \chi_\delta = \beta_0$$

The M1 transition is also a one boson operator with an isoscalar and isovector parts:

$$M = M^o + M^1, \qquad (14)$$

where

$$M^0 = g_0 \sqrt{3}(d^\dagger \hat{d})^{10} = g_0 L / \sqrt{10}$$

$$M^1 = g_1 \sqrt{2}(d^\dagger \hat{d})^{11},$$

where L is the angular momentum operator and g_1 and g_o are the isovector and isoscalar g-factor respectively, and these also can be related to the g-factors of the IBM-2 as:

$$g_\nu = \sqrt{\frac{1}{10}}(g_0 + g_1), \qquad g_\pi = \sqrt{\frac{1}{10}}(g_0 - g_1) \qquad g_\delta = \sqrt{\frac{1}{10}} g_0 \qquad (15)$$

After determining the possible best energy level fit to the experimental data, one can use the energy wave function to calculate the reduced electromagnetic transition matrix elements. For the E2 transitions four parameters have to be modified, α_n and β_n $n = 0,1$, in order to fit the measured $B(E2;2_1^+ \rightarrow 0_1^+)$ values. It has been found that $\alpha_o = \beta_o = \alpha_1 = \beta_1$ for each isotope under study, and this reduced the parameters to one. The estimated values of these parameters are shown in Table-2. The electromagnetic properties of the collective bands in the *pf* shells, and in particular the E2 transitions strengths for the inband transitions, give quite useful information about the microscopic structure of the collective states. Both the experimental and calculated B(E2) values for Ge^{64-70} isotopes are listed in Table-4. As one can see from that the overall trend is reproduced well by the IBM-3 calculations. In the case of Ge^{64}, the experimental data for the electromagnetic transitions do not exist, to compare with the model prediction. But most of values conserve the same trend with the values of the other isotopes, which increase with the neutrons number. We have a very good agreement for transitions connect state in the ground band. Small calculated value for transition $2_2^+ \rightarrow 0_1^+$ in Ge^{64} [B(E2)=0.0016×10^{-2} e^2b^2] and Ge^{66} [B(E2)=0.017×10^{-2} e^2b^2, this is in good agreement with experimental values. But in the case of Ge^{68}, the B(E2) value is 0.0300×10^{-2} e^2b^2 without experimental value, while B(E2;$2_3^+ \rightarrow 0_1^+$) =0.023×10^{-2} e^2b^2, which is in disagreement with the large predicted value by the model B(E2;$2_3^+ \rightarrow 0_1^+$)=1.23 e^2b^2, the same case with Ge^{70}. This encourages the suggestion that, there is interchange between 2_2^+ and 2_3^+ states, in the energy of the levels and 0_2^+ is not a band head of the collective beta band. The relative to the B(E2;$2_1^+ \rightarrow 0_1^+$) ratios are also calculated and listed in Table-5 together with experimental values. A small ratio for transitions from the second 2^+ gives a second indication that this state is a band head of a quasi γ- band. The ratio for transitions from the 4^+, agree well with experimental ratio. However, in all cases where the B(E2) value, in the nominator, is very small we expect to get a substantial disagreement.

To produce M1 matrix elements, the isoscaler g_o factor is taken to be zero, for all isotopes, and the isovector factor g_1 is taken to be 1.2 for all isotopes. The values of the M1 reduced transitions probability are listed in Table-4 as wisell. We can see from the table that, when the experimental value is small the model gives zero M1 matrix elements, which means that, the model assumed the state is purely symmetric in the boson space.

The theoretical $\delta((E2/M1)$ mixing ratios were calculated according to the following relation.[25]:

$$\delta_{i\rightarrow f}(E2/M1) = 0.835(E_\gamma \, in\, MeV)\frac{\langle f \parallel E2 \parallel i\rangle \quad in \;\; eb}{\langle f \parallel M1 \parallel i\rangle \quad in\, \mu_N} \tag{16}$$

The calculated reduced mixing ratios for band crossing transitions are compared with available experimental ones. The model gives zero values for M1 transition matrix elements, in most of transitions with $I_i = I_f$, which makes difficult to calculate the mixing ratio. However the model predicts the sign of mixing ratio correctly.

Table-4: Experimental [18-21] and calculated B(E2)×10^-2 in e^2b^2, B(M1)$\times10^{-2}$ in $N\mu^2$ and the mixing ratio $\delta(E2/M1)$ for G^{64-70} isotopes.

[<u>Ge^{64}</u>]

	B(E2)		B(M1)		$\delta(E2/M1)$	
$J_i^+ \rightarrow J_f^+$	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.
$2_1^+ \rightarrow 0_1^+$		0.9012				
$2_2^+ \rightarrow 0_1^+$		0.0016				
$2_3^+ \rightarrow 0_1^+$		0.000				
$2_2^+ \rightarrow 2_1^+$		1.3924		0.0000		
$2_3^+ \rightarrow 2_1^+$		0.00005		0.0000		
$2_3^+ \rightarrow 2_2^+$		0.3425		0.0000		
$1_1^+ \rightarrow 0_1^+$				0.0000		
$1_1^+ \rightarrow 2_1^+$		0.0000		0.0000		
$1_1^+ \rightarrow 2_2^+$				0.0000		
$3_1^+ \rightarrow 2_1^+$		0.0019		0.0000		
$3_1^+ \rightarrow 2_2^+$		1.0307		0.0000		

$3_1^+ \rightarrow 1_1^+$						
$4_1^+ \rightarrow 2_1^+$		1.3943				
$5_1^+ \rightarrow 3_1^+$		0.5250				
$6_1^+ \rightarrow 4_1^+$		1.4429				

[**Ge^{66}**]

$2_1^+ \rightarrow 0_1^+$	1.89(36)	1.7974				
$2_2^+ \rightarrow 0_1^+$	0.018^{+9}_{-5}	0.0172				
$2_3^+ \rightarrow 0_1^+$	0.00016	0.0001				
$2_2^+ \rightarrow 2_1^+$	2.81(11)	2.9857	0.77(35)	0.0000	-3.5^{+18}_{-26}	
$2_3^+ \rightarrow 2_1^+$		0.0067		0.0000		
$1_1^+ \rightarrow 0_1^+$				0.4397		
$1_1^+ \rightarrow 2_1^+$		0.8888		00000		
$1_1^+ \rightarrow 2_2^+$		0.1141		16.746		+0.410
$3_1^+ \rightarrow 2_1^+$		0.0263		0.0000		
$3_1^+ \rightarrow 2_2^+$		2.4935		0.0000		
$3_1^+ \rightarrow 1_1^+$						
$4_1^+ \rightarrow 2_1^+$	$\rangle$ 1.5	2.4892				
$5_1^+ \rightarrow 3_1^+$		1.7590				
$6_1^+ \rightarrow 4_1^+$		3.5295				

[**Ge^{68}**]

$2_1^+ \rightarrow 0_1^+$	2.9(3)	2.9235				
$2_2^+ \rightarrow 0_1^+$		0.0300				
$2_3^+ \rightarrow 0_1^+$	0.023(4)	1.2300				
$2_2^+ \rightarrow 2_1^+$	0.08(3)	4.9673	1.43(25)	0.0000	-0.2(0.1)	
$2_3^+ \rightarrow 2_1^+$		0.2018		13.7800		+0.07
$2_3^+ \rightarrow 2_2^+$		0.0743		0.0000		
$1_1^+ \rightarrow 0_1^+$				0.0759		
$1_1^+ \rightarrow 2_1^+$		1.2069		0.0000		
$1_1^+ \rightarrow 2_2^+$		0.1201		19.2732		+ 0.049
$3_1^+ \rightarrow 2_1^+$	0.0030(13)	0.0042	0.18(7)	0.0000	0.16(2)	
$3_1^+ \rightarrow 2_2^+$	0.063(43)	4.317	5.0(2.0)	0.0000	-0.2(0.3)	
$3_1^+ \rightarrow 1_1^+$						
$4_1^+ \rightarrow 2_1^+$	2.29(30)	4.9684				
$5_1^+ \rightarrow 3_1^+$		3.2574				
$6_1^+ \rightarrow 4_1^+$		6.0796				

			[**Ge^{70}**]		
$2_1^+ \rightarrow 0_1^+$	3.6(4)	3.1083			
$2_2^+ \rightarrow 0_1^+$		0.0640			
$2_3^+ \rightarrow 0_1^+$		1.1931			
$2_2^+ \rightarrow 2_1^+$	4.97(18.9)	0.4108	0.47(27)	0.0000	-5.0(3.0)
$2_3^+ \rightarrow 2_1^+$		3.7944		0.0000	
$2_3^+ \rightarrow 2_2^+$		4.4695		0.0000	
$1_1^+ \rightarrow 0_1^+$				2.1323	
$1_1^+ \rightarrow 2_1^+$		2.3578		0.0000	
$1_1^+ \rightarrow 2_2^+$		0.4169		0.0000	
$3_1^+ \rightarrow 2_1^+$		2.2619		0.0000	
$3_1^+ \rightarrow 2_2^+$		0.4784		0.0000	
$3_1^+ \rightarrow 1_1^+$					
$4_1^+ \rightarrow 2_1^+$	4.11(1.1)	4.0389			
$5_1^+ \rightarrow 3_1^+$		3.8393			
$6_1^+ \rightarrow 4_1^+$		5.9574			

Table-5: B(E2) ratios relative to the B(E2;2_1—0_1) transition for selected transitions in even Ge$^{64-70}$ isotopes

Nucleus →	Ge[illegible]		Ge[illegible]		Ge[illegible]		Ge[illegible]	
	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.
$\frac{B(E2;2_2-0_1)}{B(E2;2_1-0_1)}$	-	0.0018	0.802	0.0096	-	0.0103	0.4272	0.0200
$\frac{B(E2;4_1-2_1)}{B(E2;2_1-0_1)}$	-	1.5472	>0.793	1.3856	-	1.6986	1.1411	1.2991
$\frac{B(E2;2_3-0_1)}{B(E2;2_1-0_1)}$	-	0.0000	0.0084	0.0167	-	0.4210	-	0.3963

Conclusions

The nuclear structure of nuclei in region, where maximum binding energy, leads to an increased the knowledge of properties of the nucleus. The new version of the interacting boson models, IBM-3, produced a satisfactory agreement with experimental results, so the IBM-3 is preferable to IBM-2 in lighter nuclei because it ensure re good isospin quantum number. Shape coexistence from Ge^{68} to Ge^{72} has been confirmed, and the existence of the introdure excited 0^+ state in Ge^{68}, Ge^{70} and Ge^{72}.

From the calculated binding energies and the normalized energy levels, we represent electromagnetic properties of those nuclei, the properties of the 1^+,2^+ and 3^+ mixed symmetry states are well produced as well. However a definitive conclusion required more experimental information about these nuclei and the model need to be extended to find monopole matrix elements which is essential for the nuclear shape.

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التركيب من $2p_{3/2}$ الى $1f_{5/2}$ للنظائر الخفيفة في الجرمانيوم باستخدام إنموذج البوزونات المتفاعلة الثالث IBM-(3)

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المستخلص

تمت دراسة التركيب النووي لمستويات الطاقة لنظائر الجرمانيوم Ge^{64-70} مع الأخذ في الأعتبار النتائج العملية المتوفرة وذلك باستخدام التماثل في النسخة الثالثة من نموذج البوزونات المتفاعلة (IBM-3). وقد حسبت طاقة المستويات وأحتمالية الأنتقال رباعي القطب الكهربائي وثنائي القطب المغناطيسي ونسبة الخلط وقورنت مع القياسات العملية. وتمت دراسة خواص المستويات المختلطة التماثل ايضا.