Definition (3): (Variance)

Let X be a random variable with mean μ_X. The variance of X, $Var(X)$, is defined by: $Var(X) = E[(X-\mu_X)^2]$. It is also denoted $by\ \delta_X^2$.

Definition (4): (Standard Deviation)

The positive square root of the variance is called standard deviation of the random variable X, *is denoted by* δ_X. *That is* : $\delta_X = +\sqrt{\delta_X^2}$.

Theorem (2)

If X be a random variable with mean μ_X .Then the variance of X is given by:

$$\delta_X^2 = E(X^2) - \left(E(X)\right)^2 = E(X^2) - (\mu_X)^2. \blacksquare$$

Theorem (3)

If X be a random variable. Then: $Var(aX+b) = a^2 Var(X)$, where $a \,\&\, b$ *are real numbers*. $\blacksquare$

Example (7)

Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{6} & , -3 < x < 3 \\ 0 & , otherwise \end{cases} .$$

Find the mean, variance & standard deviation of X.

Solution

The mean of the random variable X is:

$$\mu_X = E(X) = \int_{-3}^{3} x f(x)dx = \frac{1}{6}\int_{-3}^{3} x dx = \frac{1}{6}\left[\frac{x^2}{2}\right]_{-3}^{3} = 0 .$$

The variance of the random variable X is:

$$Var(X) = \delta_X^2 = E(X^2) - (\mu_X)^2 .$$

Now, $E(X^2) = \int_{-3}^{3} x^2 f(x)dx = \frac{1}{6}\int_{-3}^{3} x^2\, dx = \frac{1}{6}\left[\frac{x^3}{3}\right]_{-3}^{3} = 3$.

$\therefore Var(X) = 3 - 0 = 3$.

The standard deviation of the random variable X *is*: $\delta_X = \sqrt{3}$. $\blacksquare$

Example (8)

Let X be a random variable with probability mass function

$$f(x) = \begin{cases} \frac{C_x^3\, C_{4-x}^5}{C_4^8} & , x = 0,1,2,3 \\ 0 & , otherwise \end{cases} .$$

Find the mean, variance & standard deviation of X.

Solution

The mean of the random variable X is:

$$\mu_X = E(X) = \sum_{x=0}^{3} x f(x) = 0 + f(1) + 2f(2) + 3f(3)$$

$$= \frac{1}{70}\left(C_1^3 C_3^5 + 2C_2^3 C_2^5 + 3C_3^3 C_1^5\right) = \frac{1}{70}(3 \times 10 + 2 \times 3 \times 10 + 3 \times 1 \times 5)$$

$$= \frac{1}{70}(30 + 60 + 15) = \frac{105}{70} = \frac{3}{2}.$$

The variance of the random variable X is:

$$Var(X) = \delta_X^2 = E(X^2) - (\mu_X)^2 .$$

Now, $E(X^2) = \sum_{x=0}^{3} x^2 f(x) = 0 + f(1) + 4f(2) + 9f(3)$

$$= \frac{1}{70}\left(C_1^3 C_3^5 + 4C_2^3 C_2^5 + 9C_3^3 C_1^5\right) = \frac{1}{70}(30 + 120 + 45) = \frac{195}{70} = \frac{39}{14}.$$

$$\therefore Var(X) = \frac{39}{14} - \frac{9}{4} = \frac{15}{28}.$$

The standard deviation of the random variable X is: $\delta_X = \sqrt{\frac{15}{28}}$.■