

Sheet 1

Q1:- Let $X = \{1,2,3\}$. Let a function $d : X \times X \rightarrow [0, \infty)$ be as below. Decide whether d is a metric on X . You must justify your answers.

- (i) $d(1,1)=d(2,2)=d(3,3)=0$,
 $d(1,2) = d(2,1)=2$,
 $d(2,3)=d(3,2)=4$,
 $d(1,3)=d(3,1) = 5$.
- (ii) $d(1,1)=d(2,2)=d(3,3)=0$,
 $d(1,2) = d(2,1)=2$,
 $d(2,3)=d(3,2)=4$,
 $d(1,3)=d(3,1) = 7$.

Q2:- Let $x = (1,5, -3)$ and $y = (3,8, -9)$ are two vectors in \mathbb{R}^3 . Find (a) $d_1(x, y)$, (b) $d_2(x, y)$, (c) $d_\infty(x, y)$, where d_1, d_2, d_∞ are metrics on \mathbb{R}^3 .

Q3:- Consider the metric space $(C([a, b]), d_\infty)$.

A. Let $f(x) = x^2$ and $g(x) = x^3$. Find

- (i) $d_\infty(f, g)$ in $C([0,1])$. (ii). $d_\infty(f, g)$ in $C([-1,1])$.

B. Let $f(x) = x^2$ and $g(x) = x^4$. Find

- (i) $d_\infty(f, g)$ in $C([0,1])$. (ii) $d_\infty(f, g)$ in $C([0,2])$

Q4:- Prove that $(\mathbb{R}^N, \|\cdot\|_2)$ is a normed space, where $\|\cdot\|_2$ is the Euclidean norm on \mathbb{R}^N .

Q5:- In \mathbb{R}^N we define

1. $d_1(x, y) = \sum_{i=1}^N |x_i - y_i|$,
2. $d_2(x, y) = \left(\sum_{i=1}^N |x_i - y_i|^2\right)^{\frac{1}{2}}$,
3. $d_\infty(x, y) = \max_{1 \leq i \leq N} \{|x_i - y_i|\}$.

Prove that d_1, d_2 and d_∞ are metrics on \mathbb{R}^N .

Q6:- Let d be a metric on X . Determine all constants K such that
(1) kd is a metric on X . (2) $d + k$ is a metric on X

Q7:- Inverse triangle inequality. Let $(X, \|\cdot\|)$ be a normed space. Prove that

$$\|x - y\| \geq \left| \|x\| - \|y\| \right| \quad \forall x, y \in X.$$