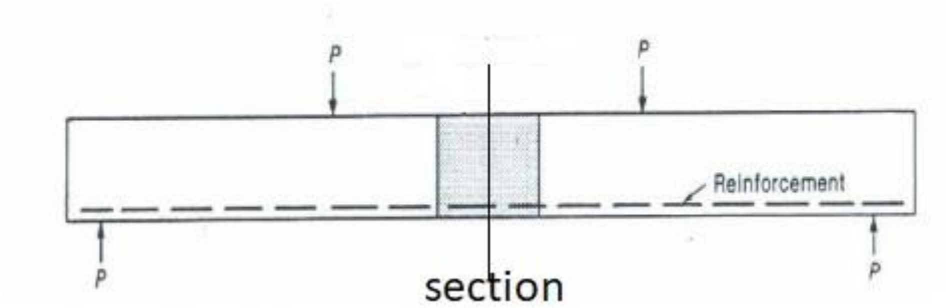
CHAPTER 2

FLEXURAL ANALYSIS OF REINFORCED CONCRETE BEAMS

BEHAVIOR OF SIMPLY SUPPORTED REINFORCED CONCRETE BEAM LOADED TO FAILURE

Concrete being weakest in tension, a concrete beam under an assumed working load will definitely crack at the tension side, and the beam will collapse if tensile reinforcement is not provided. Concrete cracks occur at a loading stage when its maximum tensile stress reaches the modulus of rupture of concrete. Therefore, steel bars are used to increase the moment capacity of the beam; the steel bars resist the tensile force, and the concrete resists the compressive force.

By consider any reinforced concrete beam carry an incrementally accumulative increase load as shown below.



The beam will pass through sex stress stages which are:

Stage 1: Elastic Uncracked Stage: The applied load on beam less than the load which cause cracking.

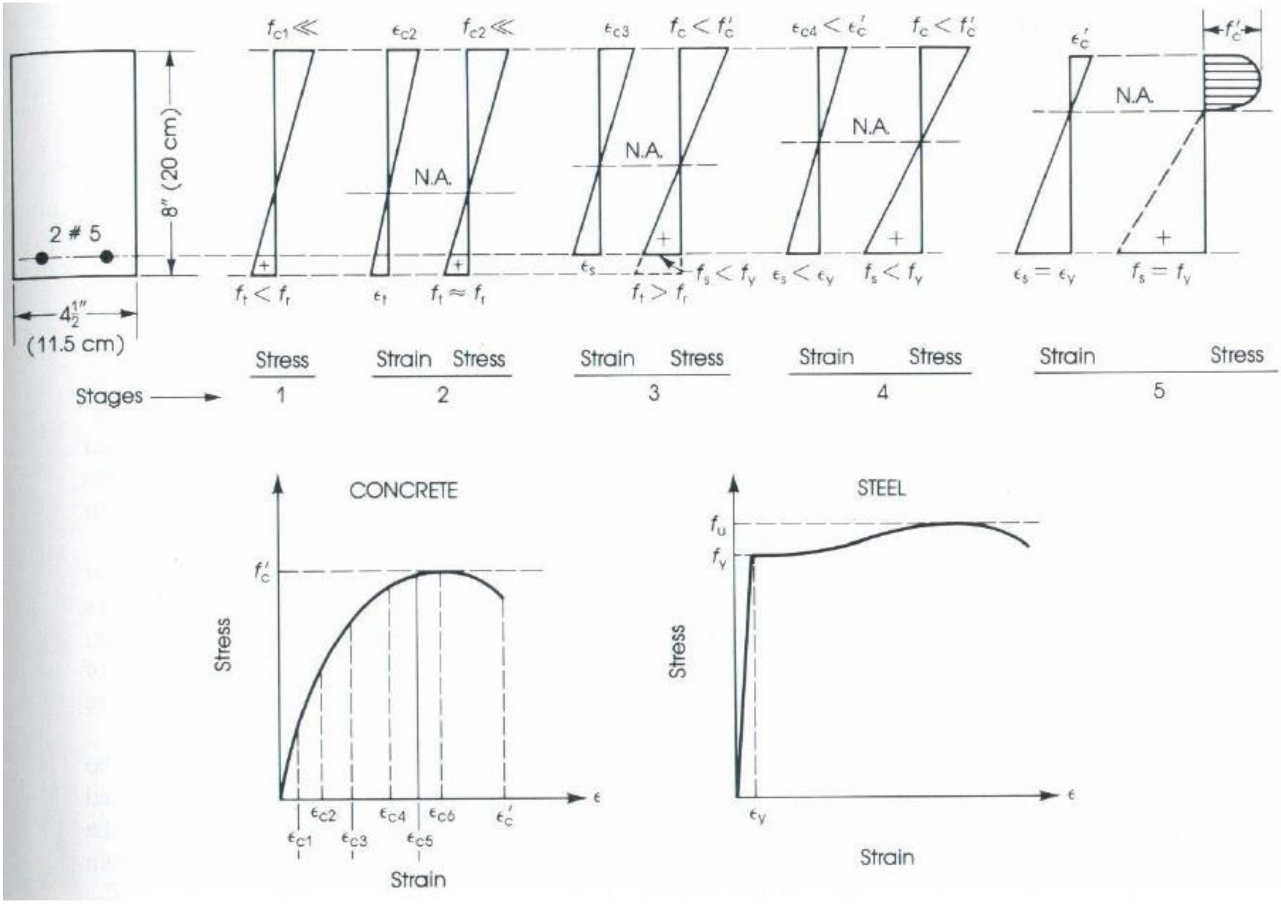
Stage 2: Elastic Cracked- threshold Stage: The applied load makes the bottom fiber stress equal to modulus of rupture of concrete f_{cr}.

Stage 3: Elastic Cracked Stage: The external applied load cause the bottom fibers to equal to modulus of rupture of the concrete. Entire concrete section was effective, steel bar at tension side has same strain as surrounding concrete.

Stage 4: Inelastic Cracking Stage : The tensile strength of the concrete exceeds the rupture fr and cracks develop. The neutral axis shifts upward and cracks extend to neutral axis. Concrete loses tensile strength and steel starts working effectively and resists the entire tensile load.

Stage 5: Ultimate Strength Stage: The reinforcement yields.

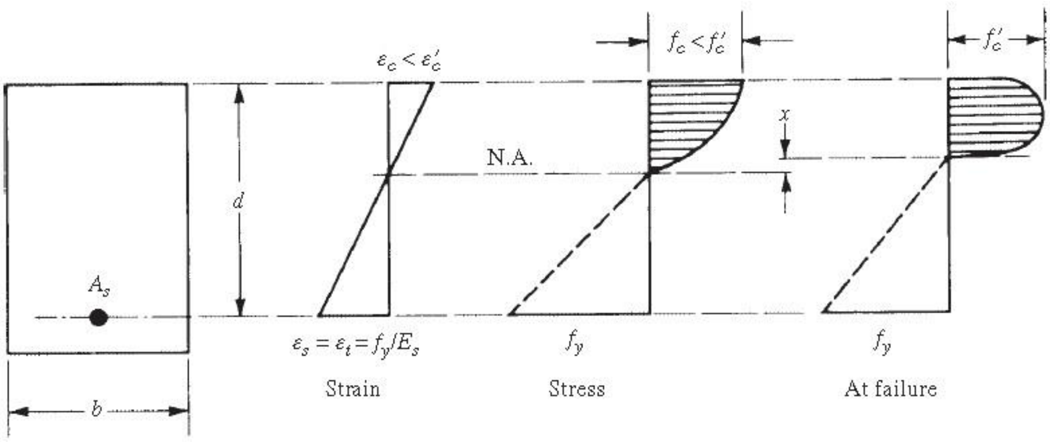
Stage 6: Failure Stage : The material stresses will be exceed its corresponding capacity .



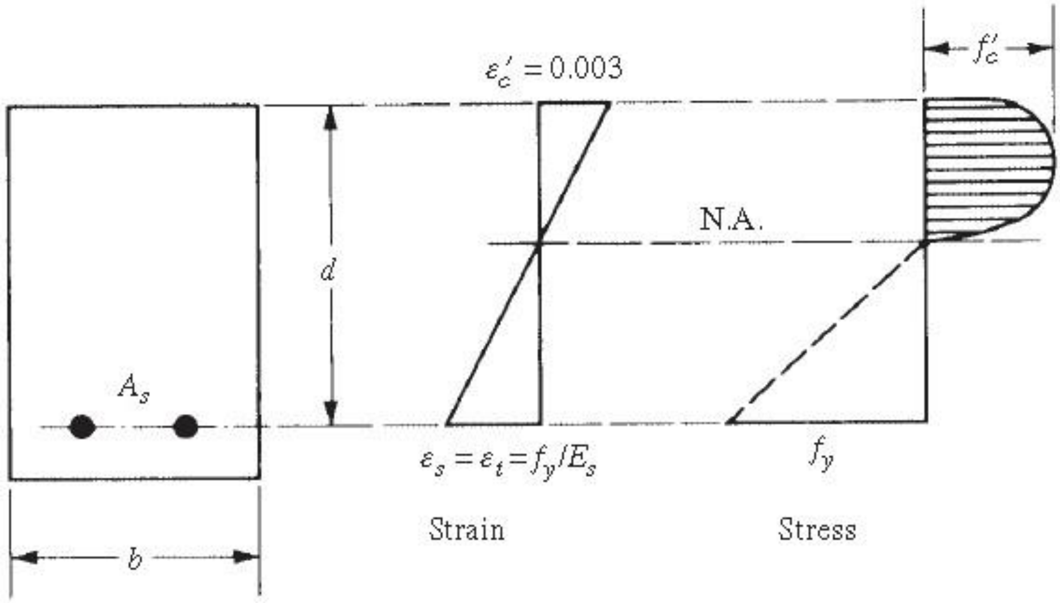
TYPES OF FLEXURAL FAILURE

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

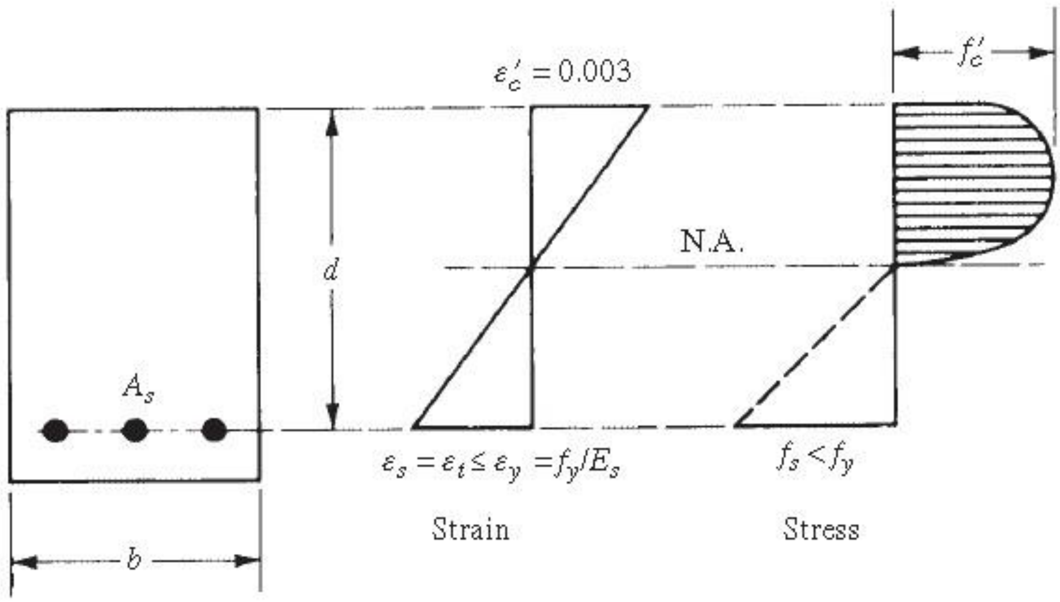
1 .Steel may reach its yield strength before the concrete reaches its maximum strength. In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than 0.005. The section contains a relatively small amount of steel and is called a tension-controlled section.



2 .Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a balanced section.



3 .Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength, that is, fs is less than fy. The strain in the steel is equal to or less than 0.002. This section is called a compression-controlled section



Analysis and Design Methods of Reinforced Concrete Structure

Working Stress Method (WSM)

Stresses are computed in both the concrete and steel using principles of mechanics that include consideration of composite behavior

Actual Stresses < Allowable Stresses

unified design method (UDM)

The Strength of members is computed at ultimate capacity Load Factors are applied to the loads Internal forces are computed from the factored loads

Required Strength < Actual Strength

Working Stress Method (WSM)

Basic assumptions for design applicable to flexural and compression members are as follows:

(a) Plane section before bending remains plane after bending.

(b) The tensile stress of concrete is neglected unless otherwise mentioned.

(c) The strain-stress relation for concrete as well as for steel reinforcement is linear.

(d) Perfect bond between steel and concrete.

Loading Stages: Uncracked section and Cracked section

Permissible Stresses

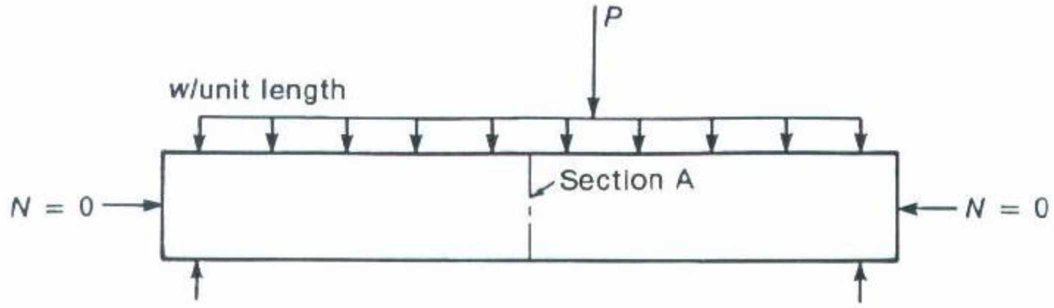
Load factors for all types of loads are taken to be unity for this design method.

Permissible stresses are defined as characteristic strength divided by factor of safety.

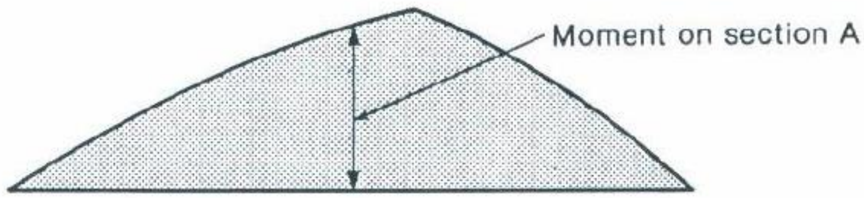
The factor of safety is not unique values either for concrete or for steel; therefore, the permissible stresses at service load must not exceed the following:

- Flexure extreme fiber stress in compression: 0.45 f'_c.
- Tensile stress in reinforcement: 0.5 fy
- Modulus of elasticity for steel reinforcement 200 GPa (29,000 psi).
- Modular Ratio n = Es /Ec.
- Transformed Section : Substitute steel area with (nAs) of fictitious concrete.
- Location of neutral axis depends on whether we are analyzing or designing a section.

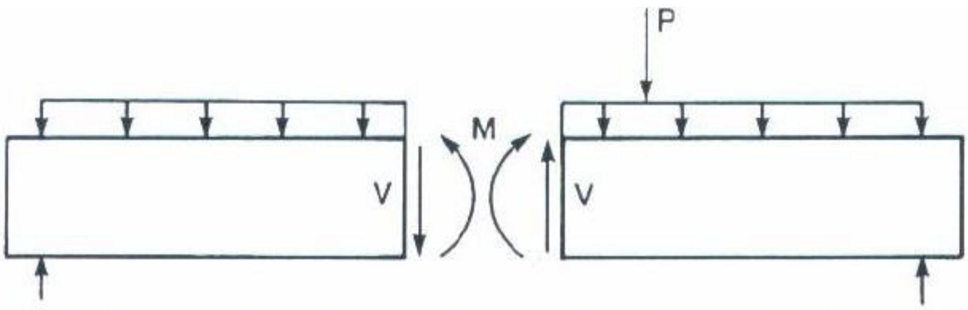
The beam is a structural member used to support the internal moments and shears



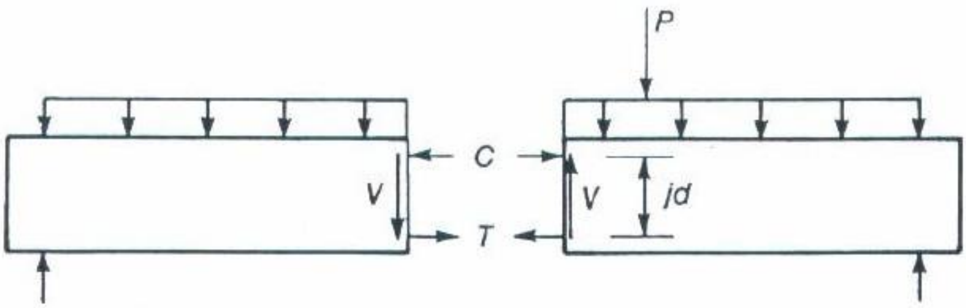
(a) Beam.



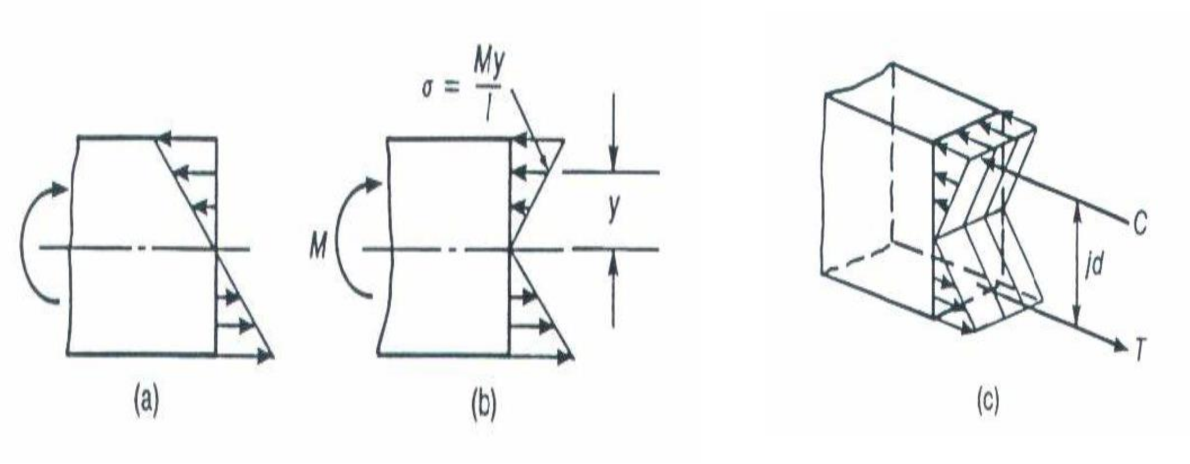
(b) Bending moment diagram.



(c) Free body diagrams showing internal moment and shear force.



(d) Free body diagrams showing internal moment as a compression–tension force couple.



The stress in the block is defined as:

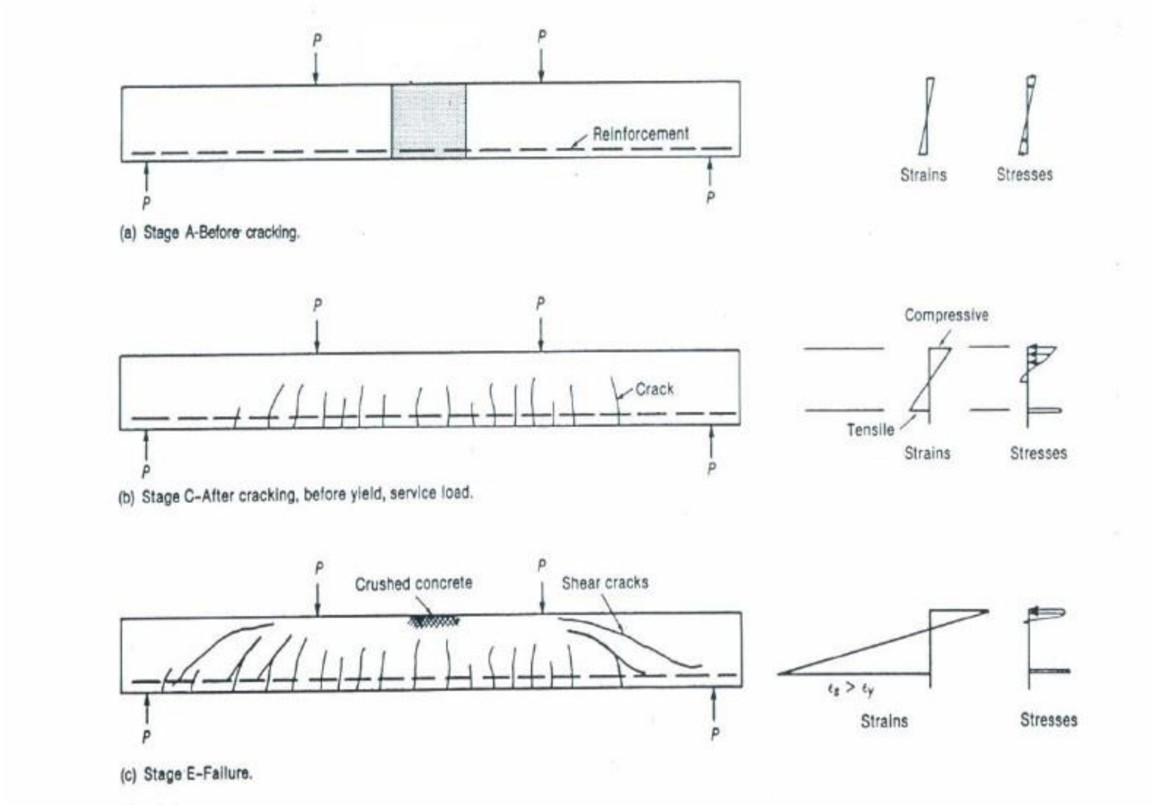
$$\sigma = (M*y) / I$$

Under the action of transverse loads on a beam strains, normal stresses and internal forces developed on a cross section are as shown below.

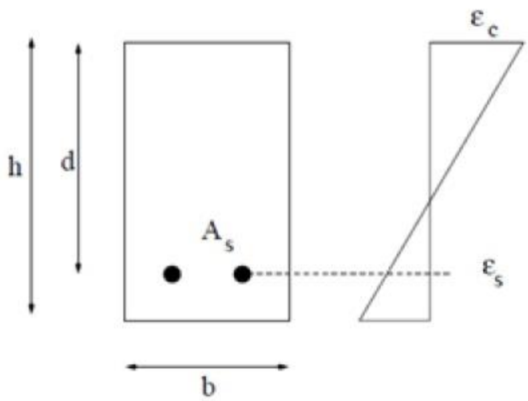
Stage 1: Before Cracking (Uneconomical),

Stage 3: After Cracking(Service Stage).

Stage 6: Ultimate (Failure).



Uncracked Section



Strain Diagram Uncracked Section

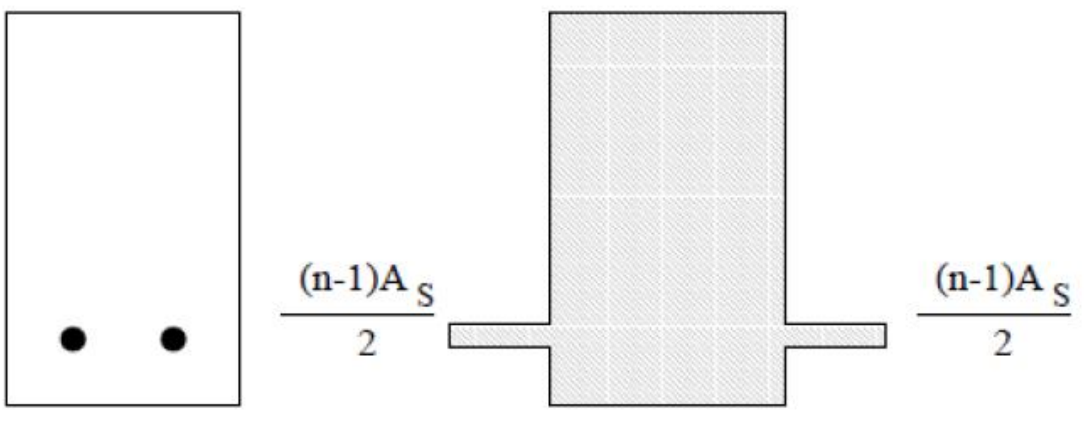
Assuming perfect bond between steel and concrete, we have εs= εc

$$\varepsilon_c = \varepsilon_s \Rightarrow \frac{f_s}{E_s} = \frac{f_c}{E_c} \Rightarrow f_s = \frac{E_s}{E_c} f_c \Rightarrow f_s = n f_c$$

Where n is the modular ratio $n = \frac{E_s}{E_c}$

Tensile force in Steel $T_s = A_s f_s = A_s n f_c$

Replace steel by an equivalent area of concrete



Transformed Section

Homogenous section & under bending

$$f_c = \frac{M\,c}{I} \Rightarrow f_s = n f_c$$

Transformed section area

$$At = (Ac\text{-}As) + n\ As \quad = Ac + (n\text{-}1)As$$

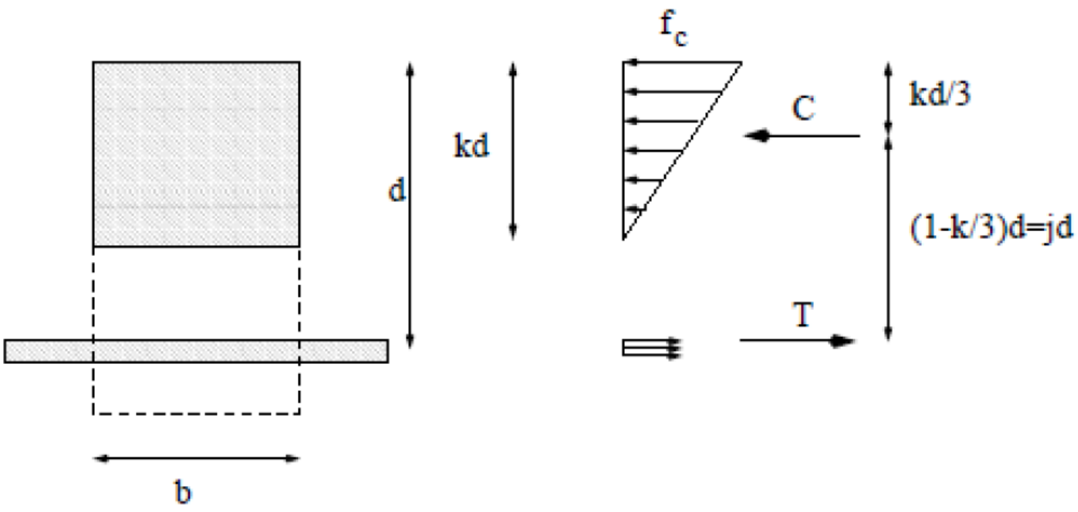
$$C = \frac{Ac \times \frac{h}{2} + (n-1)As \ \times d}{At}$$

$$I = \frac{b\,h^3}{12} + Ac\left(c - \frac{h}{2}\right)^2 + (n-1)As(d-C)^2$$

Section Cracked, Stresses Elastic

If $f_{ct} > f_r$, $f_c <\approx 0.45\, f_c'$ and $f_s < f_y$

we will assume that the crack goes all the way to the N.A and will use the transformed section,



To locate N.A. , tension force = compressive force (by def. NA) (Note, for linear stress distribution and with

$$\sum F_x = 0\,, \sigma = b\,y \Rightarrow \int b\,y\,dA = 0$$

thus

$$b \int y\,dA = 0 \text{ and } \int y\,dA = \bar{y}\,A = 0$$

by definition, gives the location of the neutral axis)

Note, N.A. location depends only on geometry & $n\left(\frac{E_s}{E_c}\right)$

Tensile and compressive forces are equal to $C = \frac{bkd}{2} f_c$ & $T = A_s\, f_s$ and neutral axis is determined by equating the moment of the tension area to the moment of compression area:

$$b\,(k\,d)\left(\frac{k\,d}{2}\right) = n\,A_s\,(d - kd) \quad 2^{\text{nd}} \text{ degree equation}$$

$$M = T\,jd = A_s\,f_s\,jd \Rightarrow f_s = \frac{M}{A_s\,jd}$$

$$M = C\,jd = \frac{b\,k\,d}{2} f_c\,jd = \frac{b\,d^2}{2} k\,jf_c \Rightarrow f_c = \frac{M}{\frac{1}{2}bd^2kj}$$

Where $j = \left(1 - \frac{k}{3}\right)$.

Or also

$$\left.\begin{aligned} \rho &= \frac{A_s}{bd} \\ b(kd)\frac{(kd)}{2} &= nA_s(d - kd) \end{aligned}\right\} \Rightarrow \boxed{k = \sqrt{2\rho n + (\rho n)^2} - \rho n}$$

Start by determining ρ ,

- If $\rho < \rho_b$ steel reaches max. allowable value before concrete, and

$$M = A_s f_s \, jd$$

- If $\rho > \rho_b$ concrete reaches max. allowable value before steel , and

$$M = f_c \frac{b\,k\,d}{2} jd$$

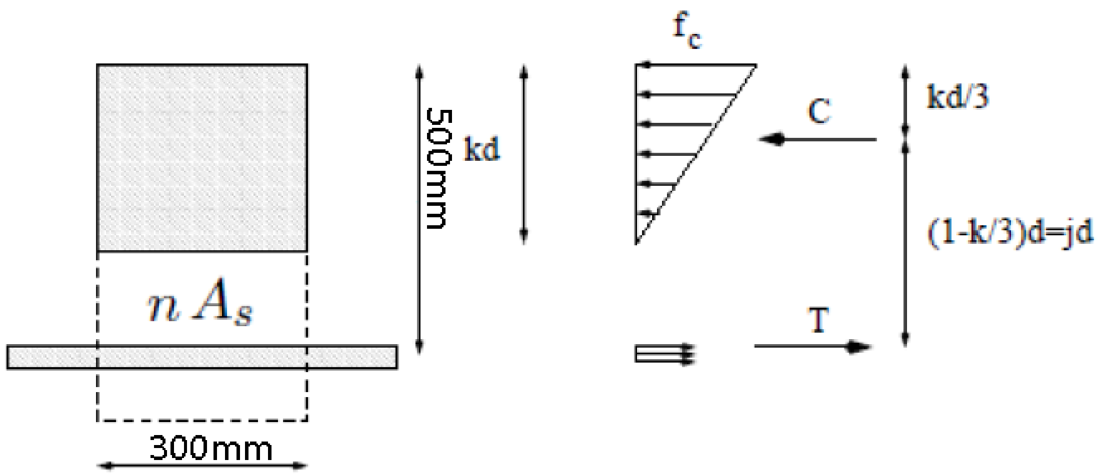
Or

$$M = \frac{1}{2} f_c \; j \; k \; b \, d^2 = R \; b \; d^2$$

Where

$$k = \sqrt{2\,\rho\,n + (\rho\,n)^2} - \rho\,n$$

Example1: The Fig. below show a section in reinforced concrete beam of width b = 300 mm and effective depth d = 500 mm , area of steel reinforcement is As = 1500 mm2 , modular ratio is n = 8.Compute the stress in the steel and concrete if the applied bending moment M= 70 kN.



Solution:

Solution by (Internal Couple Method):

$$k = \sqrt{2\,\rho\, n + (\,\rho\, n)^2} - \rho\, n \qquad \rho = \frac{1500}{300\times500} = 0.01$$

$$\rho\, n = 0.08 \quad \therefore\ k = \sqrt{2\times0.08 + (\,0.08)^2} - 0.08 = 0.328$$

$$kd = 0.328 \times 500 = 164\ mm$$

$$j = 1 - \frac{k}{3} = 0.89$$

$$jd = 445\ mm$$

$$f_c = \frac{2\,M}{kjbd^2} = \frac{2\times70\times10^6}{0.328\times0.89\times300\times(500)^2} = 6.39\ MPa$$

$$f_s = \frac{M}{A_s jd} = \frac{70\times10^6}{1500\times445} = 104.9\ MPa$$

Solution by (Transformed Section Method): as previous method we find k then compute I ,

$$I = \frac{300\times164^3}{3} + 8\times1500\times(500-164)^2 = 1795\times10^6 mm^4$$

$$f_c = \frac{M}{I} = \frac{70\times10^6\times164}{1795\times10^6} = 6.39\ MPa$$

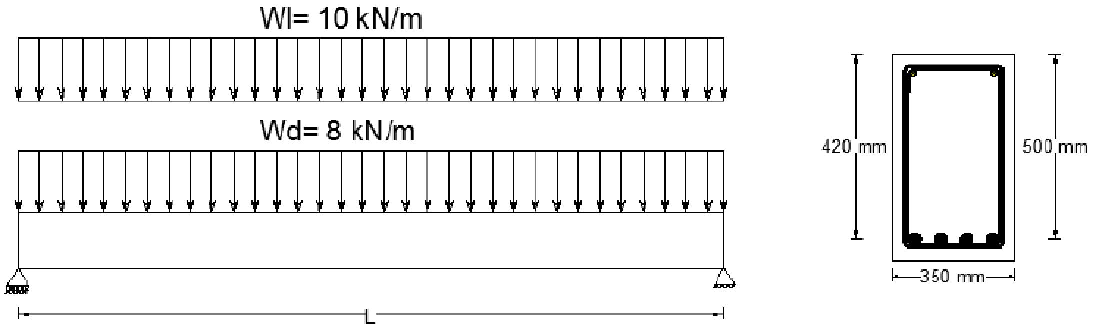
$$f_s = n\, f_{cs} = \frac{n\,M(d-kd)}{I} = \frac{8\times70\times10^6\times(500-164)}{1795\times10^6}$$
$$= 104.8\ MPa$$

Example2: For the simply supported beam shown reinforced by 4ϕ 25 bars (fy =420 MPa), the concrete strength (f'c= 21 MPa) evaluate the following:

1 .If the span of beam =4 m and dead load = 8 kN/m , live load =10 kN/m check the actual flexural stress in concrete and steel.

2 .The length of beam span that make the concrete in tension face start to crack.

3. The actual stress in concrete and steel if the span of beam =7m



Solution: 1.

Total load W= $w_d + w_l$ =8 +10 = 18 k N/m

M= $\frac{W \times L^2}{8} = \frac{18 \times 4^2}{8} = 36\, kN.m$

n= $\frac{E\ steel}{E\ concrete} = \frac{200000}{4700\sqrt{21}} = 9.28$

As =4 × Ab = 4 ×491 =1964 mm^2

Assume $f_t < f_r$

Transformed section area At = (Ac-As) + n As

= Ac + (n-1)As=500 × 350+(9.28 -1)1964

=191261.92 mm^2

$$C = \frac{Ac \times \frac{h}{2} + (n-1)As \times d}{At}$$

$$= \frac{350 \times 500 \times \frac{500}{2} + (9.28-1)1964 \times 420}{191261.92}$$

$$= 264.4\ mm$$

$$I = \frac{b\,h^3}{12} + Ac\left(c - \frac{h}{2}\right)^2 + (n-1)As(d-C)^2$$

$$I \;= \frac{350 \times\ 500^3}{12} + 350 \times 500\left(264.4 - \frac{500}{2}\right)^2 + (9.28-1)1964(420-264.4)^2$$

$$I = 4.076 \times 10^9 mm^4$$

Actual bending stress ($f\ = \frac{Mc}{I}$)

For Compression Fiber:

Actual compression concrete stress : $f_c = \frac{36\times10^6\times264.4}{4.075\times10^9} = 2.33\ MPa$

Concrete allowable compression stress: $F_c = 0.45 \times \acute{f_c} = 0.45 \times 21$

$$= 9.45\ MPa$$

$$\therefore f_c <\ F_c \quad o.k.$$

For Tension Fiber:

Actual tension concrete stress : $f_t = \frac{36\times10^6\times(500-264.4)}{4.075\times10^9} = 2.08\ MPa$

The concrete stress that make initial crack: $f_r = 0.62\ \sqrt{\acute{f_c}}$

$$f_r = 0.62\ \sqrt{21} = 2.84\ MPa$$

$\therefore f_r > f_t$ *The assumption is correct and the section is not cracked*

Actual steel stress : $f_s = 9.28\frac{36\times10^6\times(420-264.4)}{4.075\times10^9} = 12.75\ MPa$

Steel allowable stress: $F_s = 165.5\ MPa \quad for\ \ f_y = 420\ MPa$

$$\therefore f_s <\ F_s \quad o.k.$$

2. To make the concrete start to crack put the concrete tension stress at the extreme fiber equal to concrete stress at rupture ($f_t = f_r$)

$$f_r = 0.62\ \sqrt{21} = 2.84\ MPa$$

$$f_t = \frac{M_{cr}(h-c)}{I}$$

$$2.84 = \frac{M_{cr}(500 - 264.4)}{4.075 \times 10^9} \Rightarrow M_{cr} = 49.12\ kN.m$$

Any moments greater than (49.12 kN.m) will cause concrete cracks.

For maximum beam span length that make the concrete section cracked:

$$M_{cr} = \frac{WL^2}{8}$$

$$49.12 = \frac{18 \times L^2}{8} \Rightarrow L = 4.67\ m$$

3.

$$M \quad = \frac{WL^2}{8} = \frac{18 \times 7^2}{8} = 110.25\ kN.m$$

Since the moments $\mathrm{M} = 110{,}25\ \mathrm{kN.m} > M_{cr} = 49.12\ kN.m$

∴The concrete section cracked.

$k = \sqrt{2\,\rho\,n + (\,\rho\,n)^{2}} - \rho\,n \qquad \rho = \frac{1964}{350 \times 420} = 0.0134$

$\rho\,n = 0.124 \quad \therefore\ k = \sqrt{2 \times 0.124 + (\,0.124)^{2}} - \ 0.124 = 0.389$

$$kd = 0.389 \times 420 = 163.46\, mm$$

$$j = 1 - \frac{k}{3} = 0.87$$

$$jd = 365.54\, mm$$

$$f_c \ = \frac{2\,M}{kjbd^2} = \frac{2 \times 110.25 \times 10^6}{0.389 \times 0.87 \times 350 \times (420)^2} = 10.55\ MPa$$

Concrete allowable compression stress: $F_c = 0.45 \times \acute{f_c} = 0.45 \times 21$

$$= 9.45\ MPa$$

$\therefore f_c > \ F_c \qquad \Rightarrow \mathit{The\ concrete\ behavior\ not\ in\ the\ elastic\ range.}$

$$f_s = \frac{M}{A_s jd} = \frac{110.25 \times 10^6}{1964 \times 365.54} = 153.57\ MPa$$

Steel allowable stress: $F_s = 165.5\ MPa \quad for\ \ f_y = 420\ MPa$

$$\therefore f_s < F_s \quad the\ steel\ stress\ within\ the\ limit.$$

STRENGTH DESIGN APPRACH

The analysis and design of a structural member may be regarded as the process of selecting the proper materials and determining the member dimensions such that the design strength is equal or greater than the required strength. The required strength is determined by multiplying the actual applied loads, the dead load, the assumed live load, and other loads, such as wind, seismic, earth pressure, fluid pressure, snow, and rain loads, by load factors. These loads develop external forces such as bending moments, shear, torsion, or axial forces, depending on how these loads are applied to the structure.

In proportioning reinforced concrete structural members, three main items can be investigated:

1 .The safety of the structure, which is maintained by providing adequate internal design strength.

2 .Deflection of the structural member under service loads. The maximum value of deflection must be limited and is usually specified as a factor of the span, to preserve the appearance of the structure.

3 .Control of cracking conditions under service loads. Visible cracks spoil the appearance of the structure and permit humidity to penetrate the concrete, causing corrosion of steel and consequently weakening the reinforced concrete member. The ACI Code implicitly limits crack widths to 0.016 in. (0.40 mm) for interior members and 0.013 in. (0.33 mm) for exterior members. Control of cracking is achieved by adopting and limiting the spacing of the tension.

It is worth mentioning that the strength design approach was first permitted in the United States in 1956 and in Britain in 1957. The latest ACI Code emphasizes the strength concept based on specified strain limits on steel and concrete that develop tension-controlled, compression controlled, or transition conditions.

ASSUMPTIONS

Reinforced concrete sections are heterogeneous (nonhomogeneous), because they are made of two different materials, concrete and steel. Therefore, proportioning structural members by strength design approach is based on the following assumptions:

1 .Strain in concrete is the same as in reinforcing bars at the same level, provided that the bond between the steel and concrete is adequate.

2 .Strain in concrete is linearly proportional to the distance from the neutral axis.
3 .The modulus of elasticity of all grades of steel is taken as Es = 29 × 106 lb/in^2

(200,000MPa or N/mm2). The stress in the elastic range is equal to the strain multiplied by Es.

4 .Plane cross sections continue to be plane after bending.

5 .Tensile strength of concrete is neglected because (a) concrete's tensile strength is about 10% of its compressive strength, (b) cracked concrete is assumed to be not effective, and (c) before cracking, the entire concrete section is effective in resisting the external moment.

6 .The method of elastic analysis, assuming an ideal behavior at all levels of stress, is not valid. At high stresses, nonelastic behavior is assumed, which is in close agreement with the actual behavior of concrete and steel.

7 . At failure the maximum strain at the extreme compression fibers is assumed equal to 0.003 by the ACI Code provision.

8 . For design strength, the shape of the compressive concrete stress distribution may be assumed rectangular, parabolic, or trapezoidal. In this text, a rectangular shape will be assumed (ACI Code, Section 22.2).

TYPES OF FLEXURAL FAILURE AND STRAIN LIMITS

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used as explained before.

It can be assumed that concrete fails in compression when the concrete strain reaches 0.003. A range of 0.0025 to 0.004 has been obtained from tests and the ACI Code, Section 22.2.2.1, assumes a strain of 0.003.

In beams designed as tension-controlled sections, steel yields before the crushing of concrete. Cracks widen extensively, giving warning before the concrete crushes and the structure collapses. The ACI Code adopts this type of design. In beams designed as balanced or compression-controlled sections, the concrete fails suddenly, and the beam collapses immediately without warning. The ACI Code does not allow this type of design.

Strain Limits for Tension and Tension-Controlled Sections

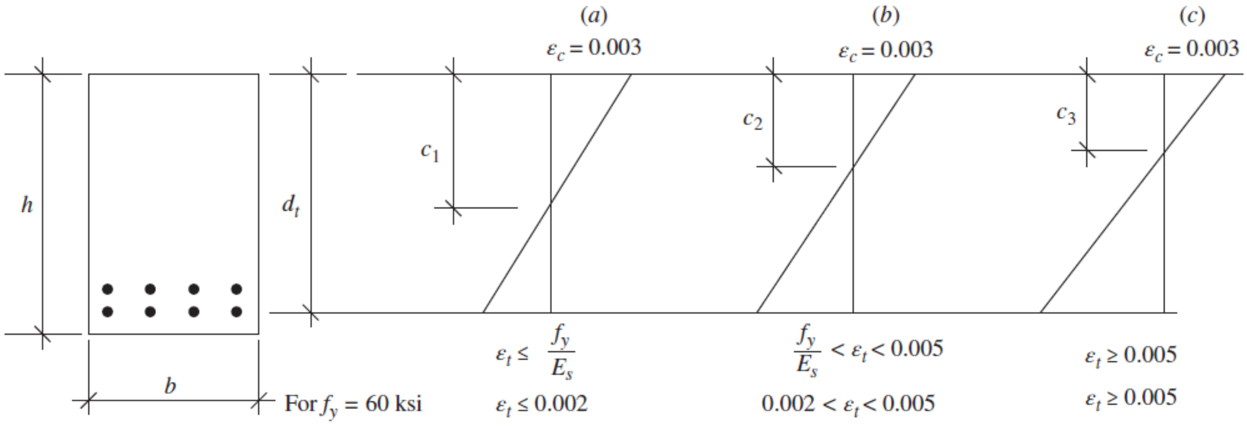
The design provisions for both reinforced and prestressed concrete members are based on the concept of tension or compression-controlled sections, ACI Code, Section 21.2. Both are defined in terms of net tensile strain (NTS), (ε_t, in the extreme tension steel at nominal strength, exclusive of prestress strain. Moreover, two other conditions may develop: (1) the balanced strain condition and (2) the transition region condition. These four conditions are defined as follows:

1 .Compression-controlled sections are those sections in which the net tensile strain, NTS, in the extreme tension steel at nominal strength is equal to or less than the compression-controlled strain limit at the time when concrete in compression reaches its assumed strain limit of 0.003, ($\varepsilon c = 0.003$). For grade 60 steel, ($fy = 420$ MPa), the compression-controlled strain limit may be taken as a net strain of 0.002, Fig. a. This case occurs mainly in columns subjected to axial forces and moments.

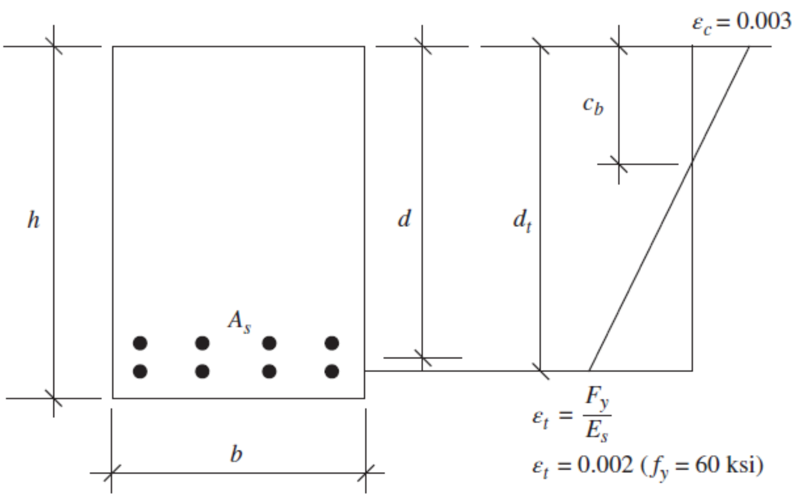
2 .Tension-controlled sections are those sections in which the NTS, εt, is equal to or greater than 0.005 just as the concrete in the compression reaches its assumed strain limit of 0.003, Fig. c.

3 .Sections in which the NTS in the extreme tension steel lies between the compression controlled strain limit (0.002 for fy = 420 MPa) and the tension-controlled strain limit of 0.005 constitute the transition region, Fig. b.

4 .The balanced strain condition develops in the section when the tension steel, with the first yield, reaches a strain corresponding to its yield strength, fy or $\varepsilon s = fy/Es$, just as the maximum strain in concrete at the extreme compression fibers reaches 0.003, Fig. d.



Strain limit distribution, $c_1 > c_2 > c_3$: (*a*) compression-controlled section, (*b*) transition region, and (*c*) tension-controlled section.



d. Balanced strain section (occurs at first yield or at distance d_t).

In addition to the above four conditions, Section 9.3.3.1 of the ACI Code indicates that the net tensile strain, εt, at nominal strength, within the transition region, shall not be less than 0.004 for reinforced concrete flexural members without or with an axial load less than 0.10 f′c Ag, where Ag =gross area of the concrete section.

Note that dt in Fig. above, is the distance from the extreme concrete compression fiber to the extreme tension steel, while the effective depth, d, equals the distance from the extreme concrete compression fiber to the centroid of the tension reinforcement, Fig. 3.5. These cases are summarized in Table below.

Table 1 Strain Limits of Figure above

Section Condition	Concrete Strain	Steel Strain	420MPa Notes (f_y = 60 ksi)
Compression controlled	0.003	$\varepsilon_t \le f_y/E_s$	$\varepsilon_t \le 0.002$
Tension controlled	0.003	$\varepsilon_t \ge 0.005$	$\varepsilon_t \ge 0.005$
Transition region	0.003	$f_y/E_s < \varepsilon_t < 0.005$	$0.002 < \varepsilon_t < 0.005$
Balanced strain	0.003	$\varepsilon_s = f_y/E_s$	$\varepsilon_s = 0.002$
Transition region (flexure)	0.003	$0.004 \le \varepsilon_t < 0.005$	$0.004 \le \varepsilon_t < 0.005$

LOAD FACTORS

For the design of structural members, the factored design load is obtained by multiplying the dead load by a load factor and the specified live load by another load factor. The magnitude of the load factor must be adequate to limit the probability of sudden failure and to permit an economical structural design. The choice of a proper load factor or, in general, a proper factor of safety depends mainly on the importance of the structure (whether a courthouse or a warehouse), the degree of warning needed prior to collapse, the importance of each structural member (whether a beam or column), the expectation of overload, the accuracy of artisanry, and the accuracy of calculations.

Based on historical studies of various structures, experience, and the principles of probability, the ACI Code adopts a load factor of 1.2 for dead loads and 1.6 for live loads. The dead-load factor load. Moreover, the choice of factors reflects the degree of the economical design as well as the degree of safety and serviceability of the structure. It is also based on the fact that the performance of the structure under actual loads must be satisfactorily within specific limits.

If the required strength is denoted by U (ACI Code, Section 5.3.1), and those due to wind and seismic forces are W and E, respectively, according to the ACI and ASCE 7-10 Codes, the required strength, U, shall be the most critical of the following factors:

1. In the case of dead, live, and wind loads,

$$U = 1.4D$$

$$U = 1.2D + 1.6L$$

$$U = 1.2D + 1.0L + 1.0W$$

$$U = 0.9D + 1.0W$$

$$U = 1.2D + (1.0L \text{ or } 0.5W)$$

2. In the case of dead, live, and seismic (earthquake) forces, E,

$$U = 1.2D + 1.0L + 1.0E$$

$$U = 0.9D + 1.0E$$

3 .For load combination due to roof live load, Lr, rain load, R, snow load, S, in addition to dead, live, wind, and earthquake load,

$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$$

$$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S$$

4. Where fluid load F is present, it shall be included as follows:

$$U = 1.4(D + F)$$

$$U = 1.2D + 1.2F + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.2F + (L + 0.5W) + 1.6(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.2F + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$U = 1.2D + 1.2F + 1.0E + L + 0.2S$$

$$U = 0.9(D + F) + 1.0E$$

STRENGTH REDUCTION FACTOR ϕ

The nominal strength of a section, say Mn, for flexural members, calculated in accordance with the requirements of the ACI Code provisions must be multiplied by the strength reduction factor, ϕ, which is always less than 1. The strength reduction factor has several purposes:

1 .To allow for the probability of understrength sections due to variations in dimensions, material properties, and inaccuracies in the design equations.

2 .To reflect the importance of the member in the structure.

3 .To reflect the degree of ductility and required reliability under the applied loads.

The ACI Code, Table 21.2.1, specifies the following values to be used:

For tension-controlled sections	$\phi = 0.90$
For compression-controlled sections	
a. with spiral reinforcement	$\phi = 0.75$
b. other reinforced members	$\phi = 0.65$
For plain concrete	$\phi = 0.60$
For shear and torsion	$\phi = 0.75$
For bearing on concrete	$\phi = 0.65$
For strut and tie models	$\phi = 0.75$

A higher ϕ factor is used for tension-controlled sections than for compression-controlled sections, because the latter sections have less ductility and they are more sensitive to variations in concrete strength. Also, spirally reinforced compression members have a ϕ value of 0.75 compared to 0.65 for tied compression members; this variation reflects the greater ductility behavior of spirally reinforced concrete members under the applied loads. In the ACI Code provisions, the ϕ factor is based on the behavior of the cross section at nominal strength, (Pn, Mn), defined in terms of the NTS, εt, in the extreme tensile strains, as given in Table 1. For tension-controlled members, $\phi = 0.9$. For compression-controlled members, $\phi = 0.75$ (with spiral reinforcement) and $\phi = 0.65$ for other members.

For the transition region, ϕ may be determined by linear interpolation between 0.65 (or 0.75) and 0.9. Figure 3.6a shows the variation of ϕ for grade 60 steel. The linear equations are as follows:

$$\phi = \begin{cases} 0.75 + (\varepsilon_t - 0.002)(50) & \text{(for spiral members)} \\ 0.65 + (\varepsilon_t - 0.002)\left(\dfrac{250}{3}\right) & \text{(for other members)} \end{cases}$$

Alternatively, ϕ may be determined in the transition region, as a function of (c/dt) for grade 60 (fy= 420 Mpa) steel as follows:

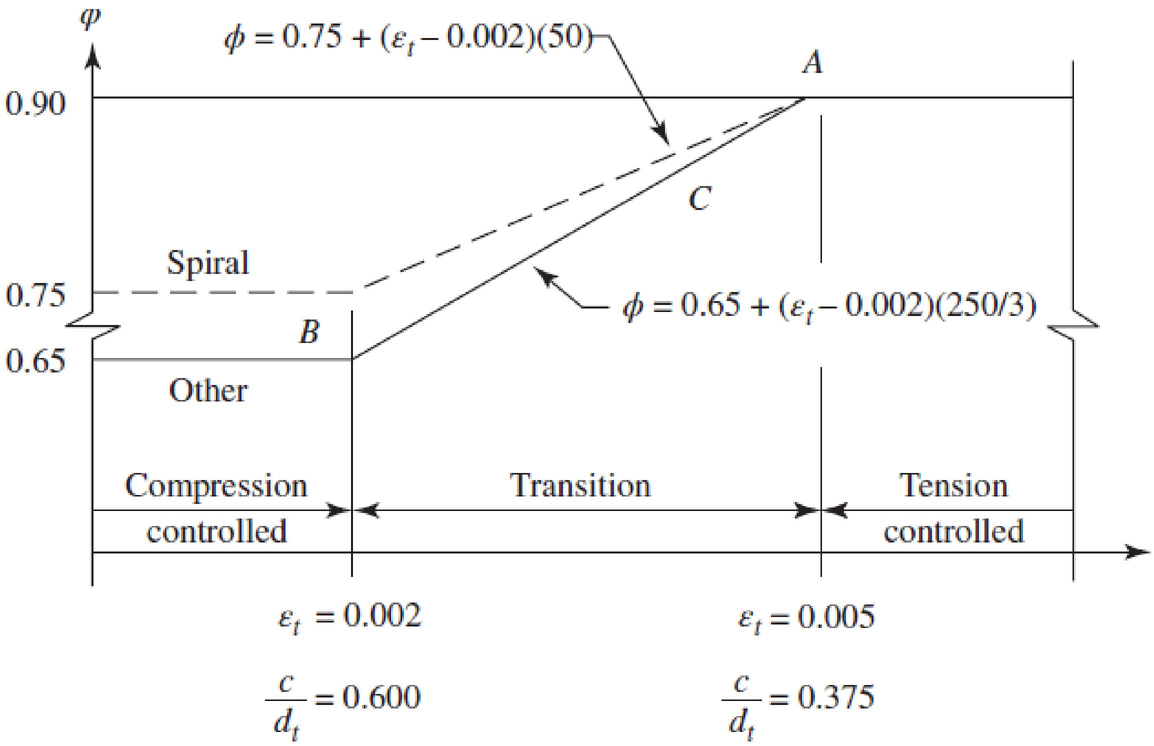
$$\phi = \begin{cases} 0.75 + 0.15\left[\dfrac{1}{c/d_t} - \dfrac{5}{3}\right] & \text{(for spiral members)} \\ 0.65 + 0.25\left[\dfrac{1}{c/d_t} - \dfrac{5}{3}\right] & \text{(for other members)} \end{cases}$$

where c is the depth of the neutral axis at nominal strength (c2 in Fig. Strain limit distribution). At the limit strain of 0.002 for grade 60 steel and from the triangles of Fig. a, c/dt = 0.003/(0.002+0.003) = 0.6. Similarly, at a strain, εt = 0.005, c/dt = 0.003/(0.005+0.003) = 0.375. Both values are shown in Fig. e.

For reinforced concrete flexural members, the NTS, εt, should be equal to or greater than 0.004 (ACI Code, Section 22.2.2). In this case,

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) = 0.82$$

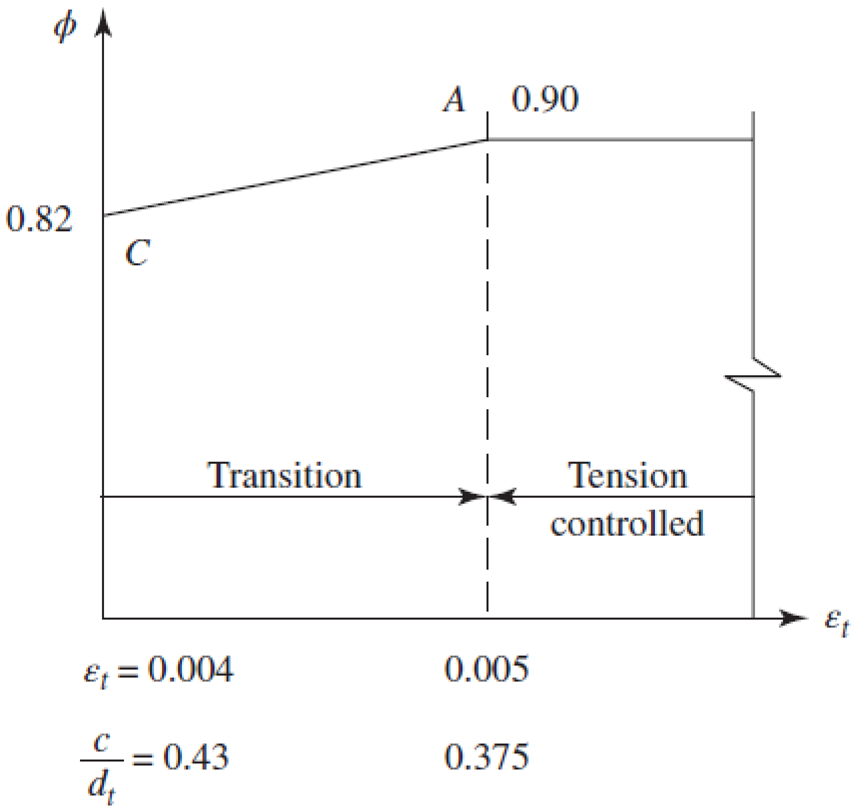
Figure f shows the range of ϕ for flexural members. For grade 60 steel (fy = 420 MPa), the range varies between 0.9 for εt $\geq$ 0.005 and 0.82 for εt = 0.004. Other values of ϕ can be obtained from above equation or by interpolation.



$$\text{Spiral } \phi = 0.75 + 0.15\left[\frac{1}{c/d_t} - \frac{5}{3}\right]$$

$$\text{Other } \phi = 0.65 + 0.25\left[\frac{1}{c/d_t} - \frac{5}{3}\right]$$

(e)



(f)

EQUIVALENT COMPRESSIVE STRESS DISTRIBUTION

The distribution of compressive concrete stresses at failure may be assumed to be a rectangle, trapezoid, parabola, or any other shape that is in good agreement with test results.

When a beam is about to fail, the steel will yield first if the section is underreinforced, and in this case the steel is equal to the yield stress. If the section is overreinforced, concrete crushes first and the strain is assumed to be equal to 0.003, which agrees with many tests of beams and columns. A compressive force, C, develops in the compression zone and a tension force, T, develops in the tension zone at the level of the steel bars. The position of force T is known because its line of application coincides with the center of gravity of the steel bars. The position of compressive force C is not known unless the compressive volume is known and its center of gravity is located. If that is done, the moment arm, which is the vertical distance between C and T, will consequently be known.

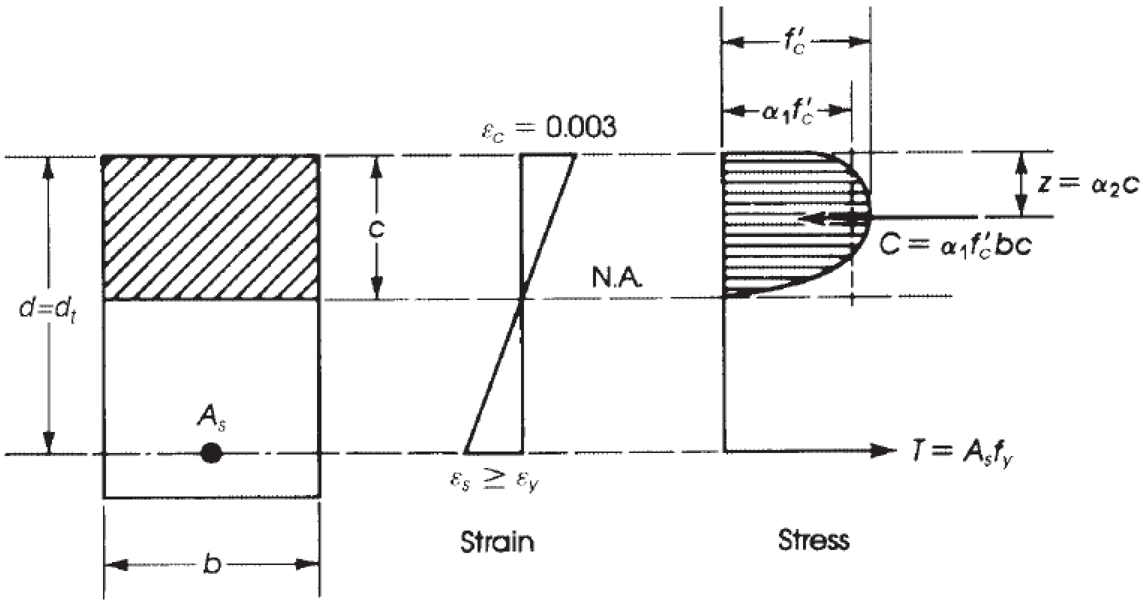


Figure 3.7 Forces in a rectangular section.

In Fig. 3.7, if concrete fails, εc = 0.003, and if steel yields, as in the case of a balanced section, fs = fy.

The compression force C is represented by the volume of the stress block, which has the nonuniform shape of stress over the rectangular hatched area of bc. This volume may be considered equal to C = bc(α1 f'c), where α1 f'c is an assumed average stress of the nonuniform stress block.

The position of compression force C is at a distance z from the top fibers, which can be considered as a fraction of the distance c (the distance from the top fibers to

the neutral axis), and z can be assumed to be equal to $\alpha 2$c, where $\alpha 2$ <1. The values of $\alpha 1$ and $\alpha 2$ have been estimated from many tests, and their values are as follows:

$\alpha 1$ = 0.72 for f'c ≤ (27.6MPa); it decreases linearly by 0.04 for every (6.9MPa) greater than (27.6 Mpa)

$$\alpha 1 = 0.72 - 0.04\,\frac{f_c' - 28}{7}$$

$\alpha 2$ = 0.425 for f'c < (27.6MPa); it decreases linearly by 0.025 for every (6.9MPa) greater than (27.6MPa)

$$\alpha 2 = 0.425 - 0.025\,\frac{f_c' - 28}{7}$$

The decrease in the value of $\alpha 1$ and $\alpha 2$ is related to the fact that high-strength concretes show more brittleness than low-strength concretes.

To derive a simple rational approach for calculations of the internal forces of a section, the ACI Code adopted an equivalent rectangular concrete stress distribution, which was first proposed by C.S. Whitney and checked by Mattock and others. A concrete stress of 0.85 f'c is assumed to be uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a line parallel to the neutral axis at a distance a=$\beta 1$c from the fiber of maximum compressive strain, where c is the distance between the top of the compressive section and the neutral axis (Fig. 3.8). The fraction $\beta 1$ is 0.85 for concrete strengths f'c ≤ (27.6MPa) and is reduced linearly at a rate of 0.05 for each (6.9MPa) of stress greater than (27.6MPa) (Fig. 3.9), with a minimum value of 0.65.

$$\beta 1 = 0.85 - 0.05\,\frac{f_c' - 28}{7} \le 0.65 \quad \text{for} \quad f'c > (28\ \text{MPa})$$

The preceding discussion applies in general to any section, and it is not confined to a rectangular shape. In the rectangular section, the area of the compressive zone is equal to (b * a) , and every unit area is acted on by a uniform stress equal to 0.85f'c , giving a total stress volume equal to 0.85f'c * a *b,

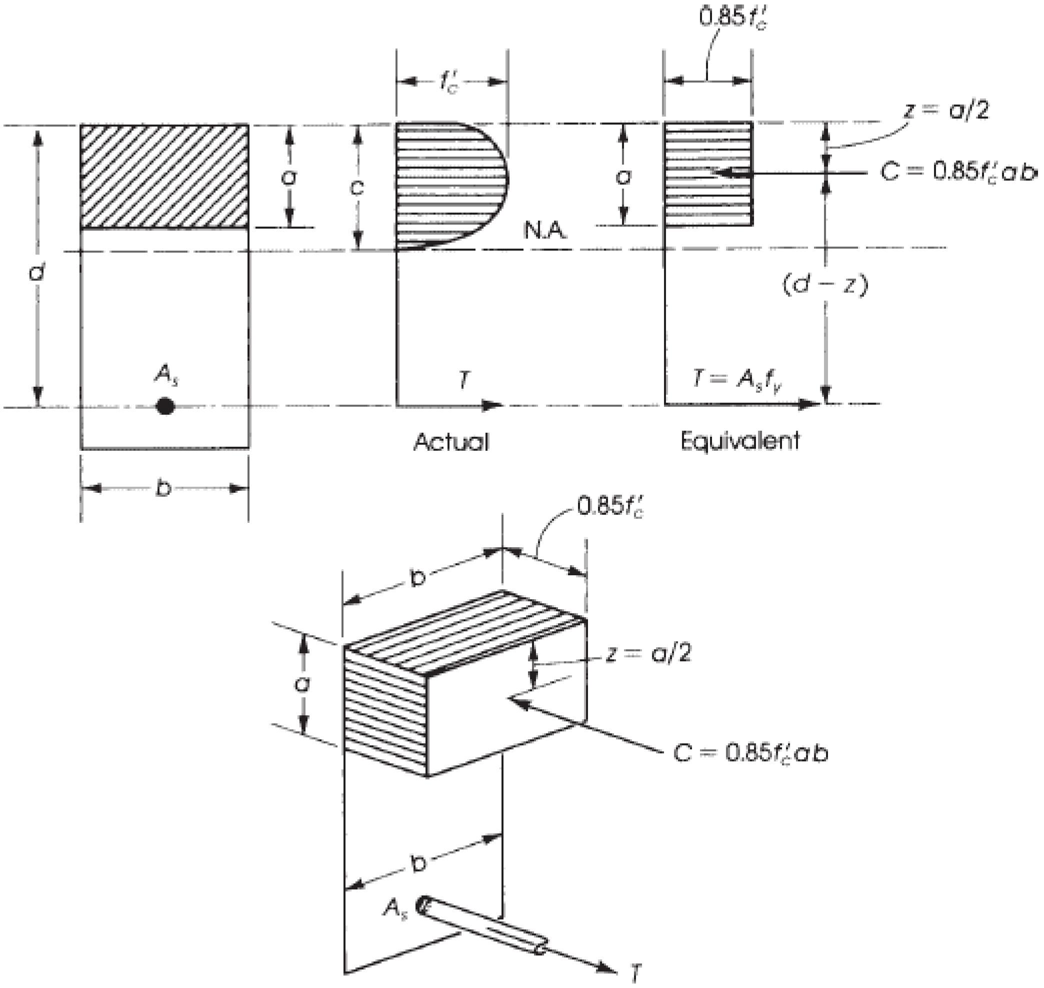
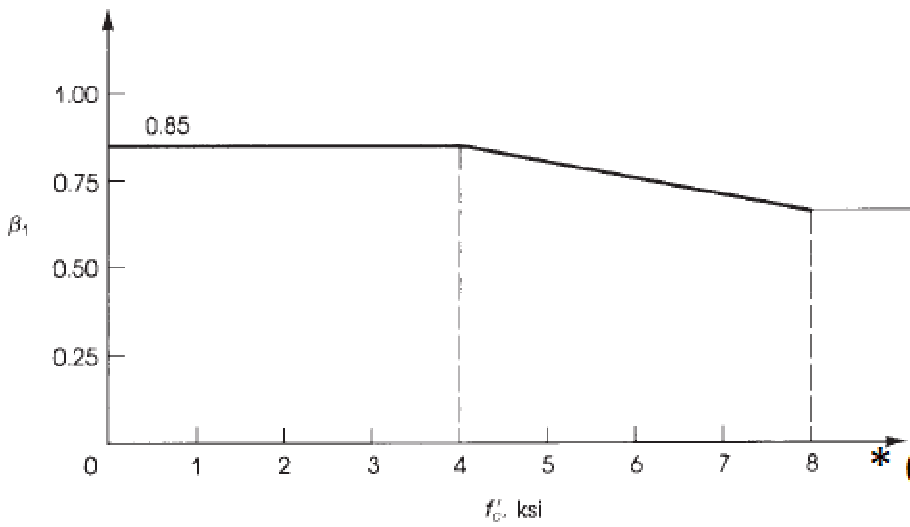


Figure 3.8 Actual and equivalent stress distributions at failure.



* 6.9 to be MPa

Figure 3.9 Values of β_1 for different compressive strengths of concrete, f_c'.

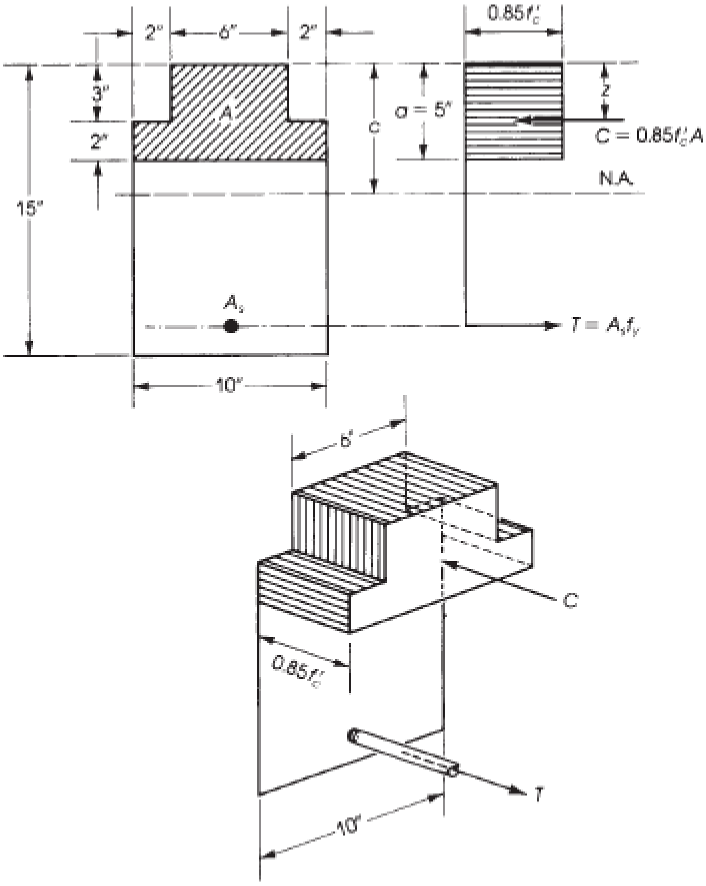


Figure 3.10 Forces in a nonrectangular section.

which corresponds to the compressive force C. For any other shape, the force C is equal to the area of the compressive zone multiplied by a constant stress equal to 0.85f'c.

For example, in the section shown in Fig. 3.10, the force C is equal to the shaded area of the cross section multiplied by 0.85f'c:

C = 0.85f'c (6 × 3 + 10 × 2) = 32.3f'c lb = 20178.5 f'c N

The position of the force C is at a distance z from the top fibers, at the position of the resultant force of all small-element forces of the section. As in the case when the stress is uniform and equals 0.85f'c , the resultant force C is located at the center of gravity of the compressive zone, which has a depth of a.

In this example, z is calculated by taking moments about the top fibers:

$$z = \frac{\left(6 \times 3 \times \frac{3}{2}\right) + 10 \times 2(1 + 3)}{6 \times 3 + 10 \times 2} = \frac{107}{38} = 2.82\text{in.}$$

SINGLY REINFORCED RECTANGULAR SECTION IN BENDING

The balanced condition is achieved when steel yields at the same time as the concrete fails, and that failure usually happens suddenly. This implies that the yield strain in the steel is reached ($\varepsilon y = fy/Es$) and that the concrete has reached its maximum strain of 0.003.

The percentage of reinforcement used to produce a balanced condition is called the balanced steel ratio, ρb. This value is equal to the area of steel, As, divided by the effective cross section, bd:

$$\rho_b = \frac{A_s(\text{balanced})}{bd}$$

where

b = width of compression face of member

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

Two basic equations for the analysis and design of structural members are the two equations of equilibrium that are valid for any load and any section:

1 .The compression force should be equal to the tension force; otherwise, a section will have linear displacement plus rotation:

$$C = T$$

2 .The internal nominal bending moment, Mn, is equal to either the compressive force, C, multiplied by its arm or the tension force, T, multiplied by the same arm:

$$Mn = C(d - z) = T(d - z)$$

(Mu = ϕMn after applying a reduction factor ϕ)

The use of these equations can be explained by considering the case of a rectangular section with tension reinforcement (Fig. 3.8). The section may be balanced, underreinforced, or overreinforced, depending on the percentage of steel reinforcement used.

Balanced Section

Let us consider the case of a balanced section, which implies that at maximum load the strain in concrete equals 0.003 and that of steel equals the first yield stress at distance dt divided by the modulus of elasticity of steel, fy/Es. This case is explained by the following steps.

Step 1. From the strain diagram of Fig. 3.11,

$$\frac{c_b}{d_t - c_b} = \frac{0.003}{f_y/E_s}$$

From triangular relationships (where cb is c for a balanced section) and by adding the numerator to the denominator,

$$\frac{c_b}{d_t} = \frac{0.003}{0.003 + f_y/E_s}$$

Substituting Es = 2×10^5 MPa,

For f_y in MPa $$C_b = \left(\frac{600}{600+f_y}\right) dt$$ ….. Eq.1

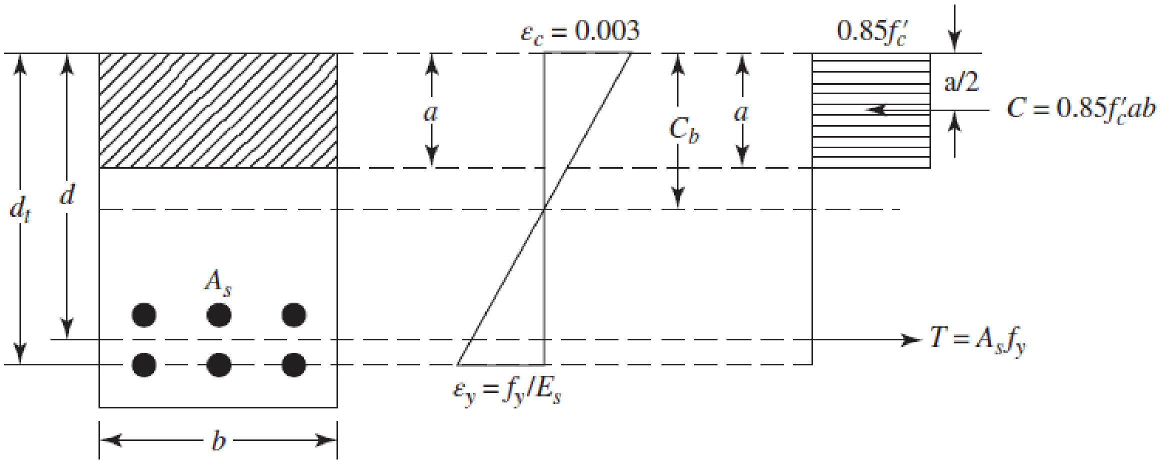


Figure 3.11 Rectangular balanced section.

Step 2. From the equilibrium equation,

C = T ➡ 0.85 f'c a b = As fy ……. Eq.2

$$a = \frac{A_s f_y}{0.85\ f'_c b}$$ …… Eq.3

Here, a is the depth of the compressive block, equal to $\beta 1$c, where $\beta 1$ = 0.85 for $f_c' \leq$ (27.6MPa) and decreases linearly by 0.05 per (6.9MPa) for higher concrete strengths (Fig. 3.9). Because the balanced steel reinforcement ratio is used,

$$\rho_b = \frac{A_s(\text{balanced})}{bd} = \frac{A_{\text{sb}}}{bd}$$

and substituting the value of Asb in Eq.2,

$$0.85 f_c' ab = f_y \rho_b bd$$

Therefore,

$$\rho_b = \frac{0.85 f_c'}{f_y d} a = \frac{0.85 f_c'}{f_y d} (\beta_1 c_b)$$

Substituting the value of cb from Eq.1, the general equation of the balanced steel ratio becomes

$$\boxed{\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \left(\frac{600}{600 + f_y} \right) \left(\frac{d_t}{d} \right)}$$

The value of d_t is equal to d when only one single layer of steel is provided.

Step 3. The internal nominal moment, Mn, is calculated by multiplying either C or T by the distance between them:

$$M_n = C(d - z) = T(d - z)$$

For a rectangular section, the distance z=a/2 as the line of application of the force C lies at the center of gravity of the area (a b), where

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$M_n = C\left(d - \frac{1}{2}a\right) = T\left(d - \frac{1}{2}a\right)$$

For a balanced or an underreinforced section, T=As fy. Then

$$M_n = A_s f_y \left(d - \frac{1}{2}a\right) \qquad \text{… Eq.4}$$

To get the usable design moment φMn, the previously calculatedMn must be reduced by the capacity reduction factor, ϕ,

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) = \phi A_s f_y \left(d - \frac{A_s f_y}{1.7 f'_c b}\right) \qquad \text{… Eq.5}$$

Equation 5 can be written in terms of the steel percentage ρ:

$$\rho = \frac{A_s}{bd} \qquad A_s = \rho bd$$

$$\phi M_n = \phi f_y \rho bd \left(d - \frac{\rho bd f_y}{1.7 f'_c b}\right) = \phi \rho f_y bd^2 \left(1 - \frac{\rho f_y}{1.7 f'_c}\right) \qquad \text{Eq.6}$$

Equation 6 can be written as

$$\phi M_n = R_u bd^2$$

where

$$R_u = \phi \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c}\right)$$

The ratio of the equivalent compressive stress block depth, a, to the effective depth of the section, d, can be found from Eq.2:

$$0.85 f'_c ab = \rho bd f_y$$

$$\frac{a}{d} = \frac{\rho f_y}{0.85 f'_c}$$

Upper Limit of Steel Percentage

The upper limit or the maximum steel percentage, ρ_{max}, that can be used in a singly reinforced concrete section in bending is based on the net tensile strain in the tension steel, the balanced steel ratio, and the grade of steel used. The relationship between the steel percentage, ρ, in the section and the net tensile strain, ε_t, is as follows:

$$\varepsilon_t = \left(\frac{0.003 + f_y/E_s}{\rho/\rho_b}\right) - 0.003 \qquad (3.24)$$

For fy = 420 MPa, and assuming fy/Es = 0.002,

$$\varepsilon_t = \left(\frac{0.005}{\rho/\rho_b}\right) - 0.003 \qquad (3.25)$$

These expressions are obtained by referring to Fig. 3.12. For a balanced section,

$$c_b = \frac{a_b}{\beta_1} = \frac{A_{sb} f_y}{0.85 f'_c b \beta_1} = \frac{\rho_b f_y d}{0.85 f'_c \beta_1}$$

Similarly, for any steel ratio, ρ,

$$c = \frac{\rho f_y d}{0.85 f'_c \beta_1} \qquad \text{and} \qquad \frac{c}{c_b} = \frac{\rho}{\rho_b}$$

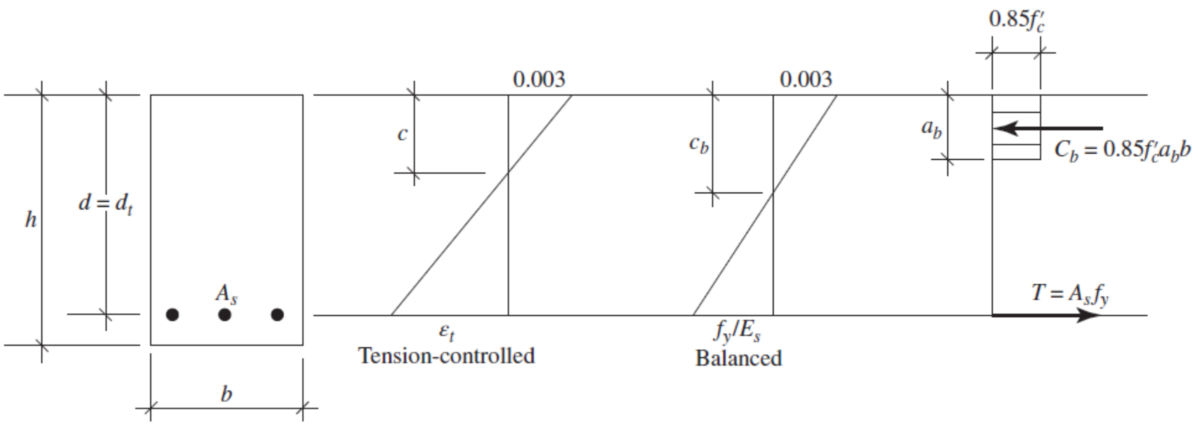


Figure 3.12 Strains in tension-controlled and balanced conditions. ($d = d_t$ one layer of steel).

Divide both sides by d to get

$$\frac{c}{d} = \left(\frac{\rho}{\rho_b}\right)\left(\frac{c_b}{d}\right) \qquad (3.26)$$

From the triangles of the strain diagrams,

$$\frac{c}{d} = \frac{0.003}{0.003 + \varepsilon_t}$$

$$\varepsilon_t = \frac{0.003}{c/d} - 0.003 \qquad (3.27)$$

Similarly,

$$\frac{c_b}{d} = \frac{0.003}{0.003 + f_y/E_s} \qquad (3.28)$$

Substituting Eq. 3.28 into Eq. 3.26

$$\frac{c}{d} = \left(\frac{\rho}{\rho_b}\right)\left(\frac{c_b}{d}\right) = \left(\frac{\rho}{\rho_b}\right)\left(\frac{0.003}{0.003 + f_y/E_s}\right) \quad \text{(From Eq. 3.26)}$$

Substitute this value in Eq. 3.27 to get

$$\varepsilon_t = \frac{0.003}{c/d} - 0.003 = \left[\frac{0.003 + f_y/E_s}{\rho/\rho_b}\right] - 0.003 \quad \text{(From Eq. 3.27)}$$

For grade 60 steel, fy = 420 Mpa, Es = 200000 MPa, and fy/Es = 0.0021, then

$$\varepsilon_t = \left(\frac{0.0051}{\rho/\rho_b}\right) - 0.003 \quad \text{(From Eq. 3.25)}$$

To determine the upper limit or the maximum steel percentage, ρ, in a singly reinforced concrete section, refer to Fig. 3.6. It can be seen that concrete sections

subjected to flexure or axial load and bending moment may lie in compression-controlled, transition, or tension-controlled zones.

When $\varepsilon_t \leq 0.002$ (or $c/d_t \geq 0.6$), compression controls, whereas when $\varepsilon_t \geq 0.005$ (or $c/d_t \leq 0.375$), tension controls. The transition zone occurs when $0.002 < \varepsilon_t < 0.005$ or $0.6 > c/d_t > 0.375$.

For members subjected to flexure, the relationship between the steel ratio, ρ, was given in Eq. 3.24:

$$\varepsilon_t + 0.003 = \frac{0.003 + f_y/E_s}{\rho/\rho_b} \qquad (3.24)$$

$$\frac{\rho}{\rho_b} = \frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t} \qquad (3.29)$$

For (fy = 60 ksi = 420 MPa) and Es = 200,000 MPa, fy/Es may be assumed to be 0.0021.

$$\frac{\rho}{\rho_b} = \frac{0.0051}{0.003 + \varepsilon_t} \qquad (3.30)$$

The limit for tension to control is $\varepsilon t \geq 0.005$ according to ACI. For $\varepsilon_t = 0.005$, Eq. 3.30 becomes

$$\frac{\rho}{\rho_b} = \frac{0.005}{0.008} = \frac{5}{8} = 0.625 \qquad (3.30a)$$

or $\rho \leq 0.63375\ \rho b$ for tension-controlled sections if $\varepsilon t = 0.0051 = fy/Es$. Both values can be used for practical analysis and design. The small increase in ρ will slightly increase the moment capacity of the section. For example, if f_c' = 4 ksi = 28 MPa and fy = 60 ksi = 420 MPa, $\rho b = 0.0283$ and $\rho \leq 0.01806$ for tension to control (as in the case of flexural members). The ϕ factor in this case is 0.9. This value is less than $\rho_{max} = 0.75\rho b = 0.0214$ allowed by the ACI Code for flexural members when $\phi = 0.9$ can be used.

Design of beams and other flexural members can be simplified using the limit of $\varepsilon_t = 0.005$.

$$\frac{\rho}{\rho_b} = \frac{0.003 + f_y/E_s}{0.008} \quad (3.31)$$

In this case, $\rho = \rho_{max}$ =upper limit for tension-controlled sections.

$$\rho_{max} = \left(\frac{0.003 + f_y/E_s}{0.008}\right)\rho_b \quad (3.31a)$$

Note that when ρ used $\leq \rho_{max}$, tension controls and ϕ = 0.9. When $\rho > \rho_{max}$, the section will be in the transition region with $\phi < 0.9$.

And for fy = 420 MPa and fy/Es = 0.0021,

$$\frac{\rho_{max}}{\rho_b} = 0.63375 \quad (3.32)$$

This steel ratio will provide adequate ductility before beam failure.

Similarly,

$$\rho_{max} = \begin{cases} 0.5474\rho_b \text{ for } f_y = 280 \text{ MPa} & (3.32a) \\ 0.5905\rho_b \text{ for } f_y = 350 \text{ MPa} & (3.32b) \\ 0.63375\cdot\rho_b \text{ for } f_y = 420 \text{ MPa} & (3.32c) \\ 0.6983\rho_b \text{ for } f_y = 525 \text{ MPa} & (3.32d) \end{cases}$$

It was established that ϕMn =R_u b d^2, where $R_u = \phi\, \rho$ fy $(1 - \rho$ fy$/1.7f_c')$.

Assume $m = \frac{f_y}{0.85\, f_c'}$ then $R_u = \emptyset\, \rho\, f_y(1 - \frac{1}{2}\, m)$

Once f_c' and fy are known, then ρb, ρ, R_u, and b d^2 can be calculated. For example, for f_c'= 28 MPa, fy = 420 MPa, ϕ = 0.9, ε_t = 0.005, and one row of bars in the section,

$$\rho_b = 0.85\beta_1\frac{f_c'}{f_y}\left(\frac{600}{600+f_y}\right)\left(\frac{d_t}{d}\right) = 0.85 * 0.85\,\frac{28}{420}\left(\frac{600}{600+420}\right)\left(\frac{1}{1}\right) = 0.283$$

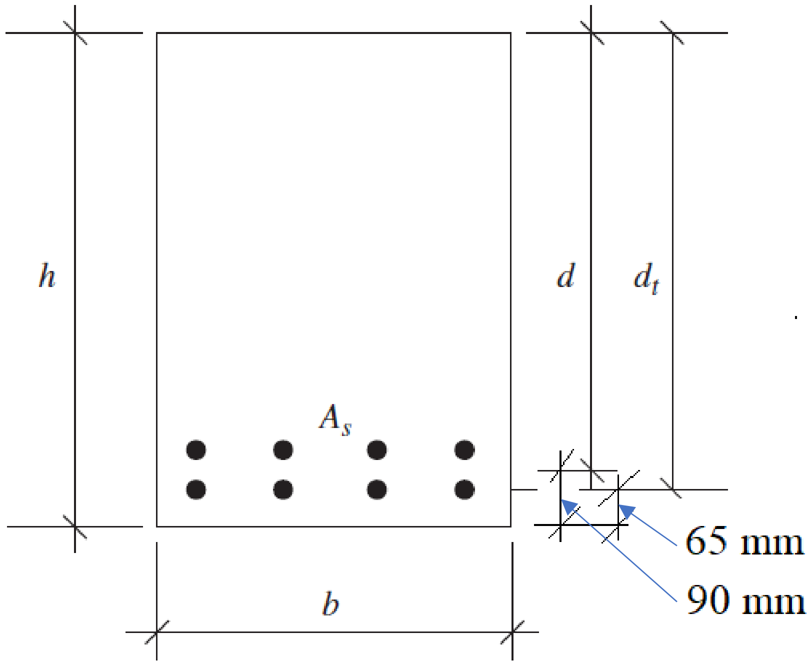
$$\rho = \frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\,\rho_b = \frac{0.003+420/200000}{0.003+0.005} * 0.283 = 0.01804$$

$$R_u = \phi \rho f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right) = 0.9 * 0.01806 * 420 \left(1 - \frac{0.01806*420}{1.7*28}\right) = 5.7 \quad MPa$$

Or from $m = \frac{f_y}{0.85\, f_c'} = \frac{420}{0.85*28} = 17.6$

$$then\ \ R_{u\,max} = \emptyset\, \rho\, f_y \left(1 - \frac{1}{2}\, m\right) = 0.9 * 0.01804 * 420 * (1 - 0.5 * 17.6) = 5.7\, MPa$$

Note that for one row of bars in the section, it can be assumed that d=d_t =h−65mm., whereas for two rows of bars, d=h−90mm, and d_t =h−65mm = d+25 mm.



For reinforced concrete flexural members with $\rho > \rho$max, ε_t will be less than 0.005. Section 9.3.3.1 of the ACI Code specifies that ε_t should not be less than 0.004 in the transition region to maintain adequate ductility and warning before failure.

For this limitation of ε_t = 0.004, the general equation (3.29)

$$\frac{\rho}{\rho_b} = \frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t} \qquad (3.29)$$

becomes

$$\frac{\rho}{\rho_b} = \frac{0.003 + f_y/E_s}{0.007} \qquad (3.33)$$

For fy = 420 MPa $\quad \frac{\rho}{\rho_b} = \frac{0.003 + 0.00207}{0.007} = 0.724 \qquad (3.34)$

and the limit in the transition region is $\quad \rho_{\max\ t} = 0.724\, \rho_b$

Note that the t here refers to the transition region. In this case from Fig. e

$$\phi_t = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right)$$

Then the limit of ϕ_t is for ε_t = 0.004 becomes

$$\emptyset_t = 0.65 + (0.004 - 0.002)\left(\frac{250}{3}\right) = 0.817 < 0.9$$

For fy = 420 MPa and f_c' = 28 Mpa, ρ_b = 0.0283

$$\rho_{\max\ t} = 0.724\,\rho_b \qquad \therefore \qquad \rho_{\max\ t} = 0.0205$$

$$and \quad m = \frac{f_y}{0.85\, f_c'} = \frac{420}{0.85 * 28} = 17.6$$

$$\therefore\ R_{n\ max\ t} = \rho_{max\ \ t}\, f_y\left(1 - \frac{1}{2}\,m\right) = 0.0205 * 420 * (1 - 0.5 * 17.6) = 7.05 \text{ MPa}$$

$\therefore$ Ru = ϕRn = 0.817(7.05) = 5.76 MPa.

This steel ratio in Eq. 3.33 is the upper limit ($\rho_{\max\ t}$) for a singly reinforced concrete section in the transition region with ϕ< 0.9.

It can be noticed that the aforementioned Ru = 5.74 MPa calculated for ε_t = 0.004 is very close to Ru = 5.7 MPa for $\rho_{\max}$ = 0.6375ρ_b and ϕ = 0.9. Therefore, adding reinforcement beyond $\rho_{\max}$ (for ε_t = 0.005) reduces ϕ because of the reduced ductility resulting in little or non-substantial gain in design strength. Adding compression reinforcement in the section is a better solution to increase the design moment, keeping the section in the tension-controlled region with ϕ = 0.9.

Example 1

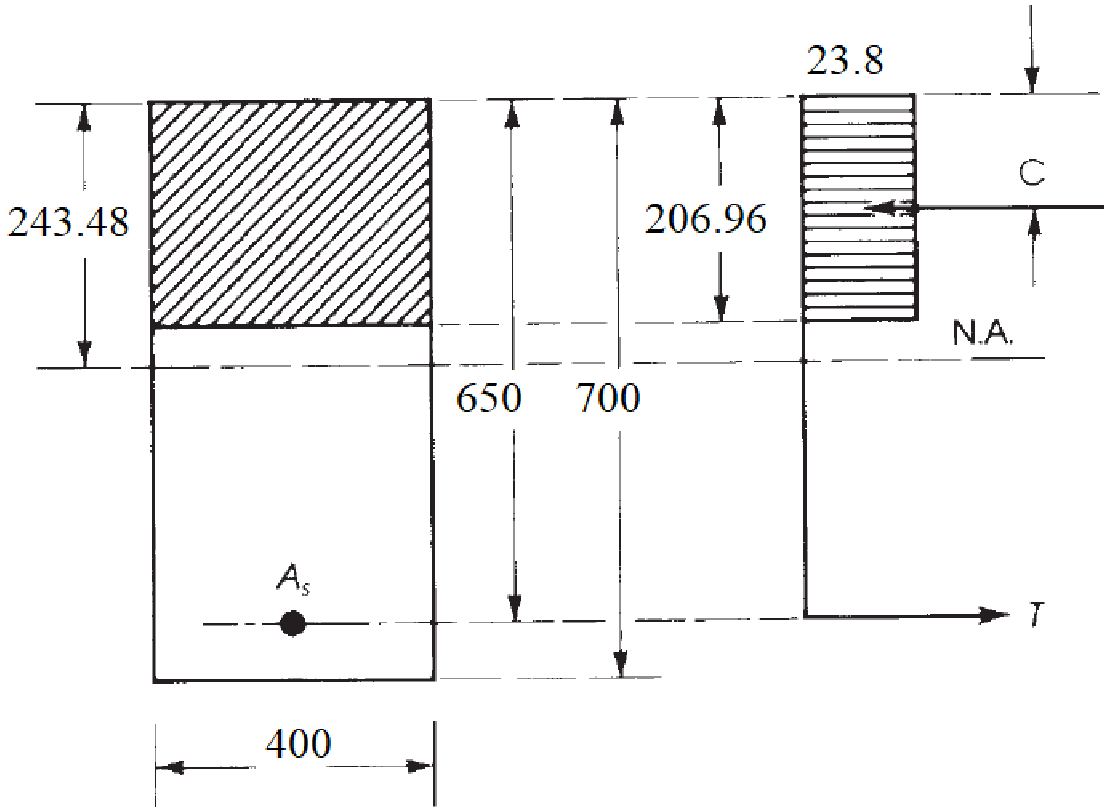
For the section shown below, calculate

a. The balanced steel reinforcement

b. The maximum reinforcement area allowed by the ACI Code for a tension-controlled section and in the transition region

c. The position of the neutral axis and the depth of the equivalent compressive stress block for the tension-controlled section in b.

Given: f_c' = 28 MPa and fy = 420 MPa.



Solution

a. $\rho_b = 0.85\beta_1 \frac{f'_c}{f_y}\left(\frac{600}{600+f_y}\right)\left(\frac{d_t}{d}\right)$ or $\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+f_y}\right)\left(\frac{d_t}{d}\right)$

$\beta 1 = 0.85$ for $f'_c \leq 28$ MPa

dt = d $\longrightarrow$ $(d_t / d) = 1$

$$m = \frac{f_y}{0.85\, f'_c} = \frac{420}{0.85 * 28} = 17.6$$

$$\rho_b = \frac{0.85}{17.6}\left(\frac{600}{600 + 420}\right)(1) = 0.0283$$

The area of steel reinforcement to provide a balanced condition (A_{sb}) is

$$A_{sb} = \rho_b\, b\, d = 0.0283 * 400 * 650 = 7358\, mm^2$$

b. from, $\rho = \frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\rho_b$ and for the maximum reinforcement area allowed by the ACI Code, at ρ_{max} the $\varepsilon_t \geq 0.005$ then

$$\rho_{\max} = \left(\frac{0.003 + f_y/E_s}{0.008}\right)\rho_b = \left(\frac{0.003+420/200000}{0.008}\right) * 0.0283 = 0.01804$$

$$A_{s\ max} = \rho_{max}\, b\, d = 0.01804 * 400 * 650 = 4690\, mm^2 \qquad \text{for } \phi = 0.9$$

For the transition region, the $\rho_{max\ t}$, at the case of ε_t = 0.004

$$\rho = \frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\, \rho_b \quad \text{then} \quad \rho_{max\ .t} = \frac{0.003 + f_y/E_s}{0.003 + 0.004}\, \rho_b$$

$$\rho_{max\ .t} = \frac{0.003 + f_y/E_s}{0.007}\, \rho_b = 0.729\ \rho_b = 0.729 * 0.0283 = 0.0219$$

$$A_{s\ max\ t} = \rho_{max\ t}\ b\, d = 0.0219 * 400 * 650 = 5694\, mm^2 \qquad \text{for } \phi = 0.817$$

c. The depth of the equivalent compressive block using $A_{s\ max}$ is

C = T

$$0.85 f_c'\, a_{max}\, b = A_{s\ max} f_y$$

$$a_{max} = \frac{A_{s\ max} f_y}{0.85 f_c' b}\, \frac{d}{d} = \rho_{max}\, m\, d = 0.01804 * 17.6 * 650 = 206.96\, mm$$

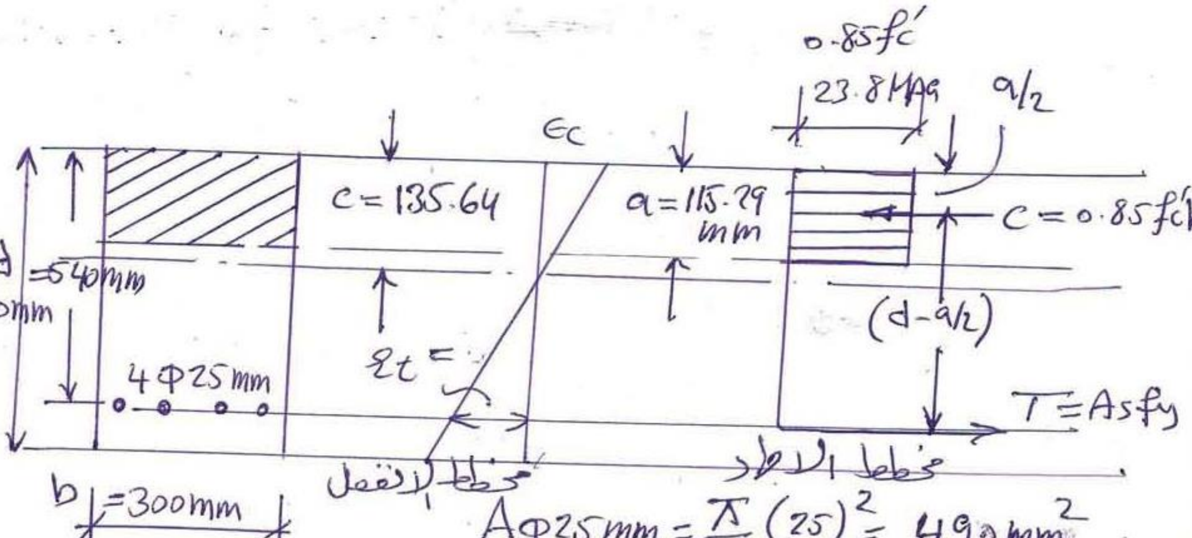
The distance from the top fibers to the neutral axis is c=a/$\beta1$. Because f_c' = 28 MPa, $\beta1$ =0.85; thus,

$$c_{max} = \frac{a_{max}}{\beta_1} = \frac{206.96}{0.85} = 243.48\, mm$$

Or,

$$\frac{c_{max}}{d} = 0.375 \rightarrow c_{max} = 0.375\, d = 0.375 * 650 = 243.48\, mm$$

Example(2) Determine the design moment strength and the postion of the neutral axis of rectangular section shown in Fig. below, if the reinforcement used is 4φ25mm, Given $f'_c = 28$ Mpa, $f_y = 420$ Mpa.



$$A_{\phi 25mm} = \frac{\pi}{4}(25)^2 = 490\ mm^2$$

$$A_s = 4\phi 25mm = 4 \times 490 = 1960\ mm^2$$

$$\rho = \frac{A_s}{bd} = \frac{1960}{300 \times 540} = 0.012098 < \rho_{max} = 0.01804$$

اذن الـ ρ هو اصغر

قيمة $f'_c = 28$ Mp , $f_y = 420$ Mpa من جدول (1) ص.

$$\rho_{max} = 0.6375\,\rho_b = 0.01804$$

$\phi = 0.9$ علماً ان الـ ρ اصغر

$$C = T \Rightarrow 0.85 f'_c b.a = A_s f_y \Rightarrow a = \frac{A_s f_y}{0.85 f'_c b}$$

$$\therefore a = \frac{1960 \times 420}{0.85 \times 28 \times 300} = 115.29mm \quad \text{or} \quad a = \rho m d = 0.012098 \times 17.65 \times 540 = 115.29\ mm$$

$$C = \frac{a}{\beta_1} = \frac{115.29}{0.85} = 135.64\ mm \quad , \quad \beta_1 = 0.85 \text{ for } f'_c \leq 28 \text{ MPa}$$

$d_t = d = 540$ (صف واحد)

من مخطط الانفعال

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right) \cdot \varepsilon_c = \left(\frac{540 - 135.64}{135.64}\right) \times 0.003 = 0.00894 > \varepsilon_t = 0.005$$

تابع

$$\therefore \phi = 0.90$$

or $\frac{c}{d_t} = \frac{135.64}{540} = 0.251 < 0.375$ ok

الشد هو البسيط

$$\phi M_n = \phi T\,(d - a/2)$$
$$= 0.9 \times A_s f_y\,(d - a/2)$$
$$= 0.9 \times 1960 \times 420\left(540 - \frac{115.29}{2}\right) = 357.37 \times 10^6 \text{ N.mm}$$
$$\therefore \phi M_n = 357.37 \text{ kN.m}$$

Example (3) Repeat Example (2) using 4 ϕ32mm as tension steel

$$A_{\phi 32} = (32)^2 \frac{\pi}{4} = 804 \text{ mm}^2$$
$$A_s = 4\phi 32 = 4 \times 804 \text{ mm}^2 = 3216 \text{ mm}^2$$

من المثال الاول

$$\rho_b = 0.0283$$
$$\rho_{max} = \left(\frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\right) = 0.01804$$
$$\rho = \frac{A_s}{bd} = \frac{3216}{300 \times 540} = 0.01985 > 0.01804$$
$$\rho_{max,t} = \left(\frac{0.003 + f_y/E_s}{0.003 + 0.004}\right)\rho_b = \frac{0.0051}{0.007}\rho_b = 0.728\,\rho_b$$
$$\rho_{max,t} = 0.728 \times 0.0283 = 0.0206 > \rho = 0.01985$$

اذن المقطع في مرحلة الانتقالية

$$\phi = 0.817 - 0.9$$
$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3216 \times 420}{0.85 \times 28 \times 300} = 189.17 \text{ mm}$$
$$c = \frac{a}{\beta_1} = \frac{189.17}{0.85} = 222.56$$
$$\varepsilon_t = \left(\frac{d-c}{c}\right)\varepsilon_c = \left(\frac{540 - 222.56}{222.56}\right) \times 0.003 = 0.00428 > 0.004$$

المرحلة الانتقالية

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right)$$
$$= 0.65 + (0.00428 - 0.002)\left(\frac{250}{3}\right) = 0.84$$
$$\phi M_n = \phi A_s f_y (d - a/2)$$
$$= 0.84 * 3216 * 420 \left(540 - \frac{189.17}{2}\right) = 505.37 \times 10^6 \ N.mm$$
$$\underline{\underline{\phi M_n = 505.37 \ kN.m}}$$

لو تم استخدام $A_{smax} = \rho_{max} b d$
$$= 0.01804 \times 300 \times 540 = 2922 \ mm^2$$
فإن هذه المساحة توفر مقاومة داخلية قدرها ϕM_n
$$\phi M_n = 0.9 A_s f_y (d - a/2)$$
$$a = \rho m d = 0.01804 \times 17.65 \times 540 = 171.94 \ mm$$
$$\phi M_n = 0.9 \times 2922 \times 420 \left(540 - \frac{171.94}{2}\right) = 501.48 \times 10^6 \ N.mm$$
$$\underline{\underline{\phi M_n = 501.48 \ kN.m}}$$

نلاحظ أن قيم ϕM_n للحالتين تقريباً متساوية
رغم أنه في الحالة الانتقالية تم استخدام $A_s = 3216 \ mm^2$
بينما عند أقصى قيمة لتسليح الشد البسيط $A_s = 2922 \ mm^2$
فرق مقدار المساحة الموجودة بين الحالتين $= 3216 - 2922 = 294 \ mm^2$
مع ذلك، لم توفر أي مقاومة إضافية، معنى ذلك
تصميم غير اقتصادي عندما $\rho > \rho_{max}$ نفضل التسليح الثنائي
doubly reinforced أي وضع تسليح إضافي في منطقة خرسانة الانضغاط

LOWER LIMIT OR MINIMUM PERCENTAGE OF STEEL

If the factored moment applied on a beam is very small and the dimensions of the section are specified (as is sometimes required architecturally) and are larger than needed to resist the factored moment, the calculation may show that very small or no steel reinforcement is required. In this case, the maximum tensile stress due to bending moment may be equal to or less than the modulus of rupture of concrete fr = $\lambda\ 7.5\sqrt{f_c'}$. If no reinforcement is provided, sudden failure will be expected when the first crack occurs, thus giving no warning. The ACI Code, Section 9.6.1, specifies a minimum steel area, As min,

$$A_{s,\min} = \left(\frac{0.25\sqrt{f_c'}}{f_y}\right) b_w d \geq \left(\frac{1.4}{f_y}\right) b_w d$$

The two minimum ratios are equal when f_c' = 31 MPa. This indicates that

$$\rho_{\min} = \begin{cases} \dfrac{1.4}{f_y} & \text{for } f_c' < 31 \text{ MPa} \\ \dfrac{0.25\sqrt{f_c'}}{f_y} & \text{for } f_c' \geq 31 \text{ MPa} \end{cases}$$

In the case of a rectangular section, use b=bw in the preceding expressions. For statically determinate T-sections with the flange in tension, as in the case of cantilever beams, the value of As, min must be equal to or greater than following equation:

$$A_{s,\min} = \left(\frac{0.25\sqrt{f_c'}}{f_y}\right)(x)(d) \geq \frac{1.4\ \ xd}{f_y}$$

where

x = 2 b_w or b_f whichever is smaller

b_w = width of web

b_f = width of flange

ADEQUACY OF SECTIONS

A given section is said to be adequate if the internal moment strength of the section is equal to or greater than the externally applied factored moment, Mu, or ϕMn ≥ Mu. The procedure can be summarized as follows:

1. Calculate the external applied factored moment, Mu.

Mu = 1.2MD + 1.6ML

2. Calculate ϕMn for the basic singly reinforced section:

a. Check that $\rho_{min} \le \rho \le \rho_{max}$.

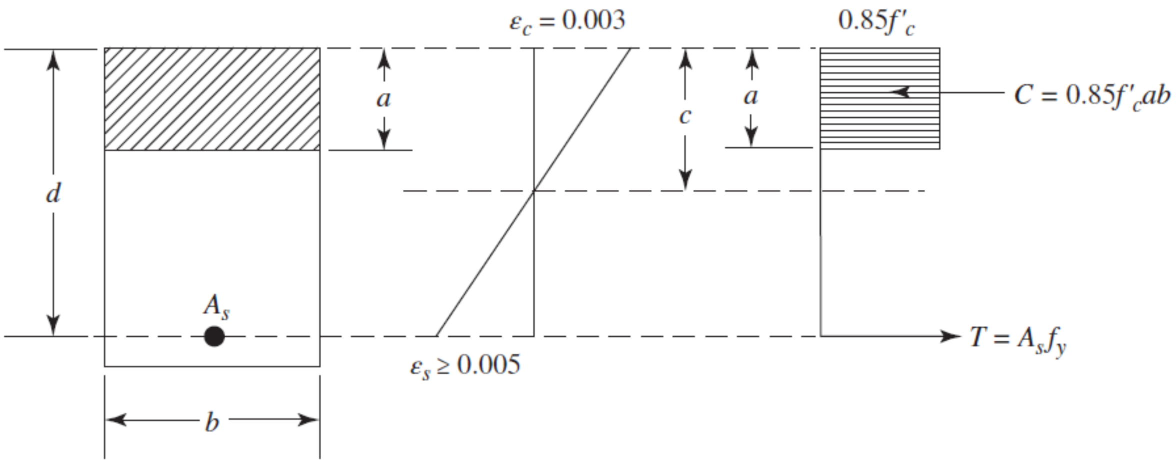


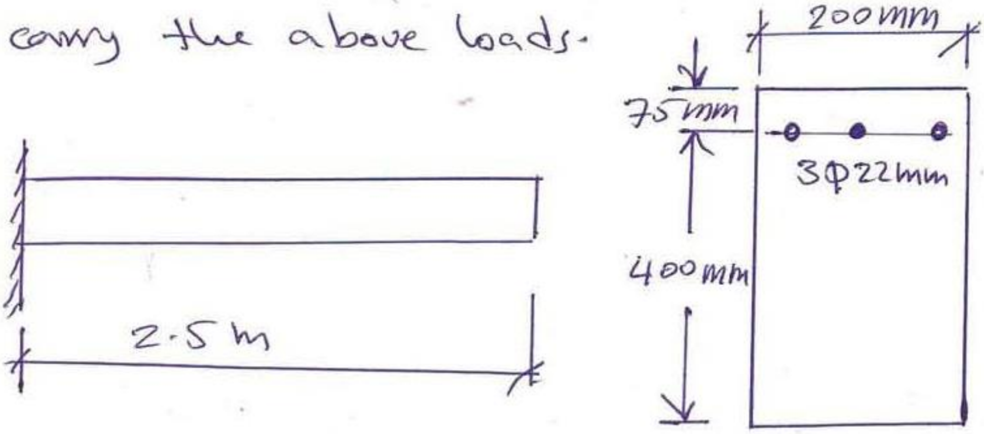
Figure 3.16 Tension-controlled rectangular section.

b. Calculate a = As fy/(0.85f_c'b) and check ε_t for ϕ.

c. Calculate ϕMn =ϕAs fy(d−a/2).

3. If ϕMn ≥Mu, then the section is adequate; Fig. 3.16 shows a typical tension-controlled section

Example (4) An 2.5 m span cantilever beam has a rectangular section and reinforcement as shown in fig below. The beam carries a dead load, including its own weight, of 22 kN/m and a live load of 13 kN/m, Using $f'_c = 28$ Mpa, and $f_y = 420$ Mpa, Check if the beam is safe to carry the above loads.



1- Calculate the external factored moment

$$W_u = 1.2D + 1.6L = 1.2(13) + 1.6(22) = 50.8 \text{ kN/m}$$

$$M_u = W_u \frac{\ell^2}{2} = 50.8 \frac{(2.5)^2}{2} = 158.75 \text{ kN.m}$$

2- Check ε_t $\quad A_{\phi 22mm} = 380 \text{ mm}^2$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3 * 380 * 420}{0.85 * 28 * 200} = 100.6 \text{ mm}$$

$$c = \frac{a}{0.85} = \frac{100.6}{0.85} = 118.35 \text{ mm}$$

$$d_t = d = 400 \text{ mm} \quad , \quad \frac{c}{d_t} = \frac{118.35}{400} = 0.296 < 0.375$$

اذن نحن في الشد مسيطر

ومن $\phi = 0.9$

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_c = \left(\frac{400 - 118.35}{118.35}\right) * 0.003 = 0.00714 > \varepsilon_t = 0.005$$

∴ احنا في الشد مسيطر $\phi = 0.9$

or Check $\rho = \frac{A_s}{bd} = \frac{3 * 380}{200 * 400} = 0.01425 < \rho_{max} = 0.01804$

لذا أيضاً الشد مسيطر ع المقطع $\quad \phi = 0.9$

3- Calculate ϕM_n

$$\phi M_n = \phi A_s f_y \left(d - a/2\right)$$

$$= 0.9 \times 3 \times 380 \times 420 \left(400 - \frac{100.6}{2}\right) = 150.69 \times 10^6 \ N.mm$$

$$\therefore \phi M_n = 150.69 \ kN.m$$

أسلوب آخر للحل

$$\rho = \frac{A_s}{bd} = \frac{3 \times 380}{200 \times 400} = 0.01425 < \rho_{max} = 0.01804$$

$$\therefore \phi = 0.9$$

$$m = \frac{f_y}{0.85 f'_c} = \frac{420}{0.85 \times 28} = 17.65$$

$$R = \rho f_y \left(1 - \frac{1}{2} \rho m\right)$$

$$= 0.01425 \times 420 \left(1 - \frac{1}{2} (0.01425)(17.65)\right)$$

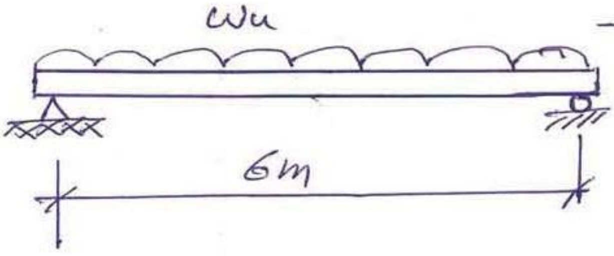
$$\therefore R = 5.232 \ N/mm^2$$

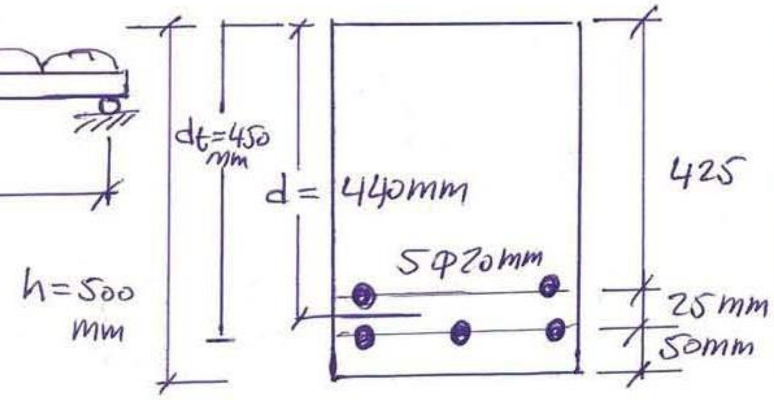
$$\phi M_n = \phi R b d^2$$

$$= 0.9 \times 5.23 \times 200 \times 400^2 = 150.69 \times 10^6 \ N.mm$$

$$\therefore \phi M_n = 150.69 \ kN.m$$

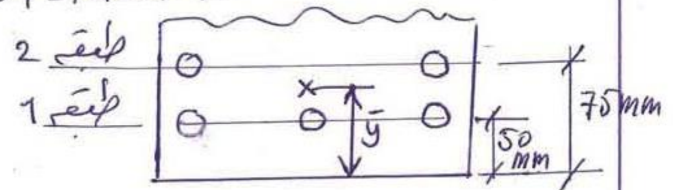
Example (5) A simply supported beam has a span of 6.0 m. If the cross section is shown below, $f'_c = 21$ MPa and $f_y = 420$ MPa; determine the allowable uniformly distributed service live load on the beam assuming the dead load is that due to the beam weight, Given $b = 300mm$, $h = 500mm$ and reinforced with $5\phi 20mm$ ($A_s = 5 \times 314 = 1570 \ mm^2$)





نحدد مركز ثقل فولاذ التسليح من الليف السفلي ، نأخذ العزوم حول الليف المحدود السفلي

$5\,A_{\Phi 20mm}\,\bar{y} = 3\,A_{\Phi 20mm} * 50 + 2\,A_{\Phi 20mm} * 75$



$$\therefore \bar{y} = \frac{3*50 + 2*75}{5}$$

$$\bar{y} = 60\ mm$$

$$d_t = h - 50 = 500 - 50 = 450\,mm$$

$$d = h - \bar{y} = 500 - 60 = 440\,mm$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+f_y}\right)\left(\frac{d_t}{d}\right)$$

$$m = \frac{f_y}{0.85 f'_c} = \frac{420}{0.85 * 21} = 23.53 \quad , \quad \beta_1 = 0.85 \quad f'_c \leq 28\,MPa$$

$$\rho_b = \frac{0.85}{23.53}\left[\frac{600}{600+420}\right]\left(\frac{450}{440}\right) = 0.02173$$

$$\rho_{max} = \left(\frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\right)\rho_b = \left(\frac{0.003+0.0021}{0.008}\right)\rho_b$$

$$\rho_{max} = 0.6375\,\rho_b = 0.6375 * 0.02173 = 0.01385$$

$$\rho = \frac{5 * 314}{300 * 440} = 0.01189 < \rho_{max}\ 0.01385$$

ok

اذن المقطع مسيطر عليه بالشد

$$\Phi = 0.9$$

وكذلك نتأكد من ρ_{min}

$$\rho_{min} = \frac{1.4}{f_y} \quad \text{for} \quad f_c' < 31.0 \text{ MPa}$$

$$\rho_{min} = \frac{1.4}{420} = 0.0033 < \rho = 0.01189 \quad \underline{ok}$$

$$\rho_{min} = 0.00333 < \rho = 0.01189 < \rho_{max} = 0.01385$$

اذن المقطع مطابق للمواصفة !

$$\phi M_n = \phi R b d^2$$

$$R = \rho f_y \left(1 - \frac{1}{2} \rho m\right)$$

$$= 0.01189 * 420 \left(1 - \frac{1}{2} (0.01189)(23.53)\right) = 4.295 \text{ MPa}$$

$$\phi M_n = 0.9 * 4.295 * 300 * 440^2 = 224.52 * 10^6 \text{ N.mm}$$

$$\therefore \phi M_n = \underline{224.52 \text{ kN.m}}$$

الوزن الذاتي للعتبة

$$w_D = 0.3 * 0.5 * 1 * 24 = 3.6 \text{ kN/m}$$

$$M_D = \frac{3.6 * 6^2}{8} = 16.2 \text{ kN.m}$$

$$M_u = \phi M_n = 224.52 \text{ kN.m}$$

$$M_u = 1.2 M_D + 1.6 M_L$$

$$224.52 = 1.2 * 16.2 + 1.6 M_L$$

$$\therefore M_l = 128.175 \text{ kN.m}$$

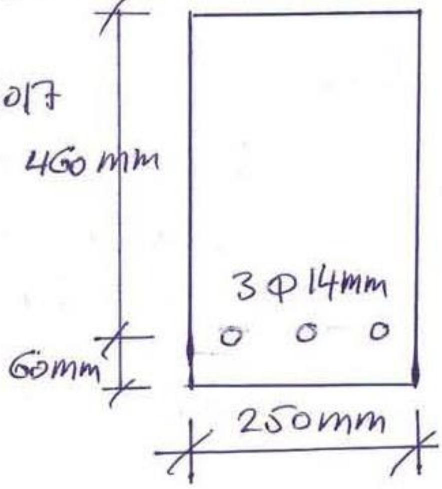
$$M_l = \frac{w_l \, l^2}{8}$$

$$128.175 = \frac{w_l * 36}{8} \Rightarrow w_l = 28.48 \text{ kN/m}$$

اذن اعلى اي الحمل التشغيلي او الحي للعتبة هو 28.48kN/m

Example (6). Check the design adequacy of the section shown below to resist a factored moment $M_u = 40.71$ kN.m, using $f'_c = 21$ Mpa and $f_y = 280$ Mpa.

Solution $A_s = 3\phi 14mm = 3 \times 154 mm = 462 mm^2$



$$\rho = \frac{A_s}{bd} = \frac{462}{250 \times 460} = 0.004017$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+f_y}\right)\left(\frac{d_t}{d}\right)$$

$d_t = d = 460$ mm

$$m = \frac{f_y}{0.85 f'_c} = \frac{280}{0.85 \times 21} = 15.69$$

$$\rho_b = \frac{0.85}{15.69}\left(\frac{600}{600+280}\right) = 0.03694$$

$$\rho_{max} = \left(\frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\right)\rho_b$$

$$= \left(\frac{0.003 + 280/200\,000}{0.003 + 0.005}\right)\rho_b = 0.55\,\rho_b$$

$$\therefore \rho_{max} = 0.55 \times 0.03694 = 0.020317$$

$$> \rho = 0.004017 \quad ok$$

check ρ_{min} required according to the ACI Code

$$\rho_{min} = \frac{1.4}{f_y} \qquad \text{for} \quad f'_c < 31.0 \text{ Mpa}$$

$$\therefore \rho_{min} = \frac{1.4}{280} = 0.005 > \rho = 0.004017$$

لذلك نستخدم

$\rho = \rho_{min} = 0.005$

$A_{s,min} = 0.005 \times 250 \times 460 = 575 mm^2$

لذلك نفضل استخدام $3\Phi 16\ mm$

$$A_{\Phi 16mm} = 200\ mm^2$$

$$\therefore A_s = 3A_{\Phi 16mm} = 3 * 200 = 600\ mm^2$$

يكفي المطلوب الآن نتأكد من المقطع المطلوب

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{600 \times 280}{0.85 \times 21 \times 250} = 37.65\ mm$$

$$\Phi M_n = \Phi A_s f_y (d - a/2) = 0.9 \times 600 \times 280 \left(460 - \frac{37.65}{2}\right)$$

$$\Phi M_n = 66.71 * 10^6\ N.mm$$

$$\Phi M_n = 66.71\ kN.m$$

$$A_s,\ \text{required for } 40.71\ kN.m = \frac{40.71}{66.71} * 600$$

المساحة المطلوبة لمقاومة العزم الخارجي $M_u = 40.71\ kN.m$

$$= 366.\ mm^2$$

حسب الكود الفقرة 9.6.1 ينص على أن أقل قيمة للتسليح أكبر بـ $\frac{1}{3}$ من المساحة المطلوبة.

$$\text{Minimum } A_s \text{ required} = 1.33 * 366 = 487\ mm^2$$

وهي تزيد عن $462\ mm^2$ الموضوعة أعلاه لذلك يجب أن نضع $3\Phi 16\ mm$ بدلاً من $3\Phi 14\ mm$

يمكن توضيح السابق كالتالي

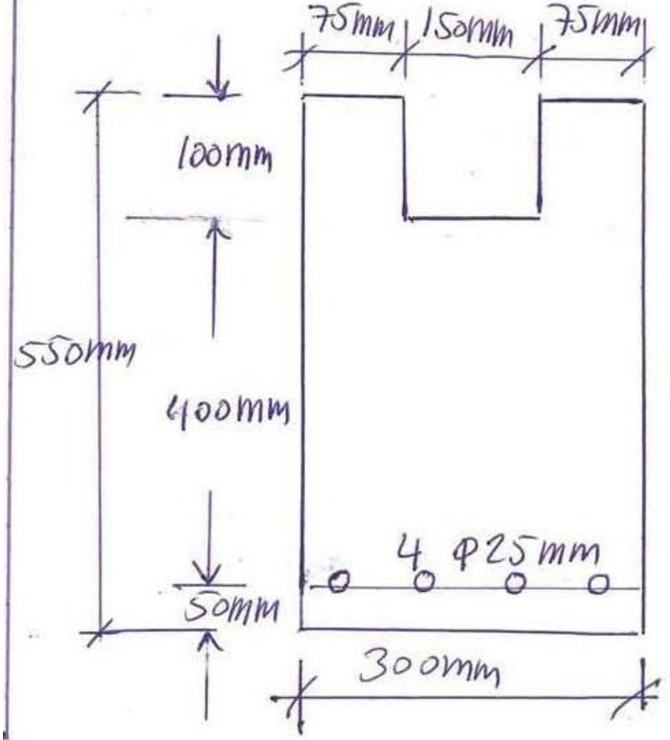
نستخرج قيمة ρ_{min}

$$\rho_{min} = \begin{cases} \frac{1.4}{f_y} & f'_c < 31\ MPa \\ \frac{0.25\sqrt{f'_c}}{\sqrt{f_y}} & f'_c \geq 31\ MPa \end{cases}$$

نستخرج قيمة ρ نتيجة تطبيق العزم الخارجي M_u هي القيمة التي يجب أن تكون $\rho_{min} < \rho_{used}$ المستخدمة.

اذا كانت $\rho < \rho_{min}$ فإن ρ_{used} المستخدمة الأصغر من ① 1.33ρ ② ρ_{min}

Example (7) Determine the design moment strength of section shown below. Given $f'_c = 28$ Mpa and $f_y = 420$ Mpa and check the specification of the section according to ACI Code.



$$A_{\phi 25mm} = \frac{\pi}{4}(25)^2 = 490 \text{ mm}^2$$

$$A_s = 3 * 490 = 1470 \text{ mm}^2$$

$$\rho = \frac{A_s}{A_{ef}, \text{المساحة الفعالة}}$$

$$A_{ef} = bd - 150 * 100$$
$$= 300 \times 550 - 150 \times 100$$
$$= 150\,000 \text{ mm}^2$$

$$\therefore \rho = \frac{1470}{150\,000} = 0.00980$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + f_y}\right)\frac{d_t}{d}$$

$d_t = d = 550$mm , $\beta_1 = 0.85$ for $f'_c \leq 28$ Mpa

$$m = \frac{f_y}{0.85 f'_c} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right) = 0.0283$$

$$\rho_{max} = \left(\frac{0.003 + f_y/E_s}{0.003 + \varepsilon_t}\right) * \rho_b = \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b$$

$$\therefore \rho_{max} = 0.6375\, \rho_b = 0.6375 \times 0.0283 = 0.01804125$$

$$\rho_{min} = \frac{1.4}{f_y} \quad \text{for} \quad f'_c < 31.0 \text{ Mpa}$$

$$\rho_{min} = \frac{1.4}{420} = 0.00333$$

$P_{min} = 0.00333 < \therefore \rho = 0.0098 < P_{max} = 0.0184$

اذن المقطع مطابق للمواصفات ويقع ضمن منطقة الشد المسيطر

$\phi = 0.9$

الآن نحسب تحمل أو مقاومة المقطع

نفرض ان عمق مستطيل الإجهاد $a > 100mm$

اذن المساحة المضغوطة من مقطع الخرسانة $A_c =$

$= ab - 100 * 150$

$C = T$

$0.85 f_c' A_c = A_s f_y$

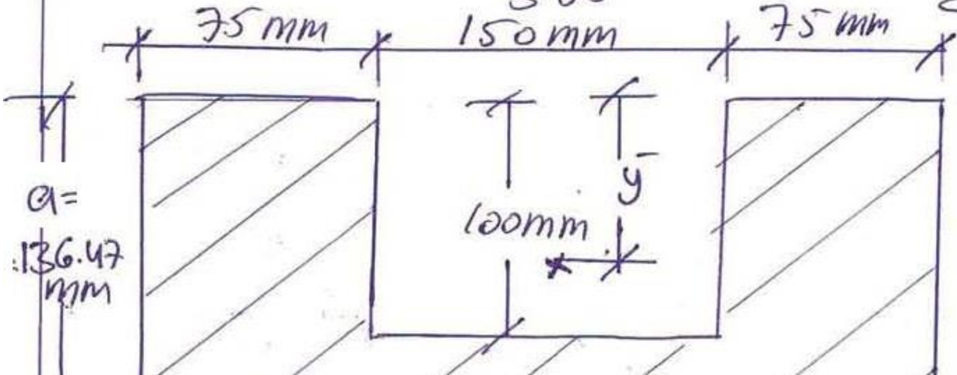
$$\therefore A_c = \frac{A_s f_y}{0.85 f_c'} = \frac{1470 * 420}{0.85 * 28} = 25941 \, mm^2$$

$A_c = ab - 15000 = a * 300 - 15000$

$$\therefore a = \frac{A_c + 15000}{300} = 136.47 \, mm > 100 \, mm$$

اذن الفرضية صحيحة

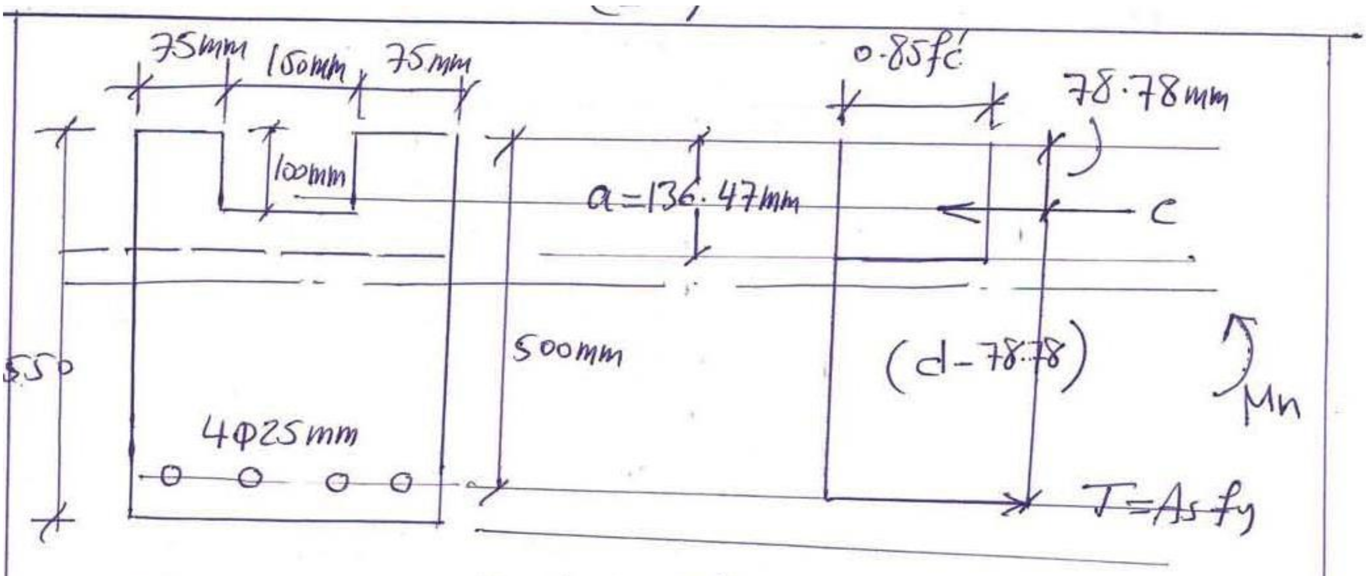
نجد مركز ثقل منطقة الضغط المؤشرة $\bar{y}$ عن الليف العلوي المضغوط



$$\bar{y} = \frac{300 * 136.47 * \frac{136.47}{2} - 150 * 100 * \frac{100}{2}}{300 * 136.47 - 150 * 100} = 78.78 mm$$

اذن ذراع العزم بين قوة الشد T و الضغط C هو

$d - \bar{y} = 550 - 78.78 = 471.22$



$$\therefore \Phi M_n = \Phi A_s f_y (d - \bar{y})$$

$$= 0.9 * 1470 * 420 (500 - 78.78) = 234.06 * 10^6 \text{ N.mm}$$

$$\Phi M_n = \underline{234.06 \text{ kN.m}}$$

BUNDLED BARS

When the design of a section requires the use of a large amount of steel, for example, when ρ_{max} is used, it may be difficult to fit all bars within the cross section. The ACI Code, Section 25.6.1.1, allows the use of parallel bars placed in a bundled form of two, three, or four bars, as shown in Fig. 3.20. Up to four bars (no. 11 or smaller) can be bundled when they are enclosed by stirrups. The same bundled bars can be used in columns, provided that they are enclosed by ties. All bundled bars may be treated as a single bar for checking the spacing and concrete cover requirements. The single bar diameter shall be derived from the equivalent total area of the bundled bars.

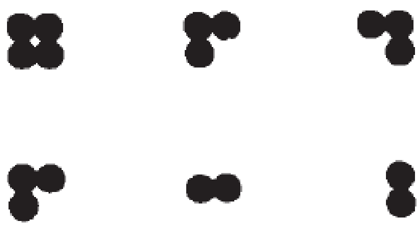


Figure 3.20 Bundled bar arrangement.

Summary: Singly Reinforced Rectangular Section

The procedure for determining the design moment of a singly reinforced rectangular section according to the ACI Code limitations can be summarized as follows:

1. Calculate the steel ratio in the section, ρ=As/bd.

2. Calculate the balanced and maximum steel ratios, for tension-controlled section. Also, calculate ρ_{min} = 1.4/fy when f_c' < 31 MPa and ρ_{min} = $0.25\sqrt{f_c'}$ ⁄ fy when f_c' ≥ 31 MPa.

3. If $\rho_{min} \leq \rho \leq \rho_{max}$, then the section meets the ACI Code limitations for tension-controlled section. If $\rho \leq \rho_{min}$, the section is not acceptable (unless a steel ratio $\rho \geq \rho_{min}$ is used). If $\rho \leq \rho_{max}$, ϕ = 0.9 ($\varepsilon_s \leq 0.005$); otherwise calculate ϕ.

4. Calculate a = As fy ⁄ 0.85f_c'b, c, εt, and ϕ.

5. Calculate ϕ Mn =ϕ As fy (d−a/2).

RECTANGULAR SECTIONS WITH COMPRESSION REINFORCEMENT

In concrete sections proportioned to resist the bending moments resulting from external loading on a structural member, the internal moment is equal to or greater than the external moment, but a concrete section of a given width and effective depth has a minimum capacity when ρmax is used. If the external factored moment is greater than the design moment strength, more compressive and tensile reinforcement must be added.

Compression reinforcement is used when a section is limited to specific dimensions due to architectural reasons, such as a need for limited headroom in multistory buildings. Another advantage of compression reinforcement is that long-time deflection is reduced. A third use of bars in the compression zone is to hold stirrups, which are used to resist shear forces.

Two cases of doubly reinforced concrete sections will be considered, depending on whether compression steel yields or does not yield.

When Compression Steel Yields

Internal moment can be divided into two moments, as shown in Fig. 3.23. Let M_{u1} be the moment produced by the concrete compressive force and an equivalent tension force in steel, A_{s1}, acting as a basic section. Then M_{u2} is the additional moment produced by the compressive force in compression steel A_s' and the tension force in the additional tensile steel, A_{s2}, acting as a steel section.

The moment M_{u1} is that of a singly reinforced concrete basic section,