5 Macroscopic Behavior of Ductile Metals:

5.1 Load versus Elongation:

The information obtained from a tensile test is the load or force required to cause a certain deformation or extension. Regardless of the specimen shape used, test measurements are invitably made on that section of the specimen throughout which the stress is assumed to be in a state of uniaxial tension. The above comments can be summarized by a general and qualitive plot of load-elongation behavior as shown below,

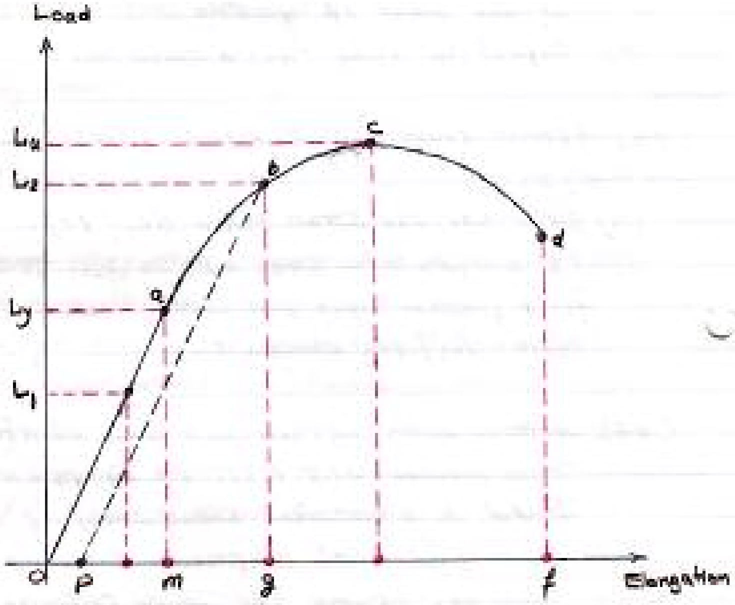


Fig.(1) Load-extension plot of a tensile test.

Many important observations result from this plot, but first we note that the elastic behavior, from 0 to a is highly exaggerated to assist in the discussion that follows. Actual experiments with real metals would show that the elastic elongation, om, is so

small in comparison with the plastic elongation, mf, that the line oa would be nearly vertical if all data were plotted to a common scale. With this in mind, we observe the following:

1. The portion of the curve shown as oa describes elastic behavior, elongation being directly proportional to the load for any point on that line; the maximum elastic elongation connected with this region is om.

2. The yield load L_y, which for our purpose is defined by point a, indicates the beginning of plastic deformation.

3. Continual increase of load above L_y follows the path abc at which point a maximum load L_u is reached.

4. When point c is reached, a condition of instability occurs and, at some cross section of the test region, a localized constriction begins. This phenomenon is known as necking and the area reduction in this region is no longer offset by the added strengthening effect of plastic deformation.

5. Once a neck has formed, practically all further plastic deformation from c to fracture at d occurs in the localized region of the neck.

6. The fracture area, A_f, is nominally taken to be the minimum area in the neck after the specimen has physically separated into two pieces.

7. If the specimen were originally loaded from o to b, and the load L_2 then fully removed, the unloading behavior could be adequately described by the line bp which is parallel to oa. Under reloading a linear relationship between load and elongation would then prevail up to point b; that is, the elastic region

would be extended as compared with oa and, until the load L_2 was exceeded, additional plastic flow would not take place. Thus, the behaviour of this specimen, which now contains some prior plastic deformation, would in effect follow the path pbcd.

5.2 Engineering or Nominal Stress and Strain:

The limitation of a load-elongation plot is that no account is taken of the physical size of the test specimen. To avoid this shortcoming, it is sensible to convert the load-elongation data into some form of stress-strain data; most often the form of engineering stress and strain is used where the following definitions prevail,

$$\text{Engineering Stress, } \sigma = \frac{L}{A_o} \qquad (5.1)$$

$$\text{Engineering Strain, } \epsilon = \frac{l - l_o}{l_o} \qquad (5.2)$$

Since the load-elongation data in Fig.(1) are merely divided by the appropriate constants A_o and l_o to produce nominal stress-strain data. Thus, a nominal stress-strain plot would be identical in form to that shown in Fig.(1). Throughout the elastic region oa, the linearity maintained between L and Δl would be duplicated in terms of stress and strain to produce

$$\sigma = E\epsilon \qquad (5.3)$$

Where E is the modulus of elasticity.

The stress at point a, which has been assumed to indicate the onset of plastic deformation, is called the yield strength and is that level of stress at which plastic flow or yielding

commences. It is defined as,

$$\sigma_y = Y = \frac{L_y}{A_o} \qquad (5.4)$$

The tensile stress level associated with the maximum load is reffered to variously as tensile strength, ultimate tensile strength, and ultimate strength. Tensile Strength will be used and is defined as,

$$\sigma_u = \frac{L_u}{A_o} \qquad (5.5)$$

Tensile strength signifies the maximum static load a particular section can support without fracturing. It is also sometimes used as a quality check.

Defining strain by eq$\underline{n}$ (5.2), the maximum elastic strain associated with oa in Fig.(1) is the strain at yielding,

$$\epsilon_y = \frac{l_y - l_o}{l_o} \qquad (5.6)$$

Consider now the strain induced by the application of load, L_2. According to the symbols used in Fig.(1),

$$\epsilon_2 = \frac{l_2 - l_o}{l_o} \qquad (5.7)$$

Which relates to the elongation og. This includes not only the strain caused by the plastic deformation from a to b but the total elastic deformation from o to b. It is the total strain induced due to load L_2.

The maximum uniform strain associated with the onset of necking is,

$$\epsilon_u = \frac{l_u - l_o}{l_o} \qquad (5.8)$$

6.3 Indications of Ductility :

The extent to which a metal can be deformed plastically is referred to as the ductility of the metal. In an attempt to place a quantitative measure on this property, two parameters are most often employed; both are based upon measurments obtained after the test specimen has fractured.

Percent elongation is nothing more than the nominal strain at fracture multiplied by 100, or.

$$\text{Percent elongation} = \left(\frac{L_f - L_o}{L_o}\right) * 100 \qquad (6.9)$$

Percent reduction of area or RA is defined by,

$$RA = \left(\frac{A_o - A_f}{A_o}\right) * 100 \qquad (6.10)$$

In comparing the ductility of two metals, the one having a larger RA will usually posses a larger percent elongation, but there are exceptions. Although there is far more information in the literature giving values of percent elongation RA is often more useful. Naturally, if necking were foreign, and fracture occured at the maximum load, these two quantities would be directly related since the deformation would be uniform for the entire test.

Example (6.1) :

A ductile metal undergoes tensile deformation until it fractures when the maximum load is reached. At that time the gage length was 60 mm while the area was 83.33 mm². Prior to loading, the gage length was 50 mm whereas the area was 100 mm². Determine the true strain at fracture using both length and area changes and find the percent elongation and reduction of area at fracture.

Solution:

$$e_f = \ln\left(\frac{l_f}{l_0}\right) = \ln\left(\frac{60}{50}\right) = 0.182$$

$$e_f = \ln\left(\frac{A_0}{A_f}\right) = \ln\left(\frac{100}{83.33}\right) = 0.182$$

They are the same because the deformation was uniform up to the point of fracture, that is, necking did not occur since fracture occured at the maximum load.

$$\text{Percent elongation} = \frac{60-50}{50} * 100 = 20\%$$

$$\text{Percent reduction of area} = \frac{100-83.33}{100} * 100 = 16.67\%$$

Assuming constancy of volume and uniform deformation,

$$V_0 = A_0 l_0 = A_f l_f = (0.833) A_0 (1.2) l_0 = A_0 l_0$$

Example (15.2):

A tensile specimen of annealed 1020 steel is pulled to fracture. The initial gage section dimensions are $l_0 = 2$ in. and $d_0 = 0.505$ in. After reaching the maximum load, a neck forms and continues to get smaller until fracture occurs. The final gage length is 2.74 in. While the smallest diameter in the neck is 0.332 in. Find the true strain at fracture using both length and area changes, and determine the percent elongation and reduction of area at fracture.

Solution:

$$e_f = \ln\left(\frac{2.74}{2}\right) = 0.315$$

$$e_f = 2\ln\left(\frac{0.505}{0.332}\right) = 0.838 = \ln\left(\frac{A_0}{A_f}\right) = \ln\left(\frac{0.2}{0.087}\right)$$

In this case, the fracture strain cannot be determined using length values since the strain is very nonuniform due to necking, thus, the 0.315 value is meaningless. The value based upon the starting area and minimum area in the neck is at least representative of the maximum strain experienced by the specimen.

$$\text{Percent elongation} = \frac{2.74-2}{2} \times 100 = 37\%$$

$$\text{Percent reduction in area} = \frac{0.2-0.087}{0.2} \times 100 = 56.5\%$$

Since nonuniform deformation occurred, the above values cannot be used to show constancy of volume as in example (5.1).

5.4 Percent Cold-work:

A common practice used to express the extent of induced plastic deformation is to introduce the term percent cold-work. This is defined for a particular reduction of area as follows,

$$\text{Reduction of area} = r = \frac{A_0 - A}{A_0} \qquad (5.11)$$

The percent cold-work is simply r multiplied by 100. In tension, the maximum value that r can attain is governed by Eqn (5.10). The maximum uniform cold-work that can be imparted by uniaxial tensile deformation is determined from Eqn (5.11) where the area at ultimate load, A_u, is used as A.

5.5 True Stress and True Strain:

For plasticity work, tensile data are most useful if the instantaneous stress and strain are associated with the current load level and total deformation that has occured. It is more useful to define a true or natural stress and its strain counterpart.
True or logarithmic strain has been discussed befor, where it was defined as:

$$de = \frac{dl}{l}$$

or after integration,

$$e = \ln\left(\frac{l}{l_o}\right) \qquad (5.12)$$

Because of volume constancy, equivalent expressions for true strain are,

$$e = \ln\left(\frac{l}{l_o}\right) = \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{1}{1-r}\right) = 2\ln\left(\frac{D_o}{D}\right) \qquad (5.13)$$

The True Stress is defined as,

$$S = \frac{L}{A} = \frac{\text{instantaneous load}}{\text{instantaneous area}} \qquad (5.14)$$

Before proceding to use these new definitions, we illustrate the additive property of true strains, which were mentioned before.
Using Equ (5.2) and assuming tensile loading, suppose $l_o = 2$ in; it is then increased in sequence by $\Delta l_1 = 0.2$, $\Delta l_2 = 0.2$, $\Delta l_3 = 0.2$, therefore,

$$\epsilon_1 = \frac{2.2 - 2}{2} = \frac{0.2}{2} = 0.1$$

$$\epsilon_2 = \frac{2.4-2.2}{2.2} = \frac{0.2}{2.2} = 0.091$$

$$\epsilon_3 = \frac{2.6-2.4}{2.4} = \frac{0.2}{2.4} = 0.083$$

$$\epsilon_t = \epsilon_1 + \epsilon_2 + \epsilon_3 = 0.274$$

If the full deformation is done in one step,

$$\epsilon_t = \frac{2.6-2}{2} = 0.300 \neq 0.274$$

Now using Eqn (5-13) and the same sequence,

$$e_1 = \ln\left(\frac{2.2}{2}\right) = \ln(1.1) = 0.095$$

$$e_2 = \ln\left(\frac{2.4}{2.2}\right) = \ln(1.09) = 0.086$$

$$e_3 = \ln\left(\frac{2.6}{2.4}\right) = \ln(1.083) = 0.080$$

$$e_t = e_1 + e_2 + e_3 = 0.261$$

If the deformation is computed in a single step,

$$e_t = \ln(2.6/2) = \ln(1.3) = 0.261$$

Which is the same as the sum of the individual values just computed. It must be cautioned here, that if changes in loading direction are severe, or plastic strain reversals are encountered, the above procedure may not accurately describe the actual true strain induced.

5.6 Work Hardening by Uniaxial Tension:

The engineering stress-strain curve and the corresponding true stress-strain curve for a general set of tensile test data are plotted in the following figure,

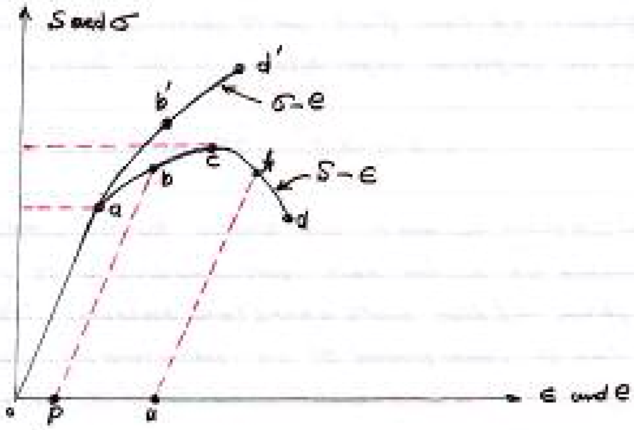


Fig. (2) Comparison of engineering (nominal) and true (logarithmic) stress-strain curves.

Although S and σ can be plotted to the same scale, the strain scale must be handled properly since Equ (5.2) and (5.12) can be used to show that,

$$e = \ln(1 + \epsilon) \qquad (5.15)$$

As the specimen loaded from o to a, the decrease in area is wholly negligible so that $A_o \simeq A_a$. Thus the yield strength would be defined by,

$$Y = \frac{L_y}{A_a} \simeq \frac{L_y}{A_o} \quad \text{or} \quad Y \simeq S_y = \sigma_y \qquad (5.16)$$

The yield stress is for all practical purposes a true stress as defined by equ (5.14) and there is no need for differentiating

between the nominal stress at yield and the true stress at yield. They are equivalent and will be referred to only as the yield strength.

With many ductile metals that contain no initial work hardening prior to plastic deformation, the fully plastic portion of the S-e curve (from a to d') can be represented by an empirical expression of the form,

$$S = Ke^n \quad \text{or} \quad \bar{S} = K\bar{e}^n \qquad (5.17)$$

Assuming K and n are known for this metal, the calculated value of S for this corresponding e is the new yield stress of the work-hardened material. Thus eqn (5.17) can be interpreted as an indication of yield stress as a function of cold-work; it could be expressed as,

$$\bar{S} = K\bar{e}^n = Y \qquad (5.18)$$

This provides a method for determining the quantitative effect of cold-working on the resulting yield stress. The new tensile strength can be found, then, using eqn (5.5), where

$$\sigma_u = \frac{L_c}{A_p} \qquad (5.19)$$

since it will reach the maximum load defined by point c and has an original area, after the prior cold-work, associated with point p. The tensile strength of this metal, when tested from a starting condition containing no prior cold-work would be,

$$\sigma_u = \frac{L_c}{A_o} \qquad (5.20)$$

where the metal has been subjected to plastic deformation

befor being tested in tension, the tensile strength of this cold-worked metal will be designated as σ_W. Using these symbols and combining Equs (5.19) and (5.20) gives,

$$\sigma_W = \frac{\sigma_u A_o}{A_p} = \frac{\sigma_u}{1-r} \qquad (5.21)$$

The magnitude of σ_u is the value usually found in handbooks, and Equ (5.21) permits predictions of the tensile strength of a cold-worked metal if the degree of cold-worked is known.

5.7 Determination of the work-Hardening Equation:

Since no simple equation describes the stress-strain behavior of real metals from the onset of loading to fracture, objections are sometimes raised to the use of equ (5.18) for describing strain-hardening behavior. Since by definition,

$$\bar{S} = \frac{L}{A} = K\bar{\epsilon}^n \quad \text{and} \quad d\bar{\epsilon} = \frac{dl}{l} = -\frac{dA}{A} \qquad (5.22)$$

at the instability point (maximum load) where $dL = 0$, it can be shown that $\bar{\epsilon}_u = n$. Although this is mathematically correct, it is very difficult to measure the true strain at the exact maximum load because of the machine sensitivity that would be required. The true stress at ultimate load can be expressed as,

$$\bar{S}_u = K\bar{\epsilon}_u^n = Kn^n \quad \text{since} \quad \bar{\epsilon}_u = n \qquad (5.23)$$

Using equs (5.5), (5.14) and (5.23),

$$L_u = \sigma_u A_o = \bar{S}_u A_u = (Kn^n) A_u \qquad (5.24)$$

There fore,

$$\sigma_u = (K n^n)\,\frac{A_u}{A_o} \qquad (5.25)$$

From equ (5.13),

$$\frac{A_u}{A_o} = e^{-e_u} \qquad (5.26)$$

So,

$$\sigma_u = K\left(\frac{n}{e}\right)^n \quad \text{since } n = e_u \qquad (5.27)$$

Where e is the base of natural logarithms in Equ (5.26) and equ (5.27).

Now once a straight line is fitted to test points, thereby defining K and n, σ_u can be calculated using Equ (5.27).

Example (5.3):

The work-hardening behavior of an initially annealed brass follows a power law expression of the form,

$$\bar{S} = 105{,}000\,\bar{e}^{0.5}$$

If a piece of this metal were subjected to 30% cold-work in a uniform manner, what would be the expected yield strength and tensile strength of the plastically strained piece?

Solution:

From equ (5.13), the induced strain can be found since $r = 0.3$,

$$\bar{e} = \ln\left(\frac{1}{1-0.3}\right) = 0.357$$

The expected yield strength is determined from equ (5.18) as,

$$Y = \bar{\sigma} = K\bar{\epsilon}^n = 105{,}000(0.357)^{0.5}$$
$$= 45{,}030 \text{ psi}$$

For this situation, the tensile strength of the cold-worked piece is found from eqn (5.21) to be,

$$\sigma_w = \frac{45{,}030}{1-0.3} = 64{,}330 \text{ psi}$$

Example (5.4):

If the material specified in example (5.3) had been uniformly cold-worked to 60% reduction in area, find the expected yield and tensile strengths of the strained piece.

Solution

$$\bar{\epsilon} = \ln(1/(1-0.6)) = 0.916$$

$$Y = 105{,}000(0.916)^{0.5} = 100{,}500 \text{ psi}$$

Since the induced strain, 0.916, is greater than the strain-hardening exponent 0.5, from the following equation,

$$\boxed{\sigma_w \cong Y = K\bar{\epsilon}^n \qquad \text{where} \quad \bar{\epsilon} \geqq n} \qquad (5.28)$$

the tensile strength of the strained piece would approximate the calculated value of Y. i.e.,

$$\sigma_w = Y = 100{,}500 \text{ psi}$$

5.8 Ductility of Prior Worked Metals :

Suppose a tensile test is performed with a metal that has not been cold worked prior to this test, and an RA of 60% results. Since this is based upon the smallest section in the neck of the specimen after fracture, the maximum strain experienced by this metal during this test is ,

$$\bar{e}_f = \ln\left(\frac{1}{1-0.6}\right) = 0.916 \quad \left(r_{max} = \frac{RA}{100} \right) \qquad (5.29)$$

A second specimen, identical in all aspects to the first one, might be cold-worked in tension, say to 10% reduction in area (i.e. $r = 0.1$). Using the concept of true strain, the answer becomes more apparent; with equ(5.29), the maximum strain this metal would experience in tension is 0.916. Due to the 10% cold-work initially imparted, the metal has experienced an initial strain of ,

$$e = \ln\left(\frac{1}{1-0.1}\right) = 0.105$$

and since true strains are additive, an additional strain of 0.916 − 0.105 or 0.811 should cause fracture. Thus, the RA value for this prior worked specimen is computed from,

$$e = \ln\left(\frac{1}{1-r}\right) \quad \text{or} \quad \frac{1}{1-r} = \exp(e)$$

The additional strain to cause fracture is 0.811 so that,

$$\exp(0.811) = \frac{1}{1-r} \quad \text{and} \quad r = 0.555$$

or,

$$RA = 55.5\ \%$$

Simple laboratory experiments will verify this concept.

Example (5.5);

Consider a ductile metal containing no prior effects of work hardening. Using a particular mode of deformation the piece fractures after being cold-worked 70%. If another piece of the same annealed metal were first cold-worked 40% then machined into a tensile specimen, what reduction of area at fracture would be expected when the tensile test was completed?

Solution;

At first glance it might seem that the added could-worked to cause tensile fracture would be,

$$(70-40) = 30\% \quad or \quad r = 0.3$$

That this is not correct can be best shown by working with true strains, assuming that homogeneous deformation occurs regardless of the method of loading.
Using the maximum cold-work as 70%, the maximum strain at fracture is,

$$e_f = \ln\left(\frac{1}{1-r}\right) = \ln\left(\frac{1}{1-0.7}\right) = 1.20$$

The strain induced initially in the second piece is,

$$e_i = \ln\left(\frac{1}{1-0.4}\right) = 0.51$$

Thus the addational strain that would be expected at fracture is,

$$e = e_f - e_i = 1.20 - 0.51 = 0.69$$

To induce the strain of 0.69 requires a reduction of area of,

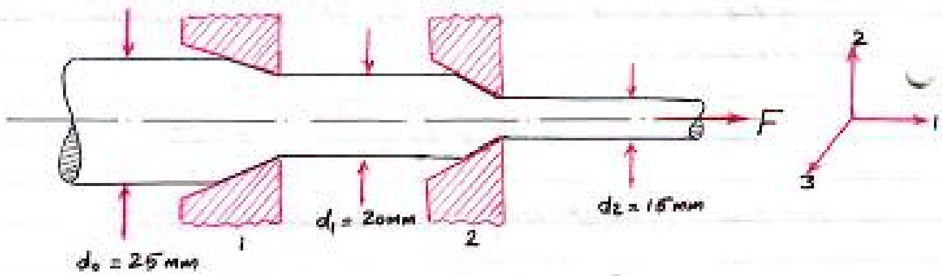
$$0.69 = \ln\left(\frac{1}{1-r}\right) \quad \text{or} \quad e^{0.69} = \frac{1}{1-r}$$

$$\therefore \quad r = \frac{0.994}{1.994} = 0.498$$

Which does not agree with the value proposed earlier.

Example (15-6):

A round bar of an annealed metal, whose strain-hardening behavior is expressed by $\bar{S} = 200{,}000\,\bar{\epsilon}^{0.5}$, is originally 25 mm in diameter. By a series of two consecutive reductions with cylindrical dies, the initial diameter is reduced to 20 and 15 mm respectively as indicated in the sketch. Determine the plastic work per unit volume for each reduction and relate these findings to a graphical plot of the true stress-true strain curve.



Solution:

The plastic work per unit volume is equated to the area under the true stress-strain curve.

$$dW_{ep} = \bar{\sigma}\, d\bar{\epsilon} = \bar{S}\, d\bar{\epsilon}$$

Note that the total W_v for this entire process is,

$$W_{v_1} + W_{v_2} = 39,714 + 97,842 = 137,556 \text{ in-lb/in}^3.$$

Which would also be found if the overall reduction had been completed in one step,

$$\bar{\epsilon} = 2 \ln\left(\frac{25}{15}\right) = 1.021 \quad \text{and,}$$

$$\int_0^{1.021} \bar{\sigma}\, d\bar{\epsilon} = 137,556 \text{ in-lb/in}^3.$$

Problems:

(5.1) The strain-hardening behavior of a metal is given by $\bar{\sigma} = 100,000\,\bar{\epsilon}^{0.2}$.

(a) If a piece of initially annealed material were uniformly cold-worked 10%, what would be the yield strength and tensile strength of this material.

(b) Repeat (a) if the material had been cold-worked 40% instead of 10%.

(5.2) An annealed tensile specimen of alpha brass, with $A_0 = 0.2 \text{ in}^2$ carries a maximum load of 12,000 lb and the initial area has been reduced 40% at that instant. If a second annealed specimen of identical starting dimensions were pulled untill the induced true strain was half the magnitude of the strain hardening exponent, what load in pounds would be required to reach this condition?

(5.3) A material deforms according to $\bar{\sigma} = 150,000\,\bar{\epsilon}^{0.3}$.

(a) Calculate the force necessary to extrude the solid from a 6 in diameter bar to a 1 in diameter bar.

(b) Calculate the force necessary to draw the solid from a 6 in to a 1 in diameter (drawing is pulling).